

Muon $g - 2$ /EDM experiment at J-PARC

日本 μ 子反常磁矩 ($g - 2$) 与电偶极矩测量实验

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- **Introduction**

- Muon anomalous magnetic moment
- Recent result from Fermilab

- **Muon $g - 2$ /EDM experiment at J-PARC**

- Overview
- R&D highlights
- Roadmap

- **Summary**

Muon anomalous magnetic moment

- The Hamiltonian for the spin 1/2 particle (charge e and mass m) in the external electromagnetic field is

$$H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

Magnetic dipole moment

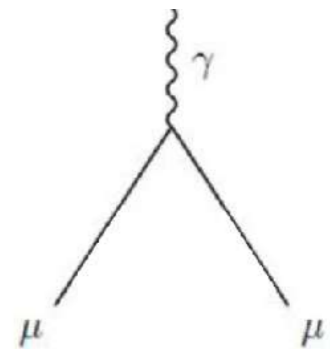
$$\vec{\mu} = g \frac{e\hbar}{2m} \vec{S}$$

Electric dipole moment

$$\vec{d} = \eta \frac{e\hbar}{2m} \vec{S}$$

- $\vec{\mu}$ is proportional to gyromagnetic ratio (g), which is predicted to be $g = 2$ by Dirac equation.
- But quantum fluctuations gives **the anomaly of muon** a_μ :

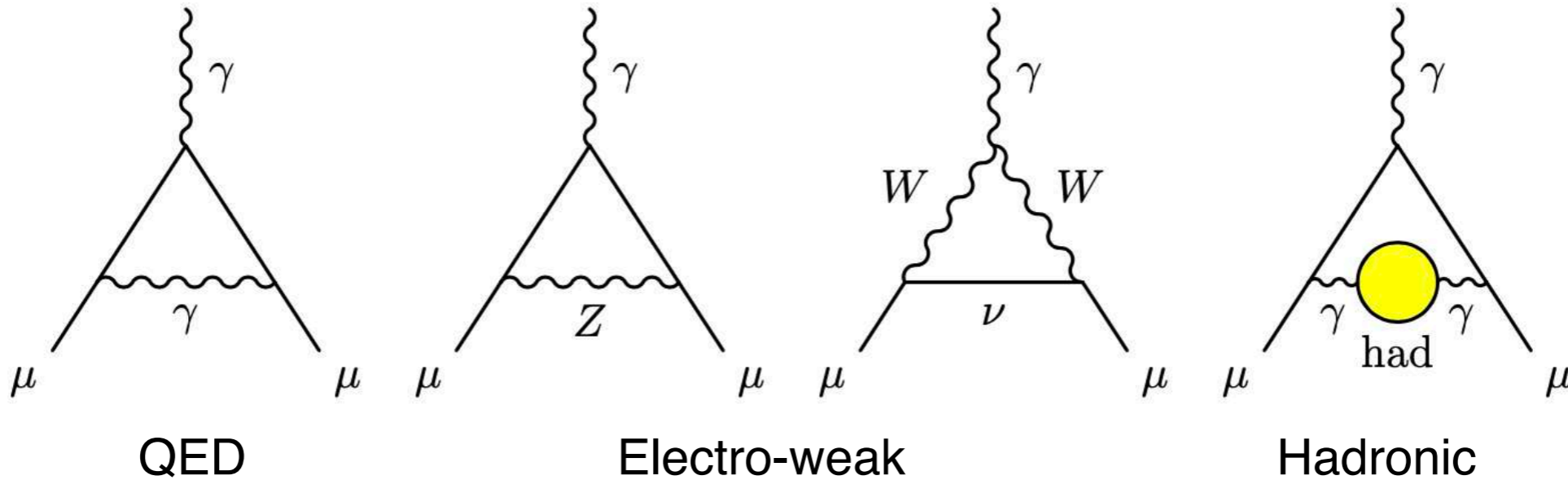
$$a_\mu \equiv \frac{g_\mu - 2}{2}$$



Muon anomaly

- In the standard model, muon anomaly a_μ is calculated from each contributions:

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}}$$



First-order QED correction:

$$a_\mu^{(2)} = \frac{\alpha}{2\pi} = 0.001\,161\,40\dots$$

[J. Schwinger, *Phys Rev* 73 (1948) 416]

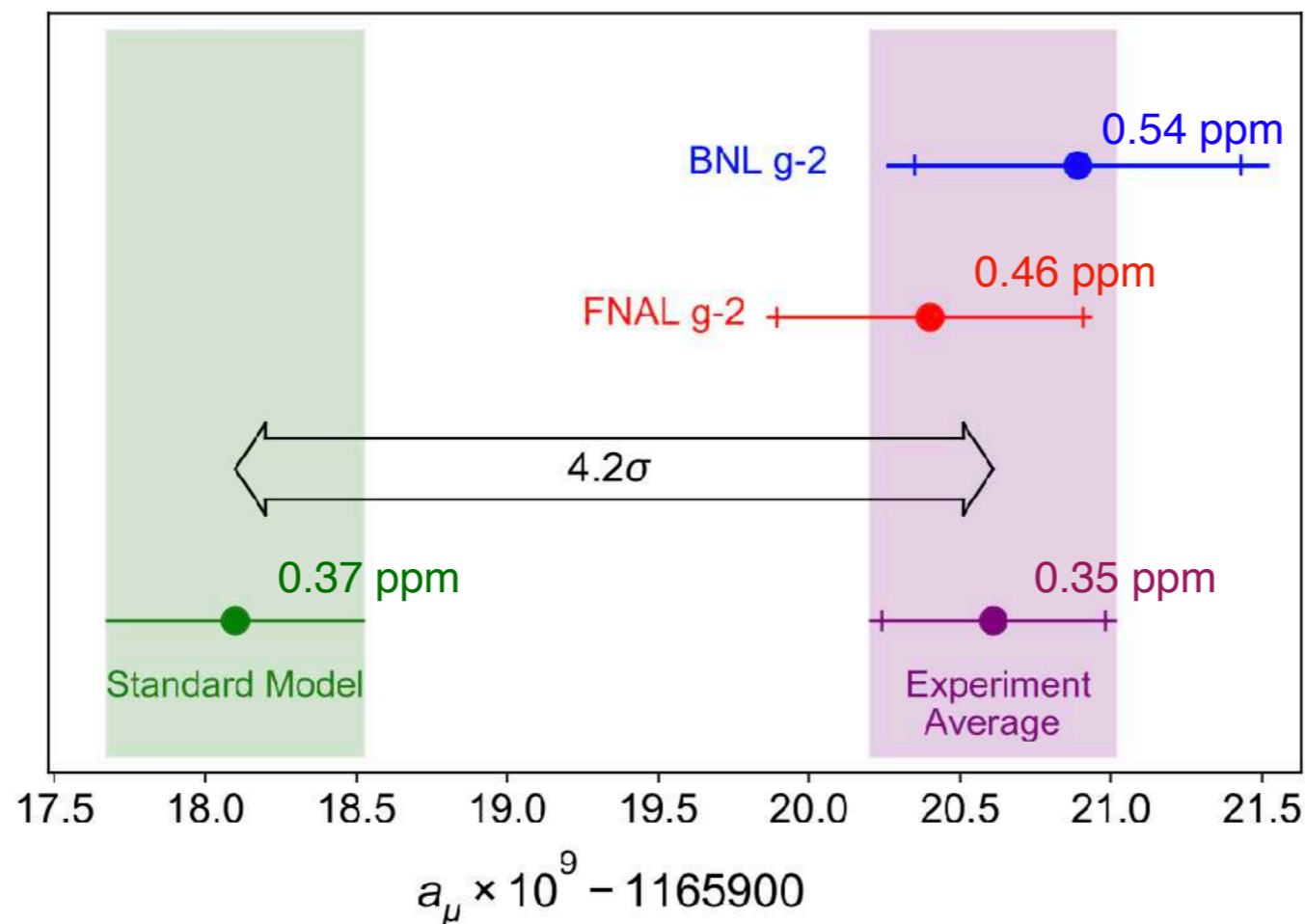


Muon anomaly

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$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}}$$

- Recent result from Fermilab shows 4.2σ tension between a_μ^{EXP} and a_μ^{SM}



$a_\mu(\text{BNL})$	$116592092(63) \times 10^{-11}$	0.54 ppm
$a_\mu(\text{FNAL})$	$116592040(54) \times 10^{-11}$	0.46 ppm
$a_\mu(\text{EXP})$	$116592061(41) \times 10^{-11}$	0.35 ppm
$a_\mu(\text{SM})$	$116591810(43) \times 10^{-11}$	0.37 ppm

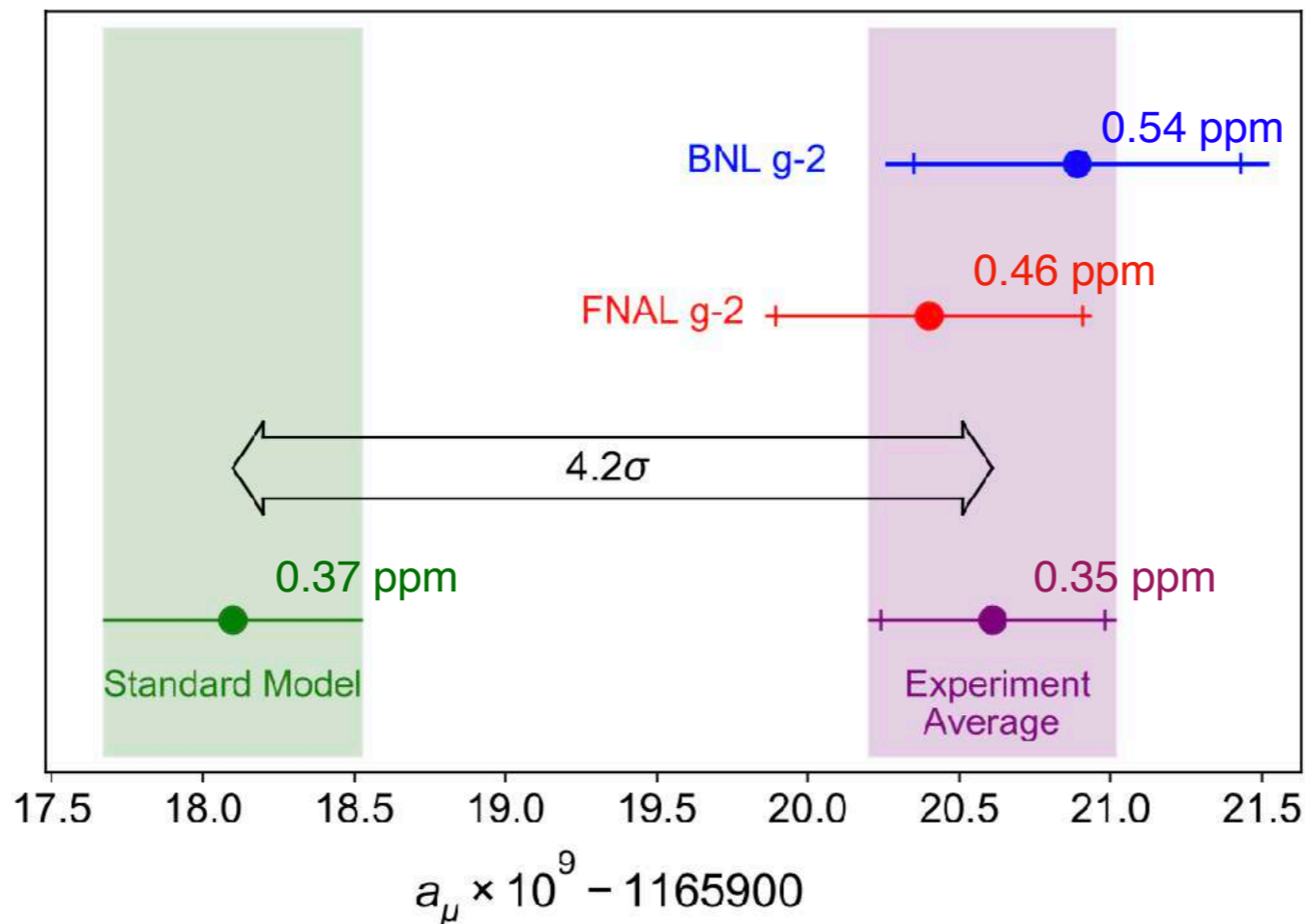
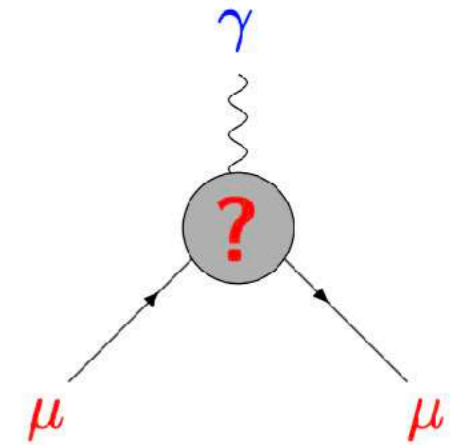
Phys Rev Lett.126.141801

Muon anomaly

- **New physics** contribution to a_μ ?

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}} + a_\mu^{\text{NP}}?$$

Unknown particles
SUSY, MSSM...



Why muon?

New physics effects enhanced by

$$\delta a_\ell \propto m_\ell^2 / M_{\text{NP}}^2$$

Muon is more sensitive by a factor

$$(m_\mu / m_e)^2 \approx 4.3 \cdot 10^4$$

Phys Rev Lett.126.141801

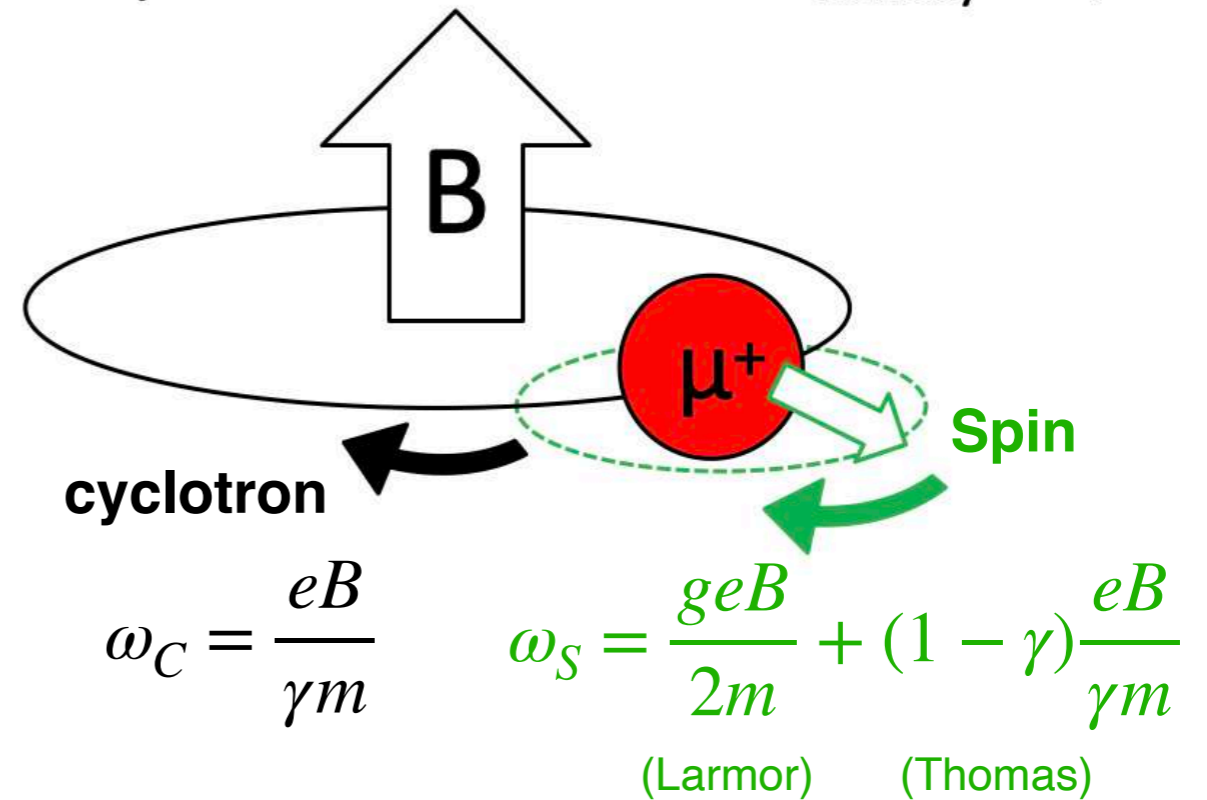
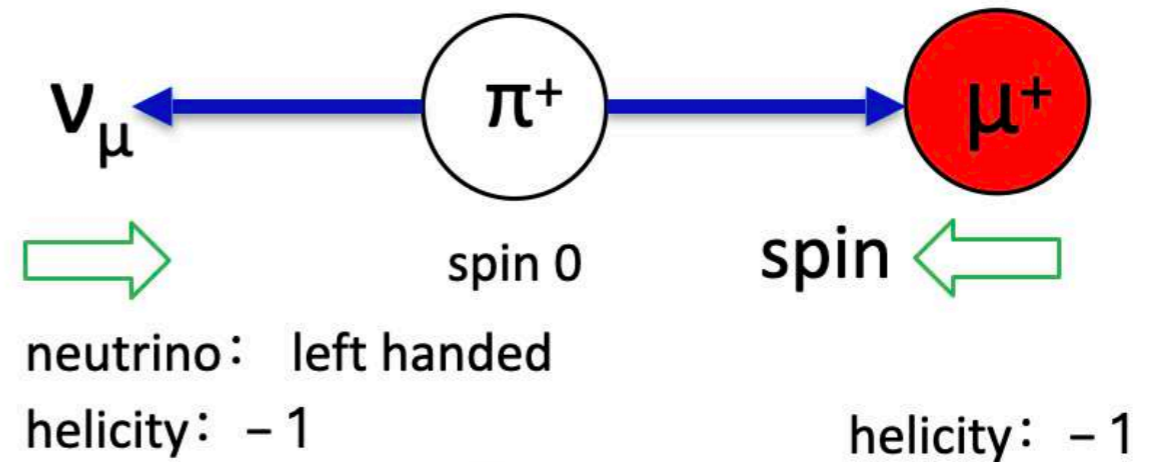
Experimental determination of a_μ

1. Prepare a polarized muon beam.

2. Store in a magnetic field (muon's spin precesses)

$$\omega_a = \omega_S - \omega_C = -\underbrace{\frac{1}{2}(g-2)}_{a_\mu} \frac{eB}{m}$$

Measure two quantities: ω_a, B



Experimental determination of a_μ

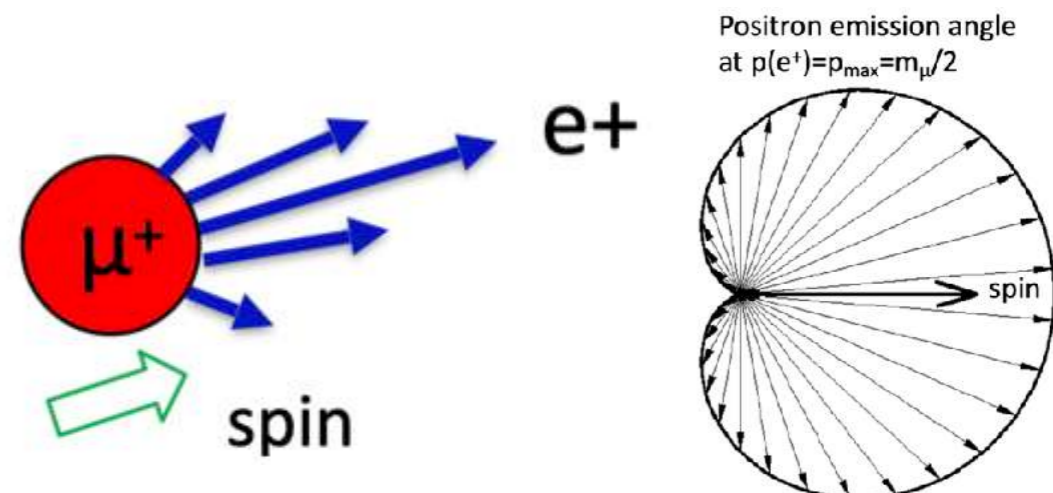
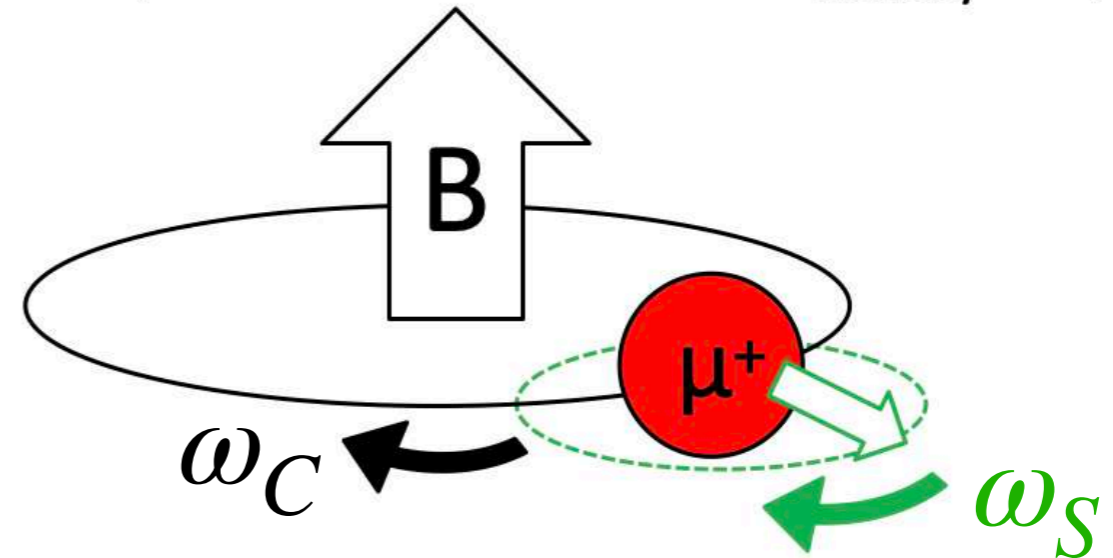
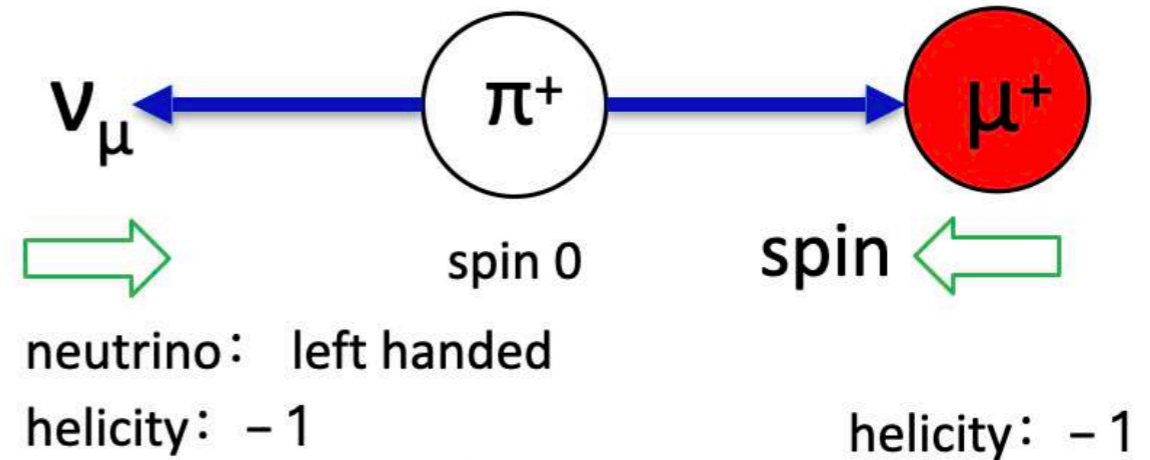
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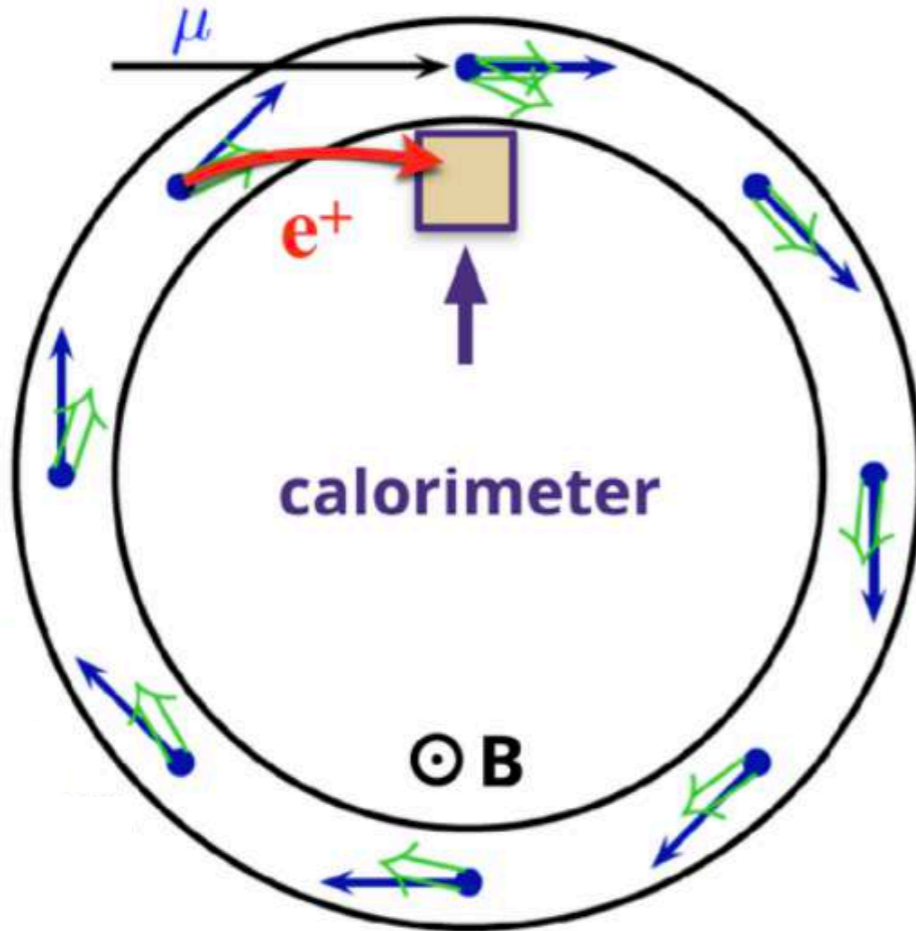
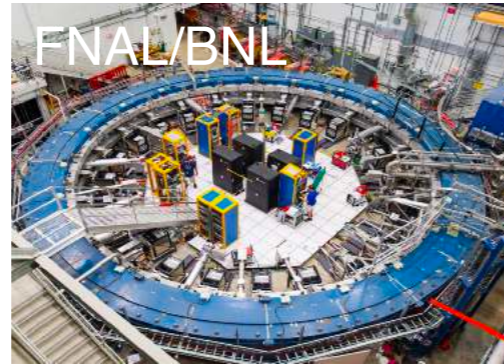
3. Detect a decay positron

(Higher energy positron tends to emit along muon's spin direction).

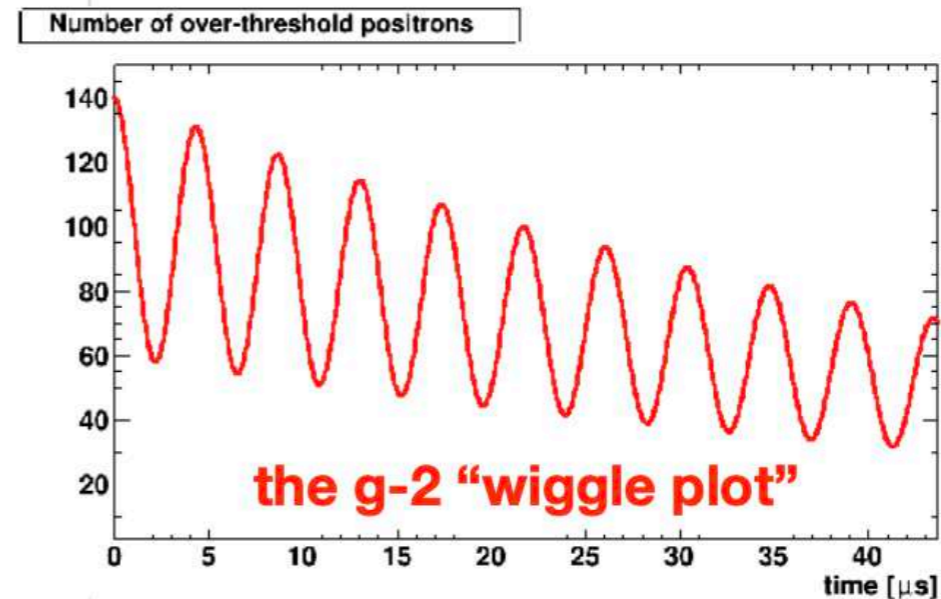
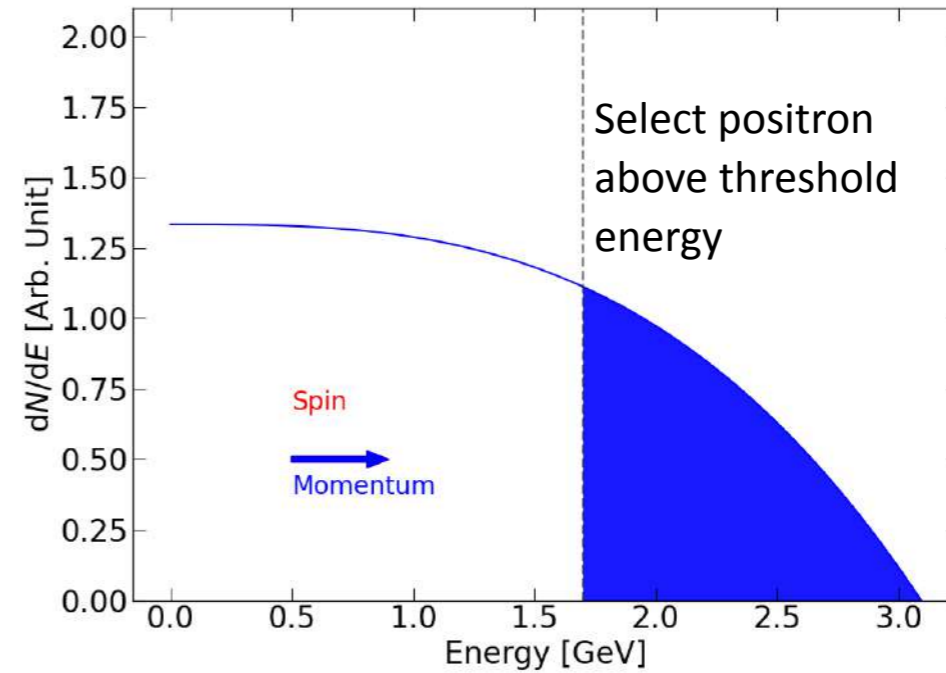


Frequency ω_a extraction in Fermilab

$$\omega_a = a_\mu \frac{eB}{m}$$



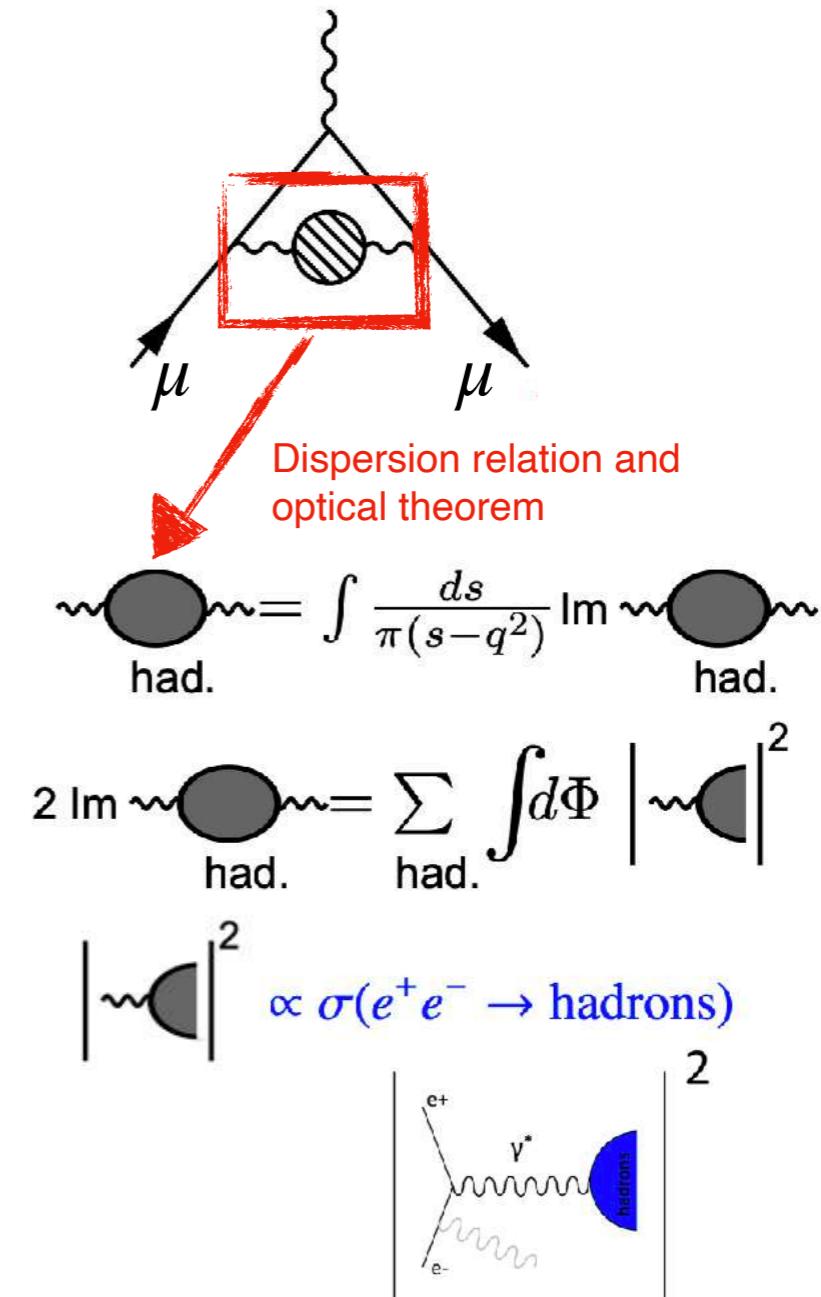
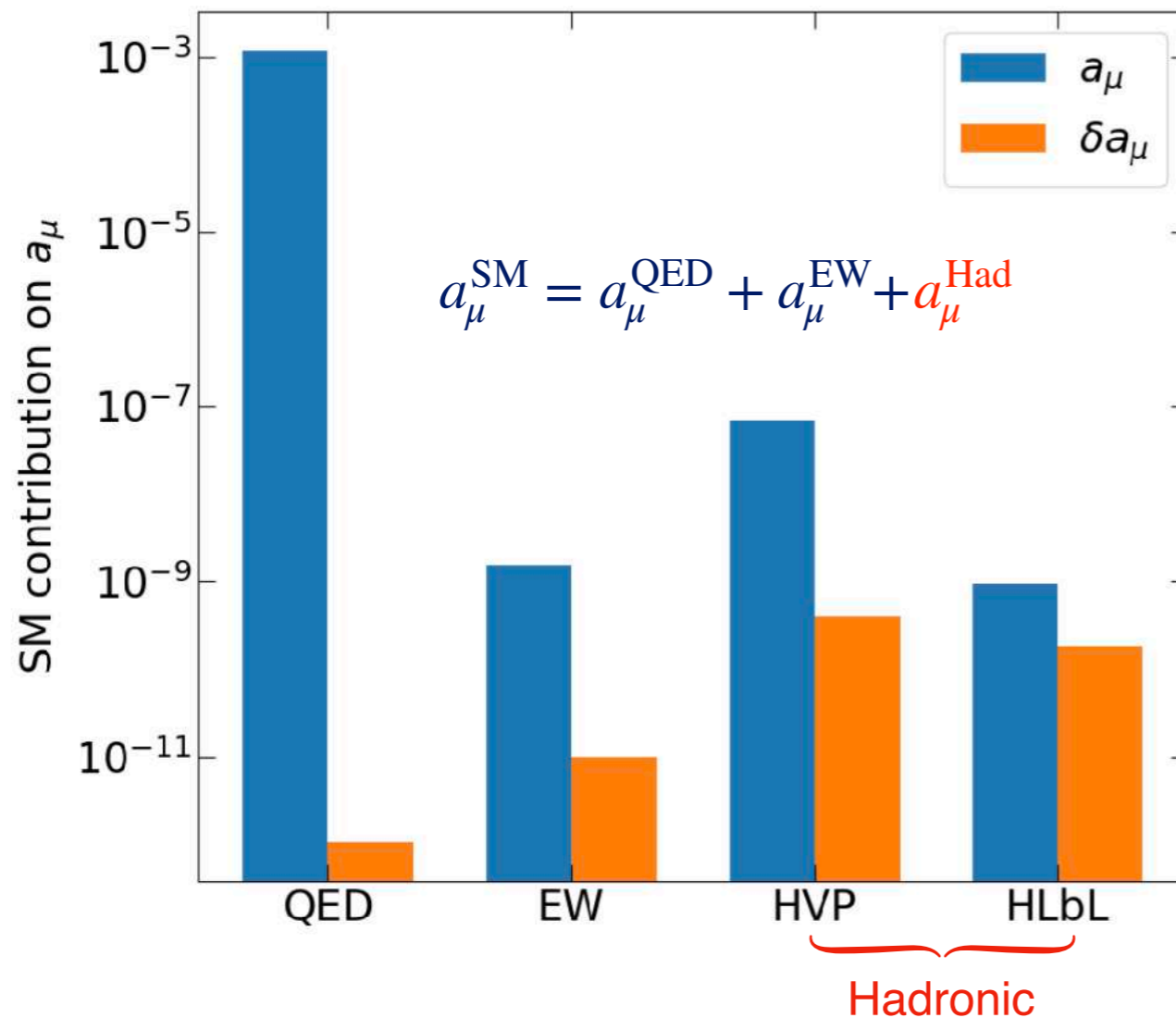
spin
 momentum
Rotation by ω_s
Rotation by ω_c



$$N(t) = N_0 e^{-t/\tau} \left[1 + A_\mu \cos(\omega_a t + \phi) \right]$$

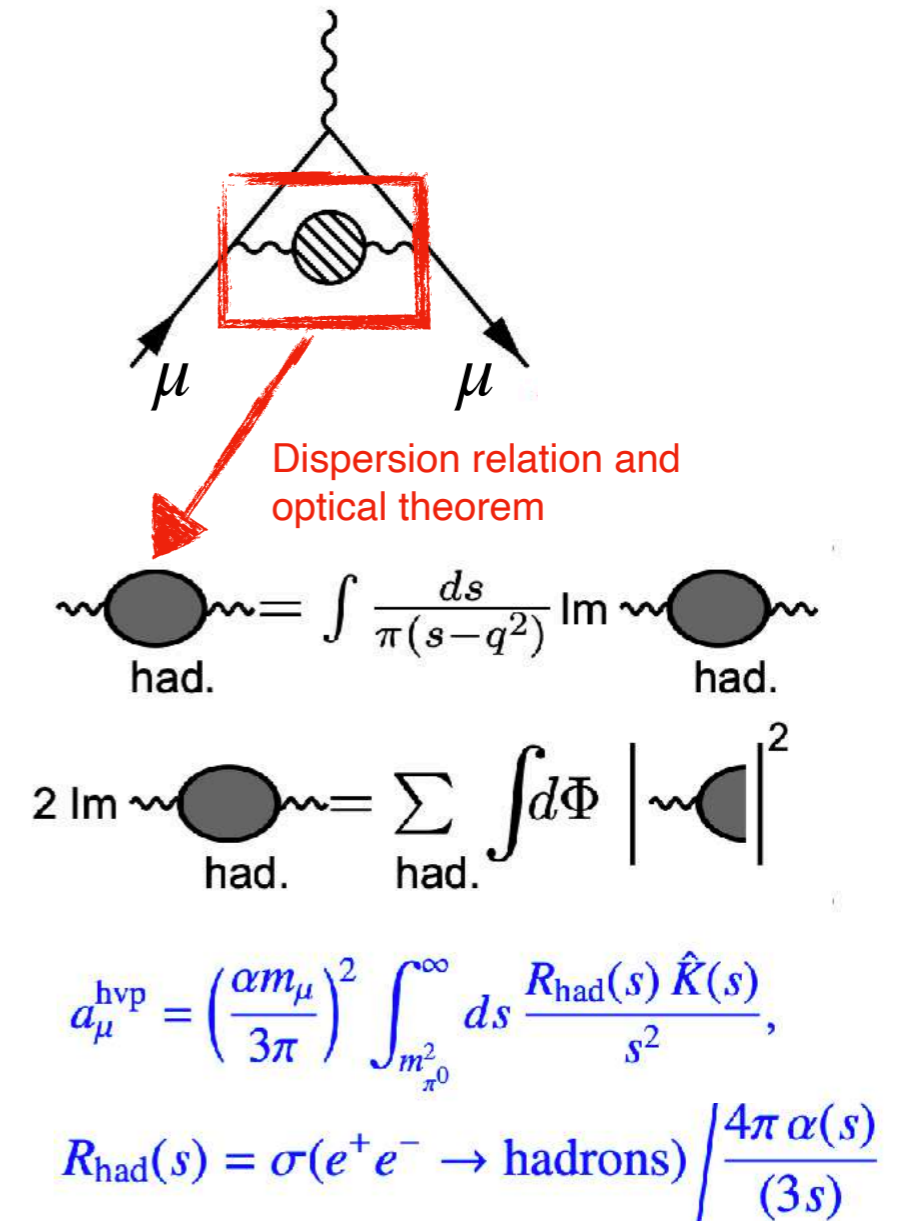
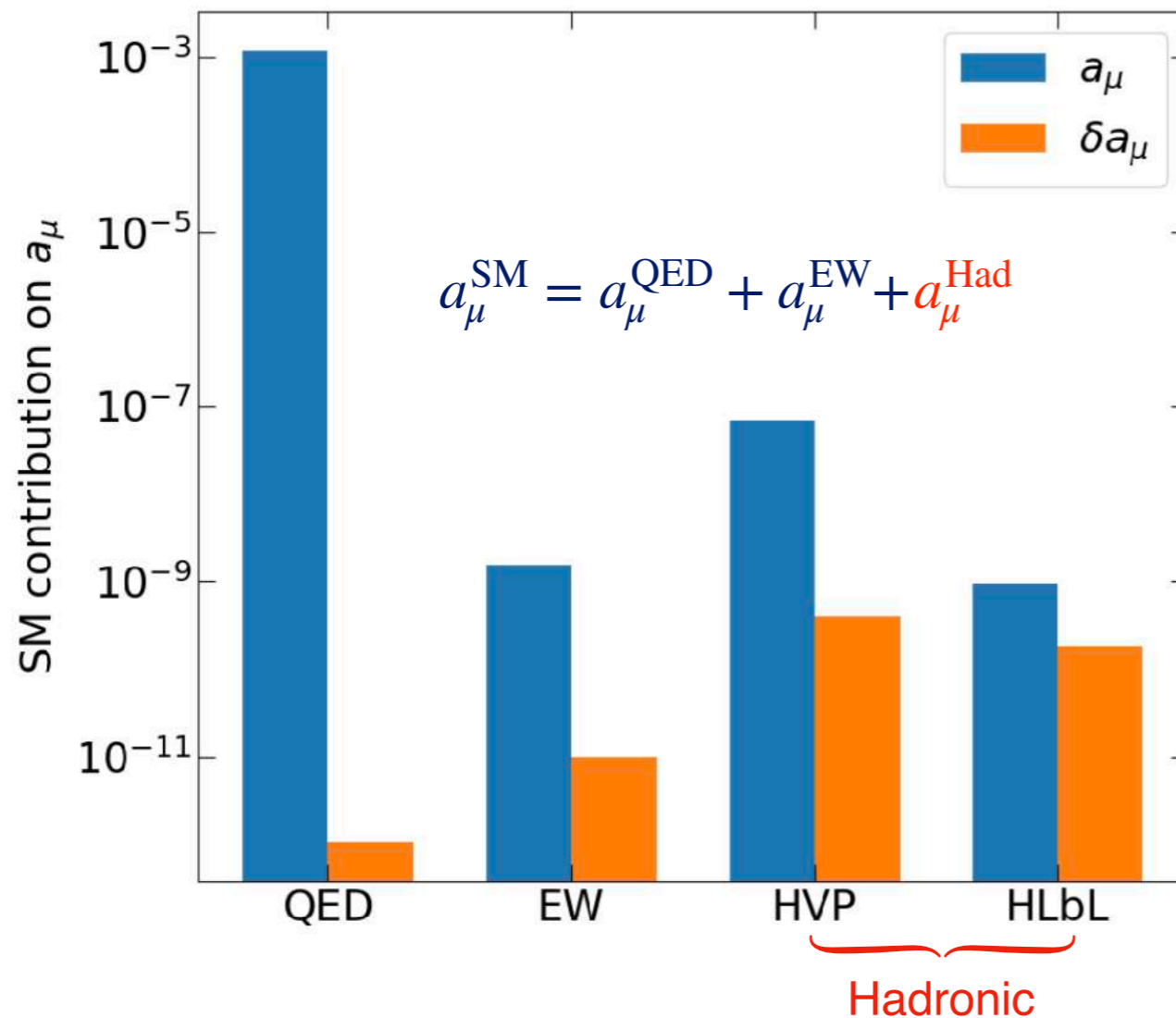
Theory initiative to determine a_μ^{SM}

- In the theory a_μ^{SM} , hadronic contribution is the most challenging part
- At low energy pQCD is not useful, either LQCD or exp. data is needed.

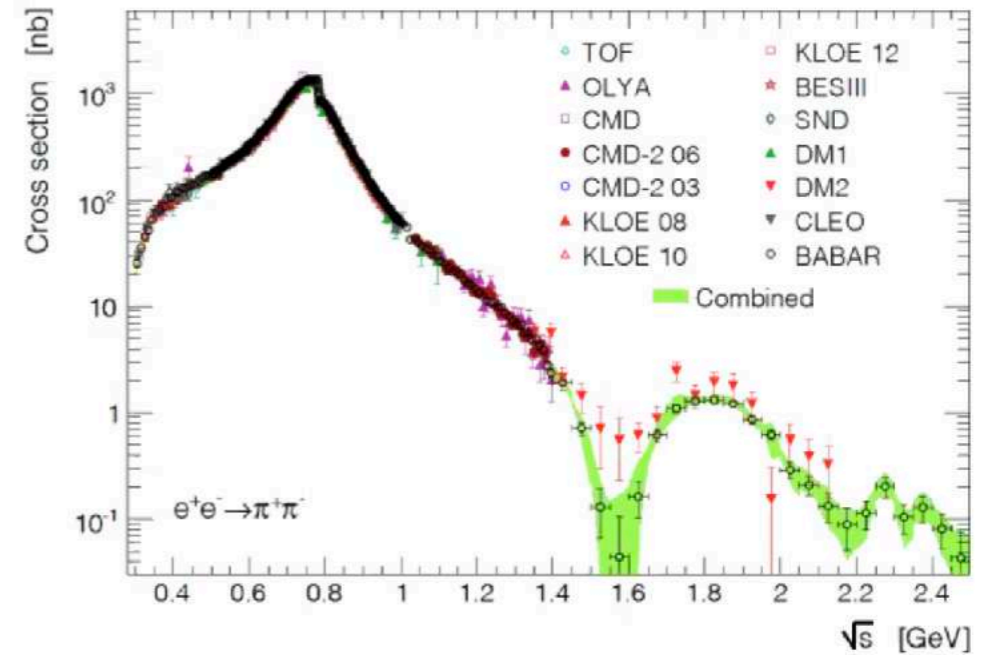
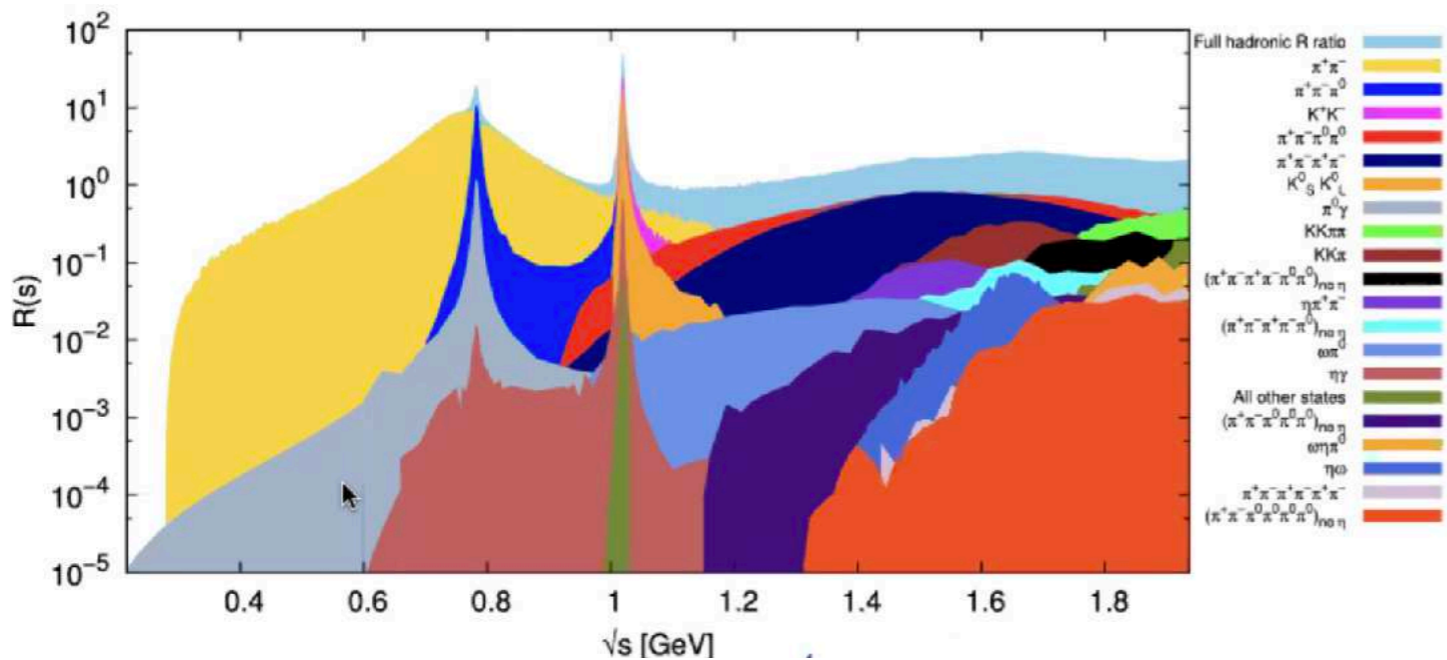


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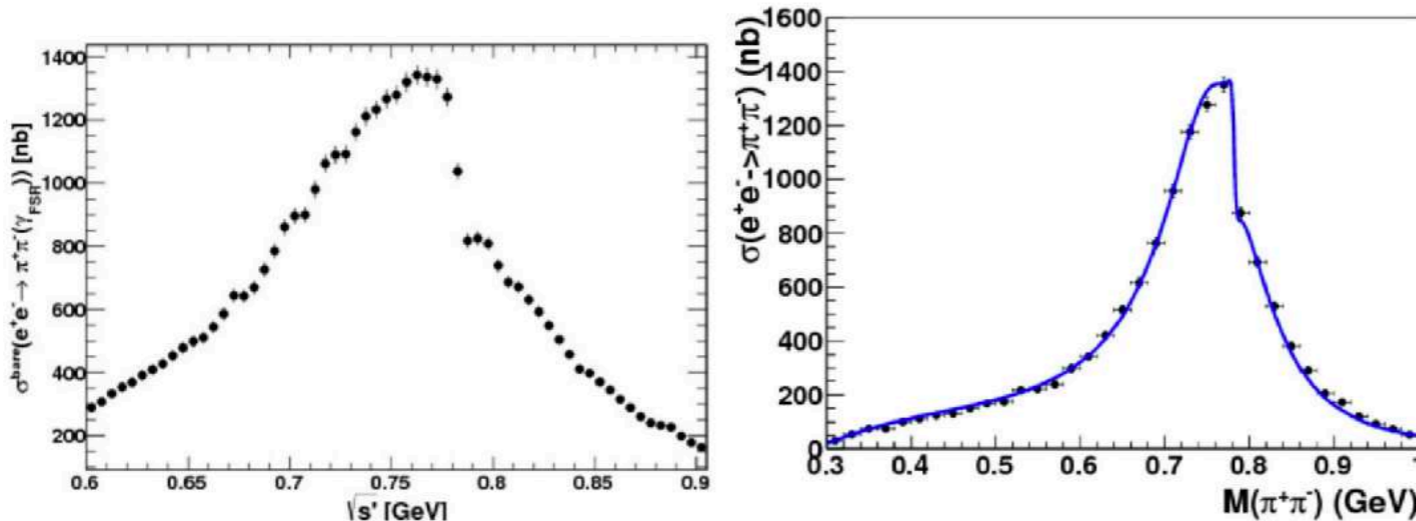


Theory initiative to determine a_μ^{SM}

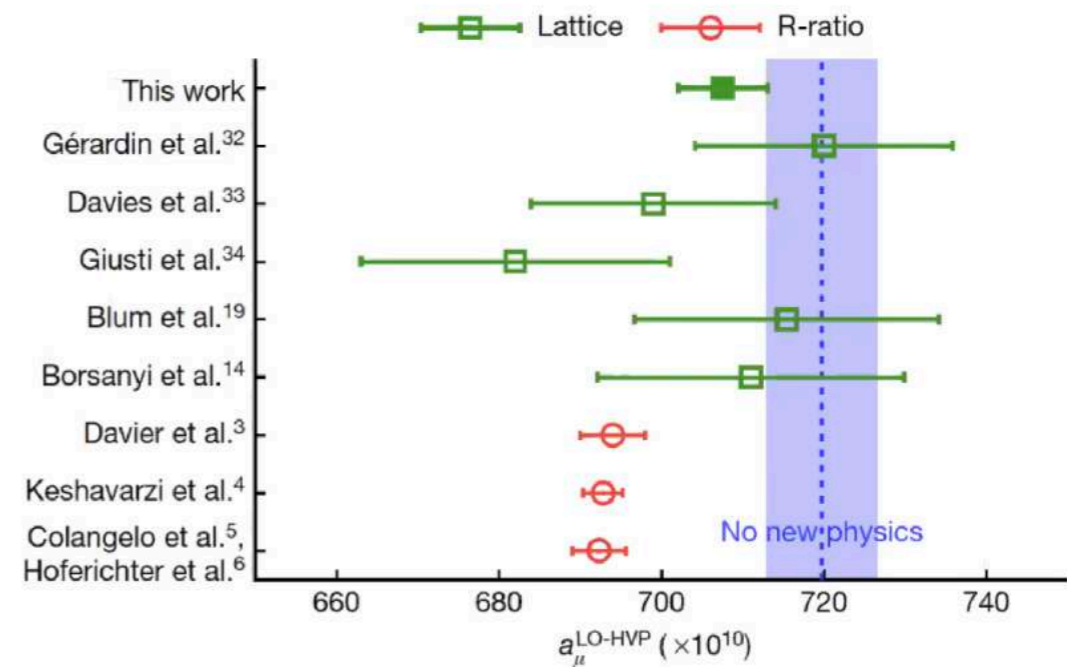


Use experimental data for cross section ratio $R_{\text{had}}(s)$

Various Theoretical Calculations



BESIII data
CLEO-c data
 $\pi^+\pi^-$ data from BESIII and CLEO-c is critical



Theory initiative to determine a_μ^{SM}

The muon g-2 theory initiative

A group of 170 experts came to a consensus on **a single value** of muon g-2 in the standard model.

The white paper

Phys. Rep. 887 (2020) 1-166

(Submitted 15 June

Accepted 29 July

Published 14 Aug)



2017 workshop in Fermilab

2018 workshop in Mainz

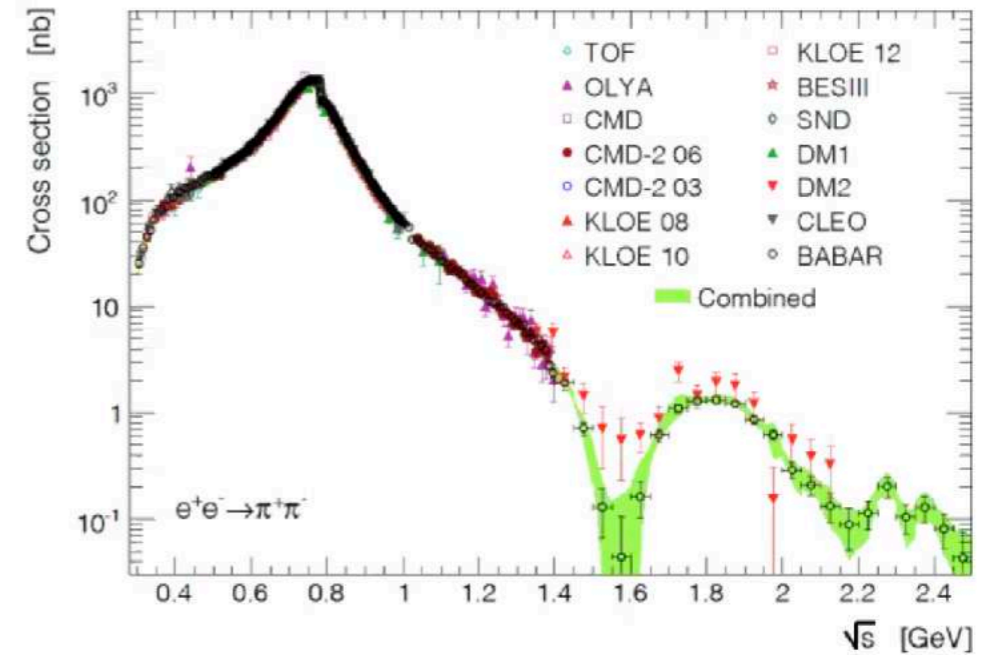
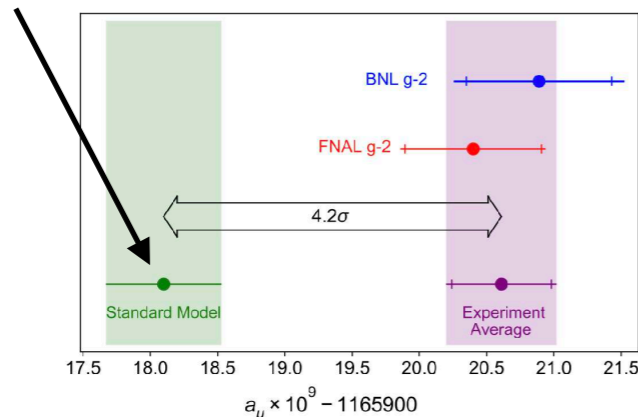
2019 workshop in Seattle

2020 → 2021 workshop in KEK (online)

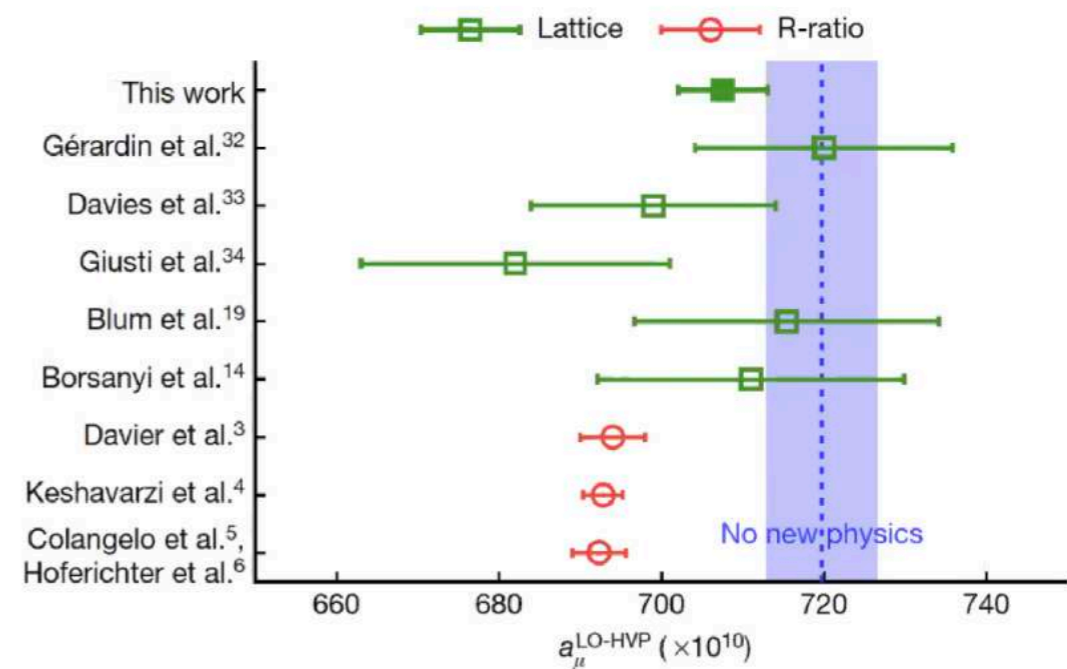
Supported by the KEK-IPNS theory center

<https://www-conf.kek.jp/muong-2theory/index.html>

$$a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}$$



Various Theoretical Calculations



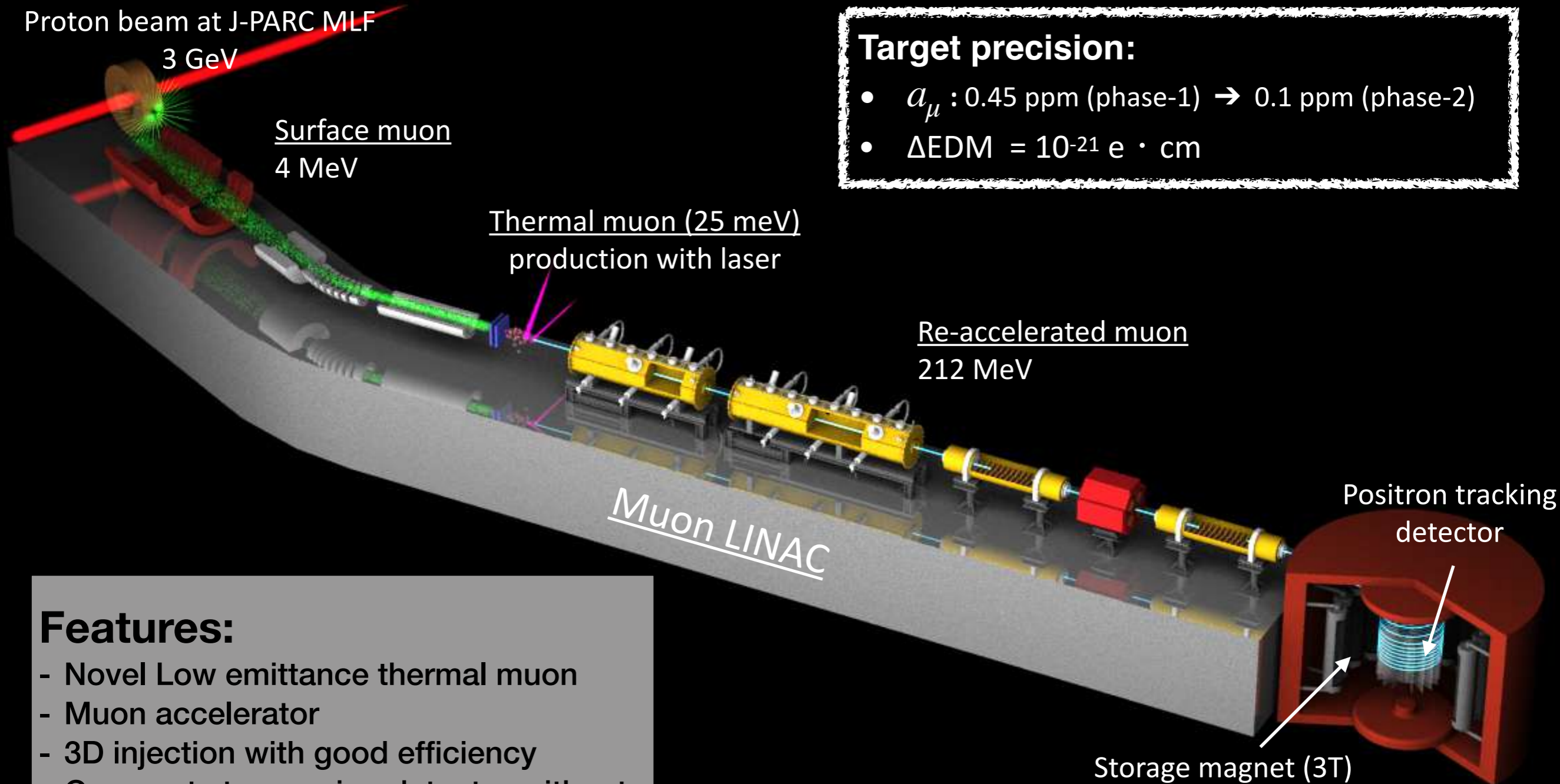
Borsanyi et al (BMWc), Nature 2012

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New muon $g - 2$ /EDM experiment in Japan



Target precision:

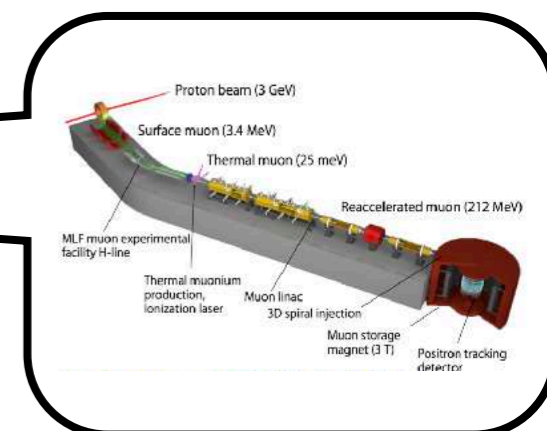
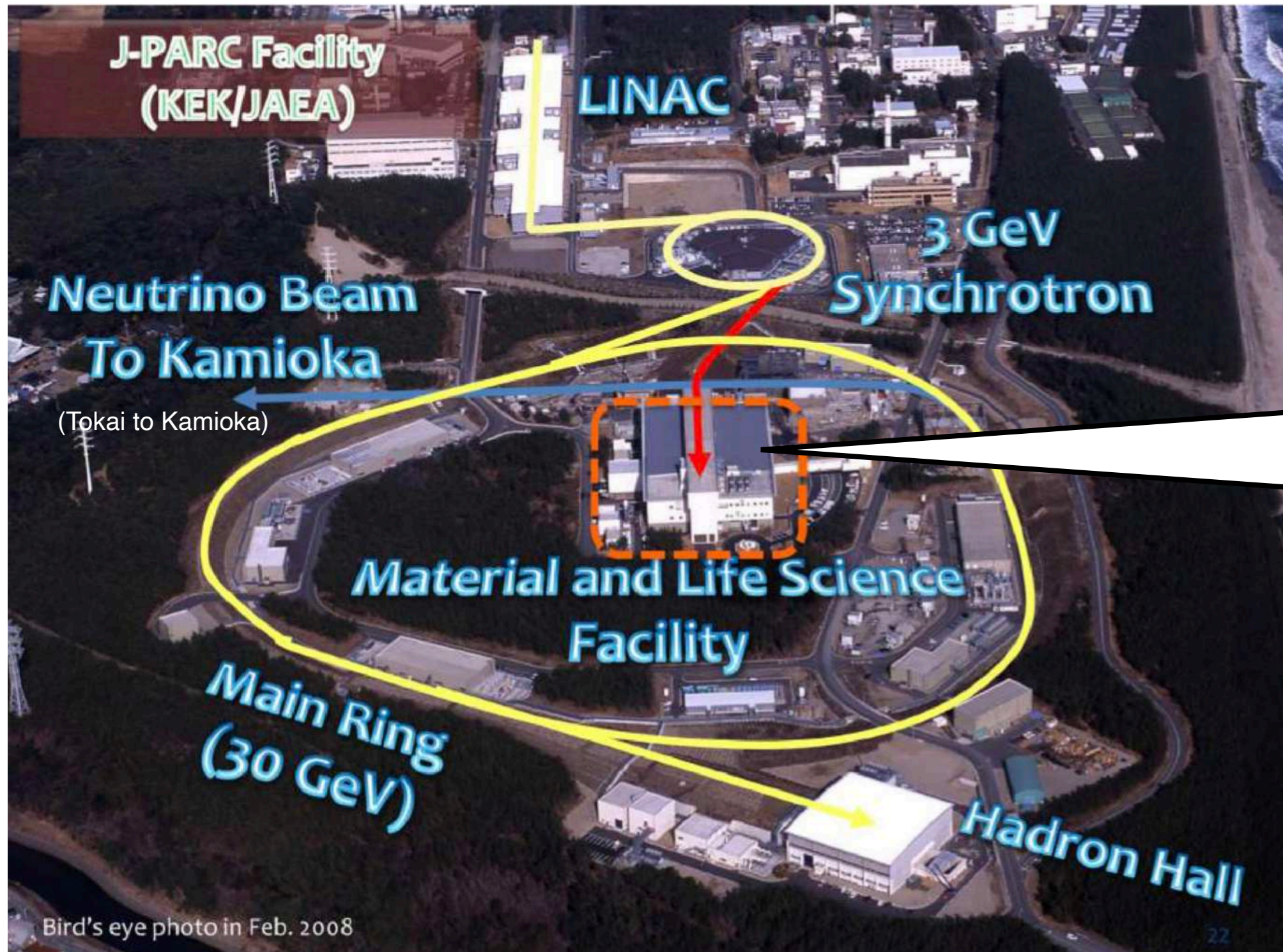
- a_μ : 0.45 ppm (phase-1) \rightarrow 0.1 ppm (phase-2)
- $\Delta\text{EDM} = 10^{-21} \text{ e} \cdot \text{cm}$

Features:

- Novel Low emittance thermal muon
- Muon accelerator
- 3D injection with good efficiency
- Compact storage ring detector without strong focusing

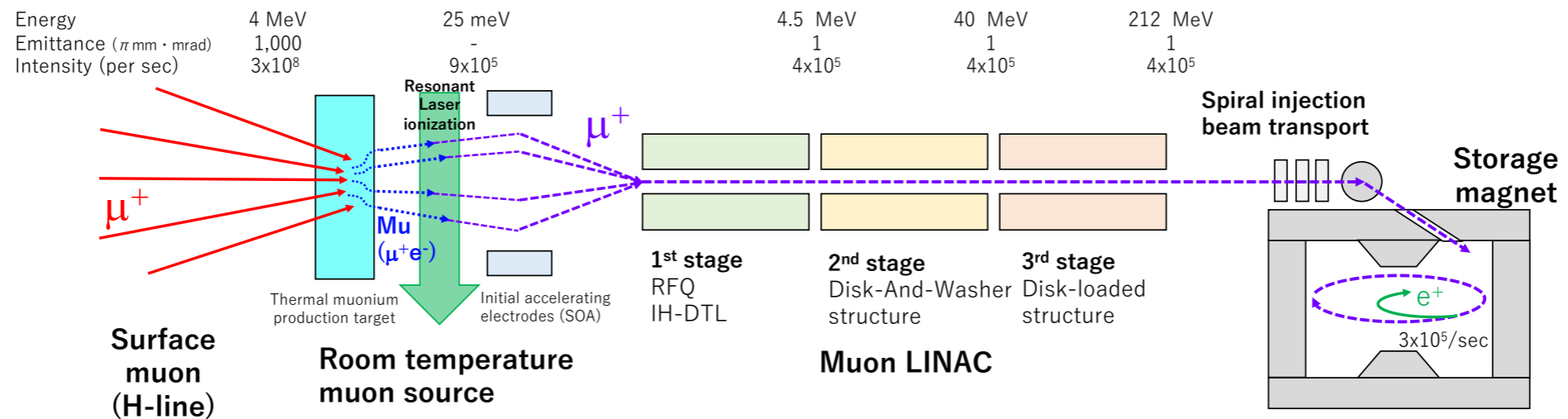
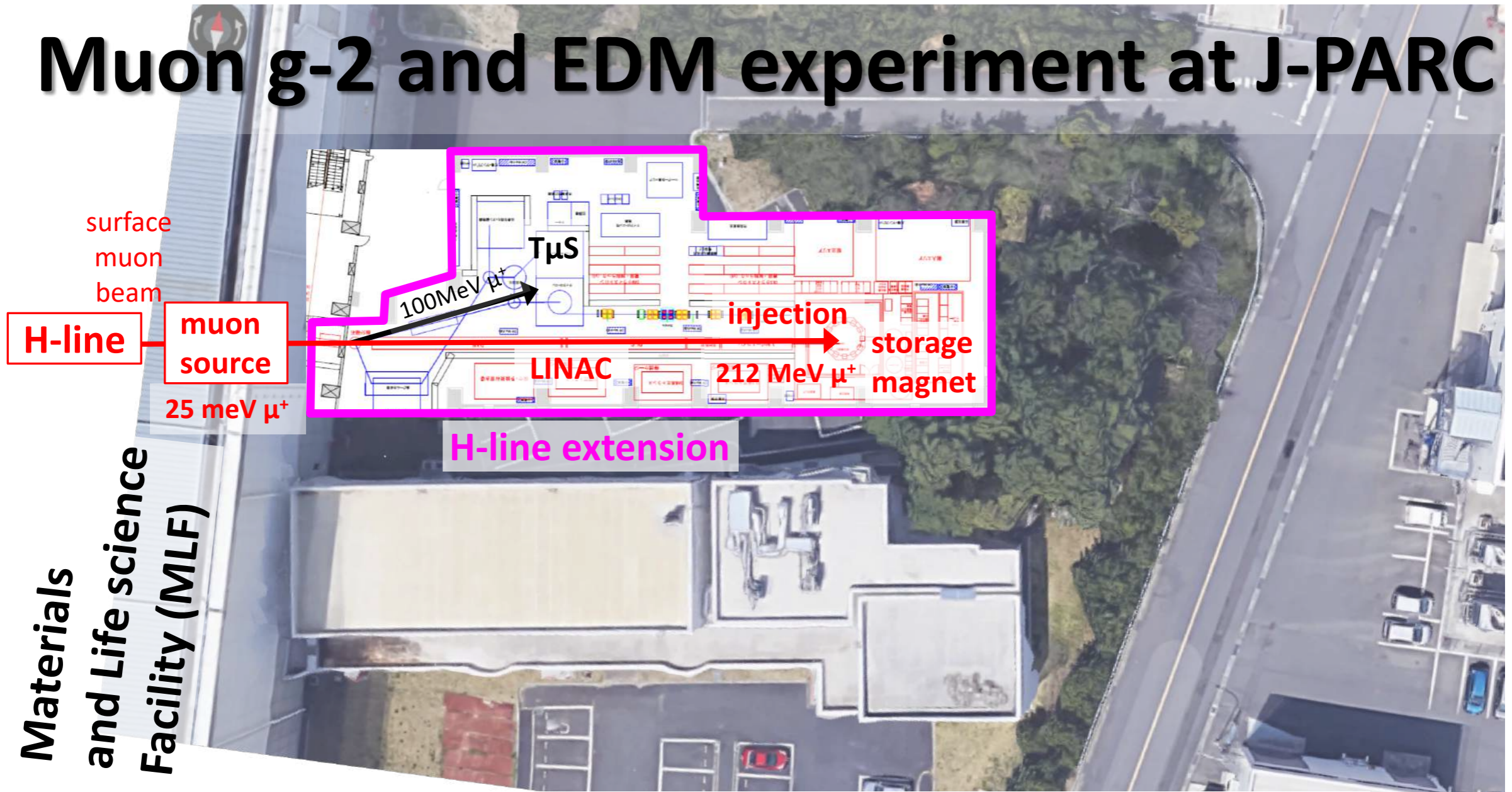
Prog. Theor. Exp. Phys. 2019, 053C02

Japan proton accelerator research complex (J-PARC)



Location of our experiment

Muon g-2 and EDM experiment at J-PARC



New muon $g - 2$ /EDM experiment in Japan

- Our experiment is completely different from BNL/FNAL approach!
- In the E-B field, the more general form of spin precession vector:

$$\vec{\omega}_a = -\frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2} \left(\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} \right) \right]$$

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BNL E821 approach
 $\gamma=30$ ($P=3$ GeV/c)

Magic " γ ": $a_\mu = \frac{1}{\gamma^2 - 1}$

$$\vec{\omega}_a = -\frac{e}{m} \left[a_\mu \vec{B} + \frac{\eta}{2} \left(\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} \right) \right]$$

FNAL E989

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Directly remove E field!

BNL E821 approach
 $\gamma=30$ ($P=3$ GeV/c)

J-PARC approach
 $E = 0$ at any γ

$$\vec{\omega}_a = -\frac{e}{m} \left[a_\mu \vec{B} + \frac{\eta}{2} \left(\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} \right) \right]$$

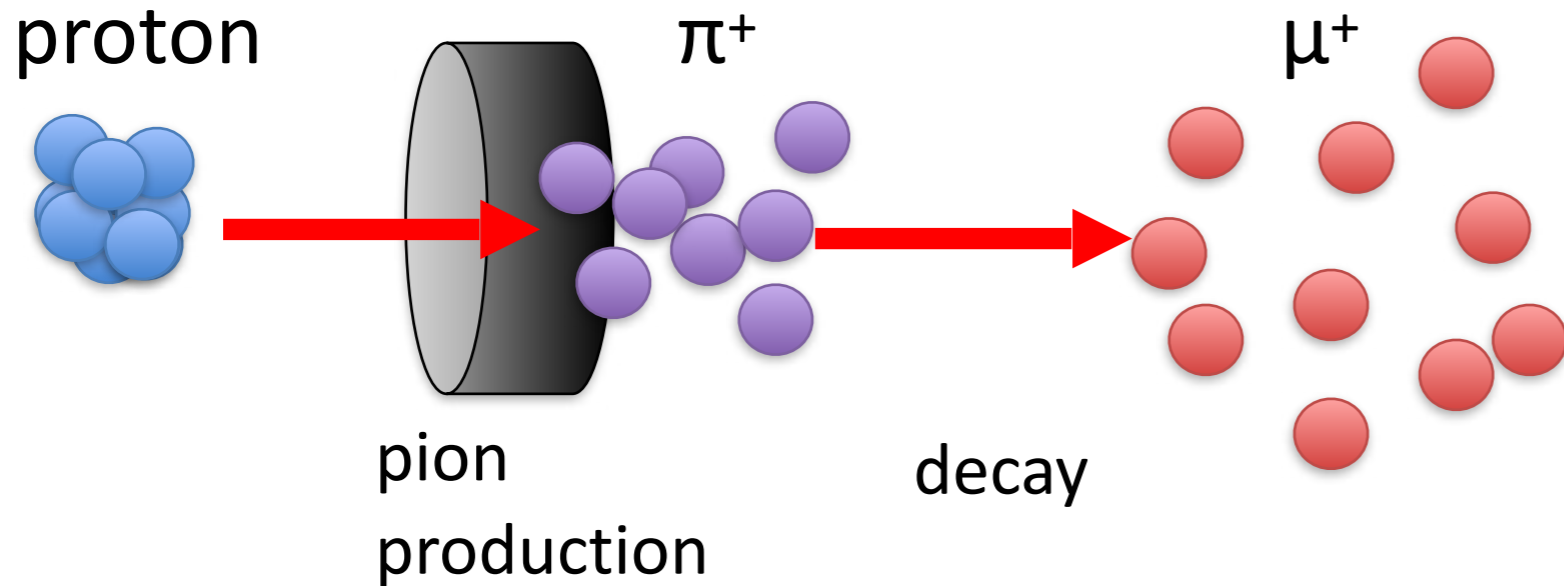
FNAL E989

$$\vec{\omega}_a = -\frac{e}{m} \left[a_\mu \vec{B} + \frac{\eta}{2} \left(\vec{\beta} \times \vec{B} \right) \right]$$

J-PARC E34

Muon beam at BNL/FNAL

Conventional muon beam



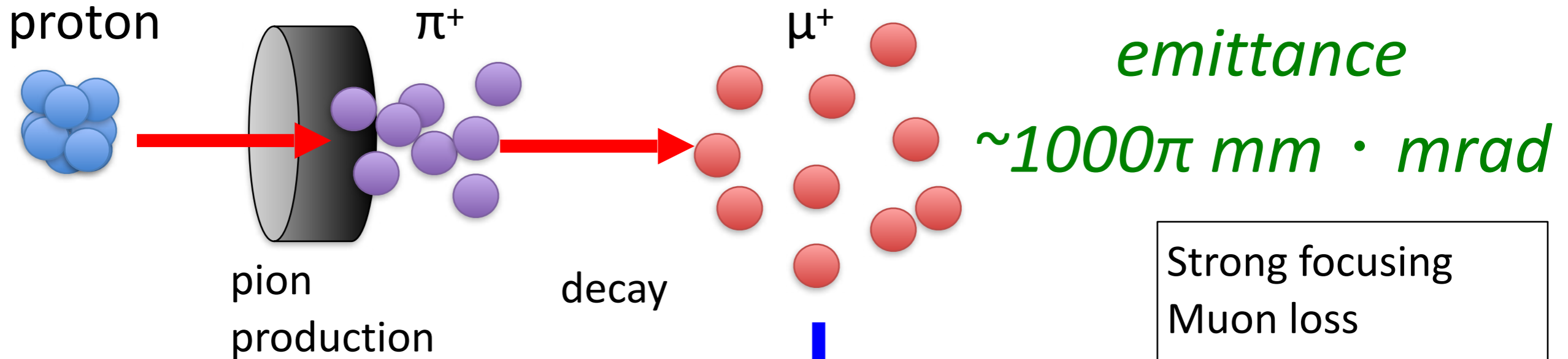
emittance
 $\sim 1000\pi \text{ mm} \cdot \text{mrad}$

Strong focusing
Muon loss
BG π contamination



Novel thermal muon beam at J-PARC

Conventional muon beam



emittance

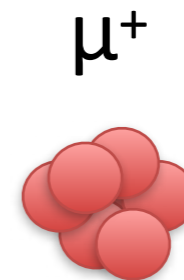
$\sim 1000\pi \text{ mm} \cdot \text{mrad}$

Strong focusing
Muon loss
BG π contamination



cooling

**Thermal
muon beam**



emittance

$1\pi \text{ mm} \cdot \text{mrad}$

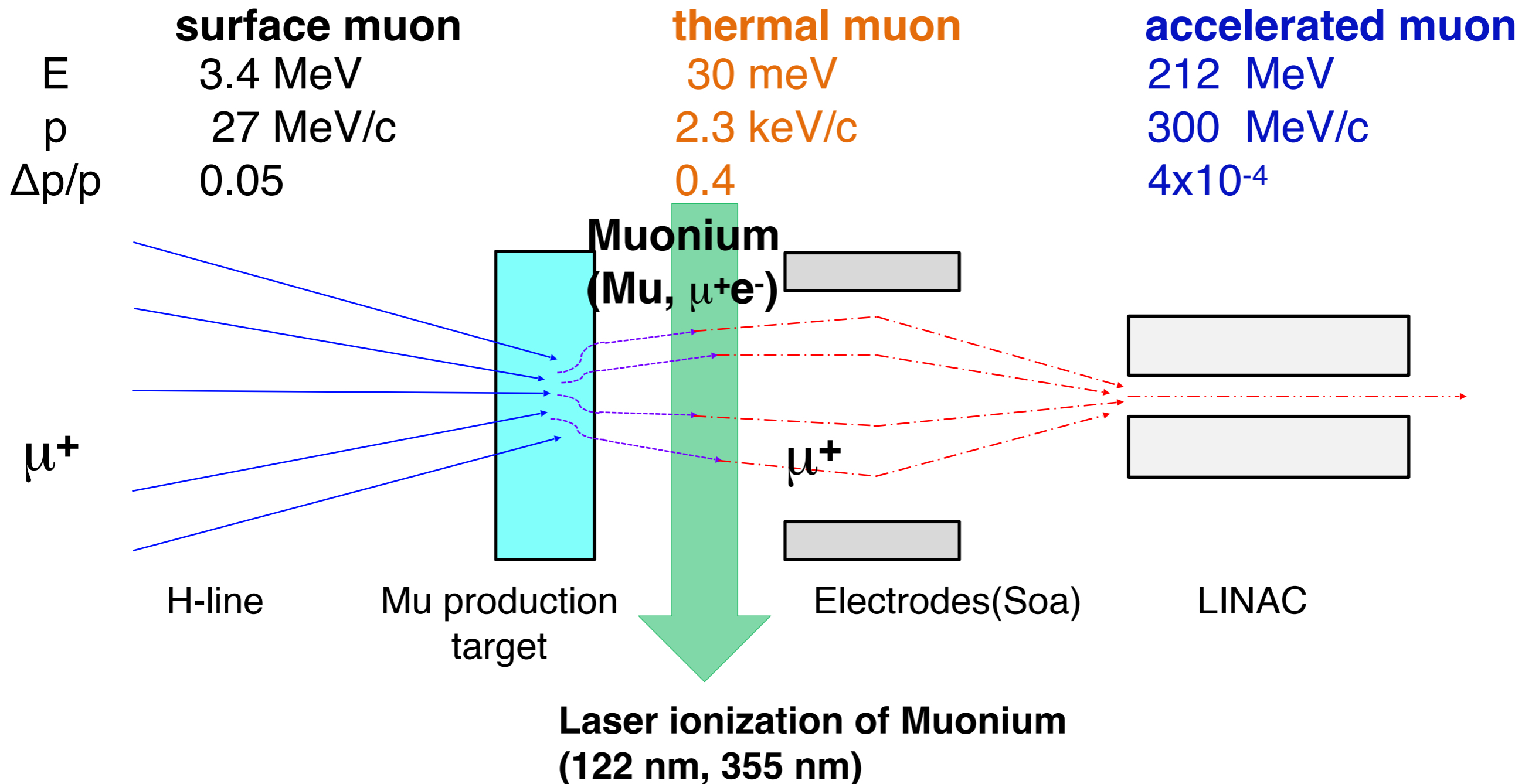
Free from any of these

Re-accelerated thermal Muon beam

Requirement for zero E-field:

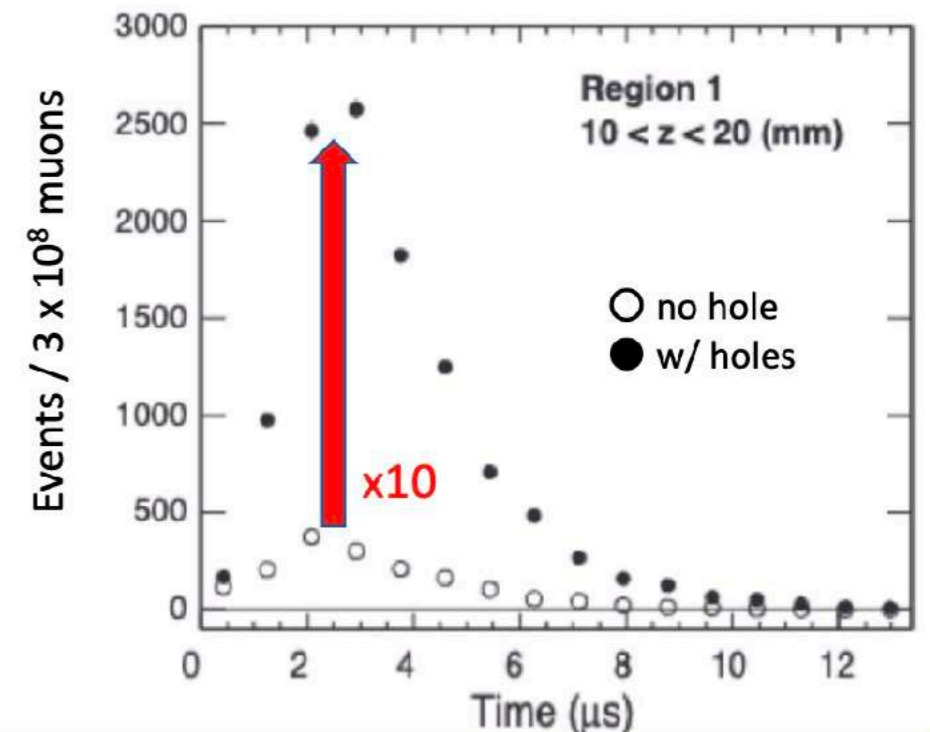
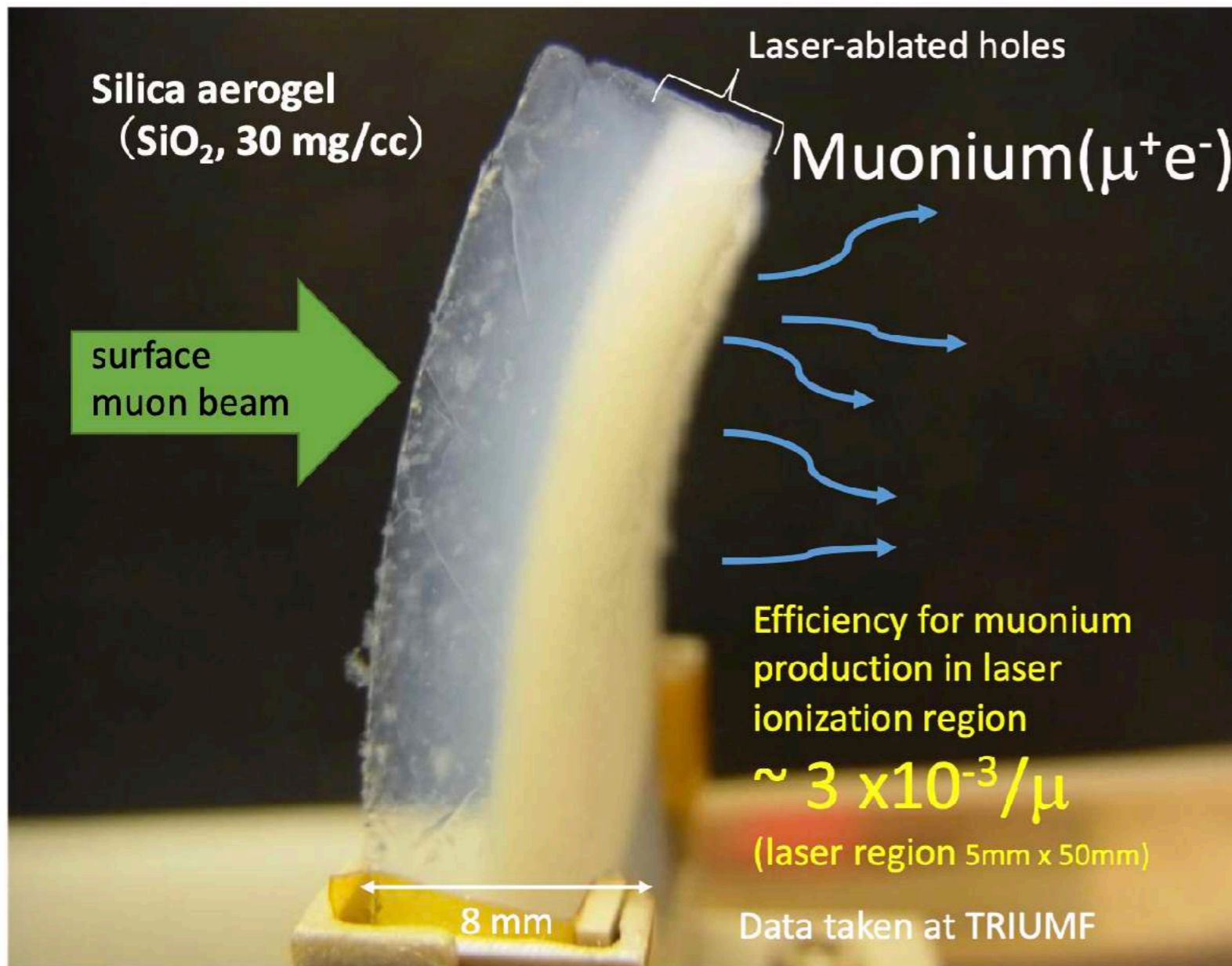
Muons should be kept stored without E-focusing

→ Beam with ultra-small transverse dispersion, i.e. $\Delta p_T/p \sim 0$



Development of thermal muon source

- Muonium yield from silica aerogel has been confirmed
- Novel technique of laser-ablated aerogel was used to enhance the Mu yield

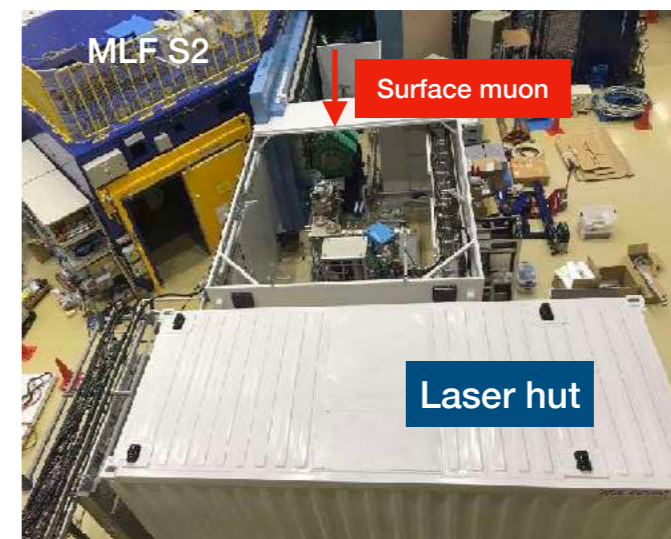
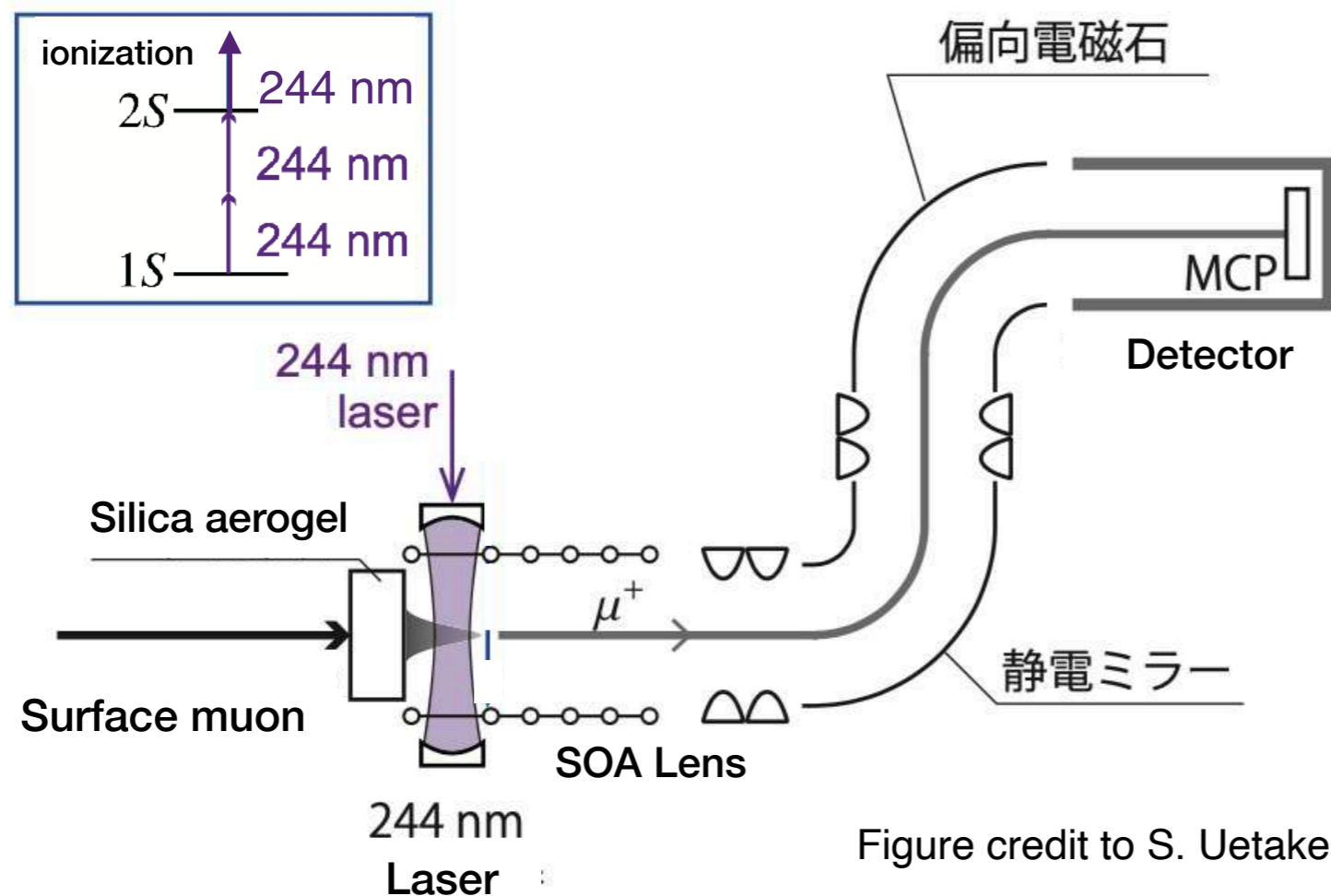


Need muonium to diffuse out of target before decay. Yield increased to 3% !

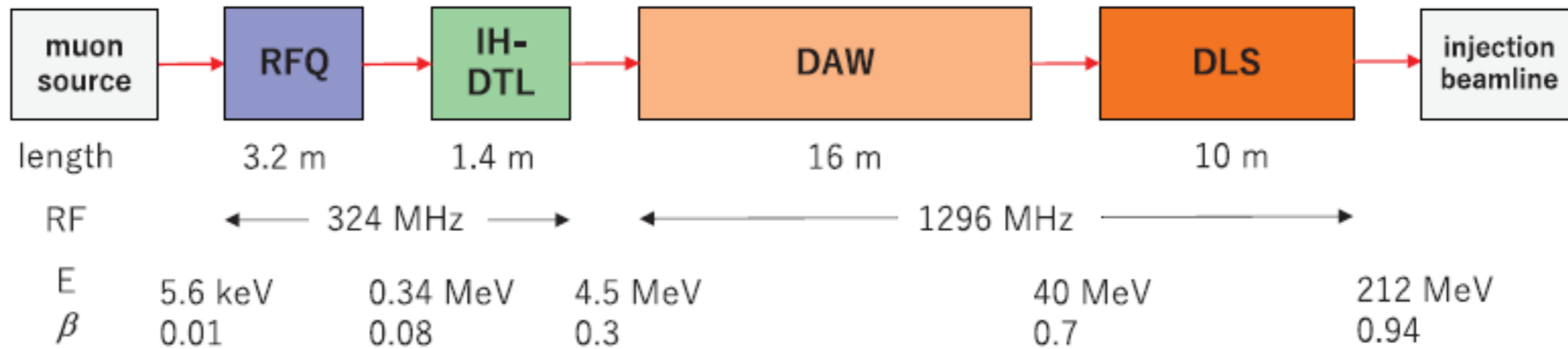
Sample	Laser-ablated structure (pitch)	Vacuum yield (per 10^3 muon stops)
Flat	none	3.72 ± 0.11
Flat (Ref. [7])	none	2.74 ± 0.11
Laser ablated	500 μm	16.0 ± 0.2
Laser ablated	400 μm	20.9 ± 0.7
Laser ablated	300 μm	30.5 ± 0.3

Development of thermal muon source

- Now, a demonstration experiment 244 nm laser is being prepared at J-PARC
- It will be world's first time to generate thermal muon by this approach!
- Except for the muon g-2, such thermal muon source has wide application for other muon-related precision experiment



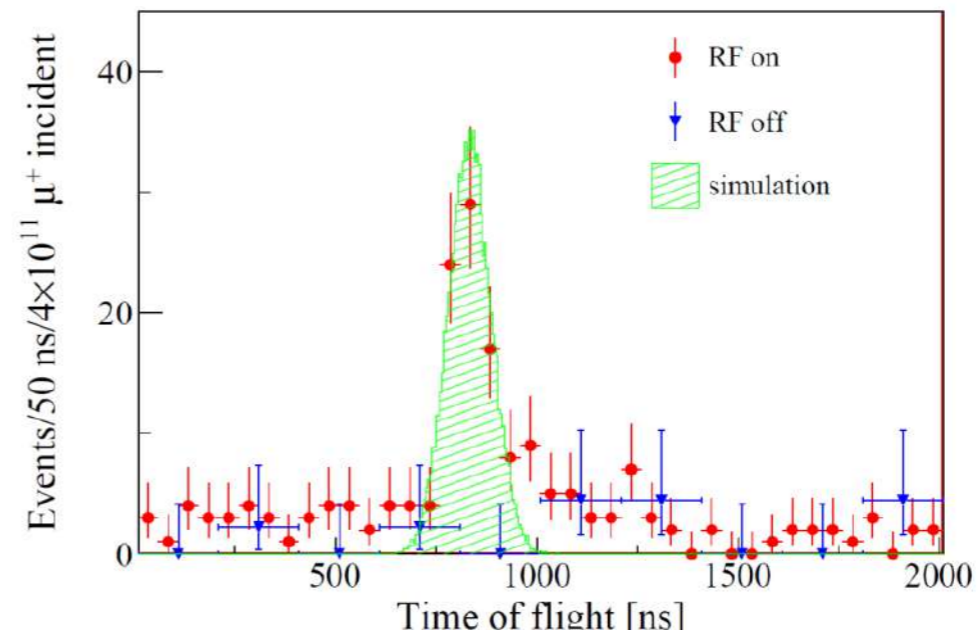
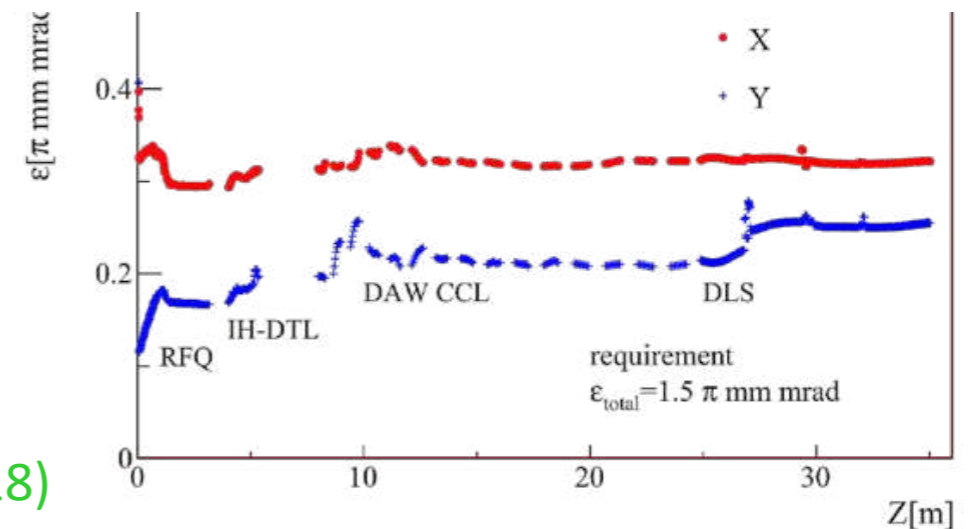
Muon acceleration (LINAC)



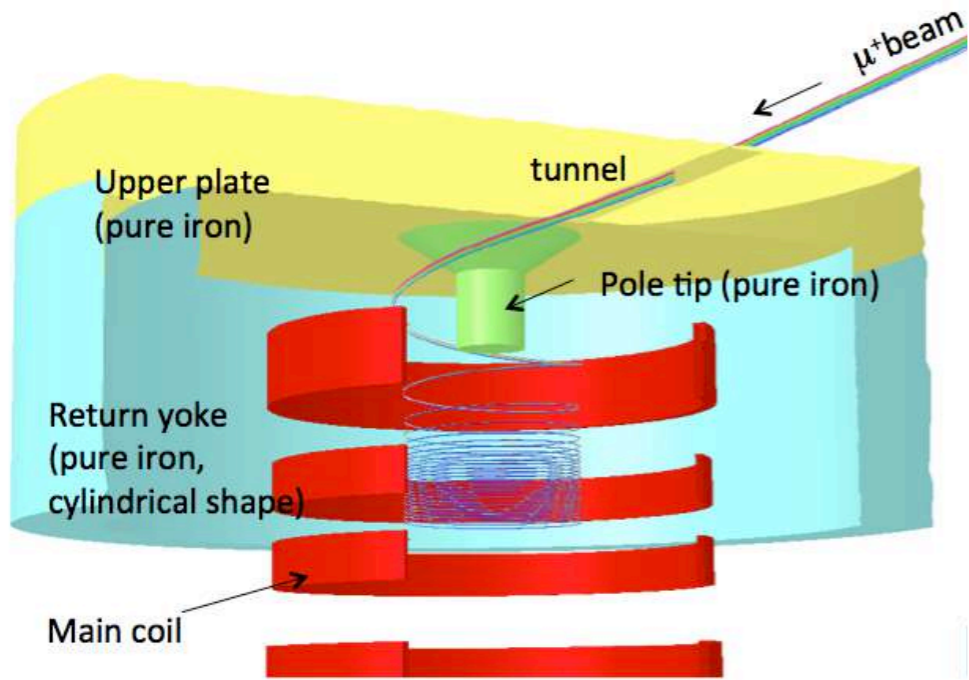
End to end simulation
 transmission loss $\sim 7\%$ + decay loss
 emittance growth is small

First muon acceleration with RFQ!

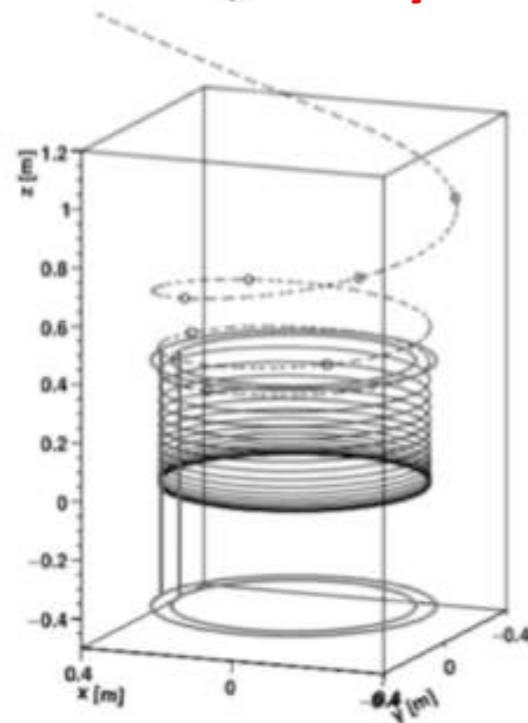
S. Bae et al., Phys. Rev. Accel. Beams 21, 050101 (2018)



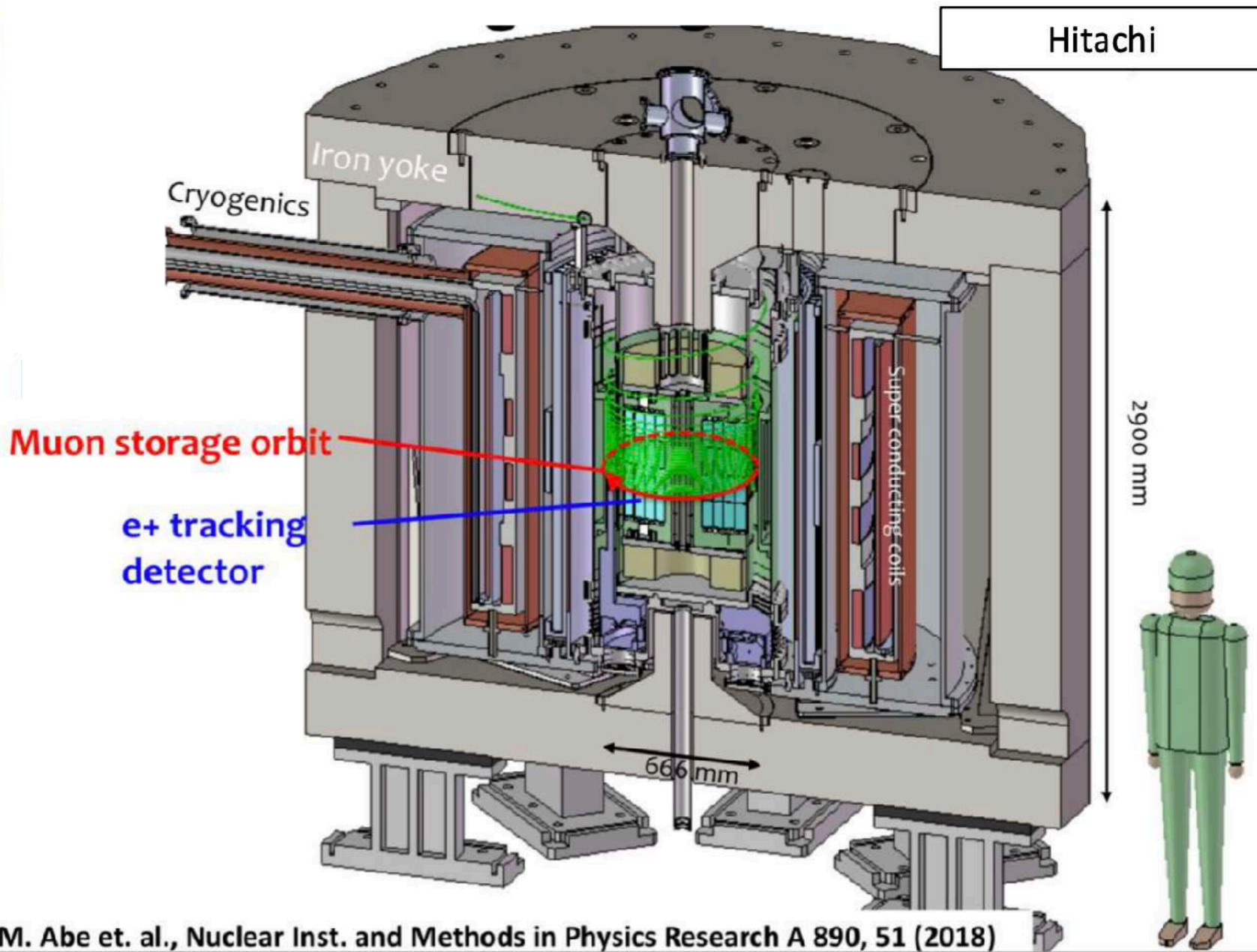
Muon injection and storage



Injection efficiency : ~85%



H. Inuma et al., Nucl. Instr. And Methods. A 832, 51 (2016)

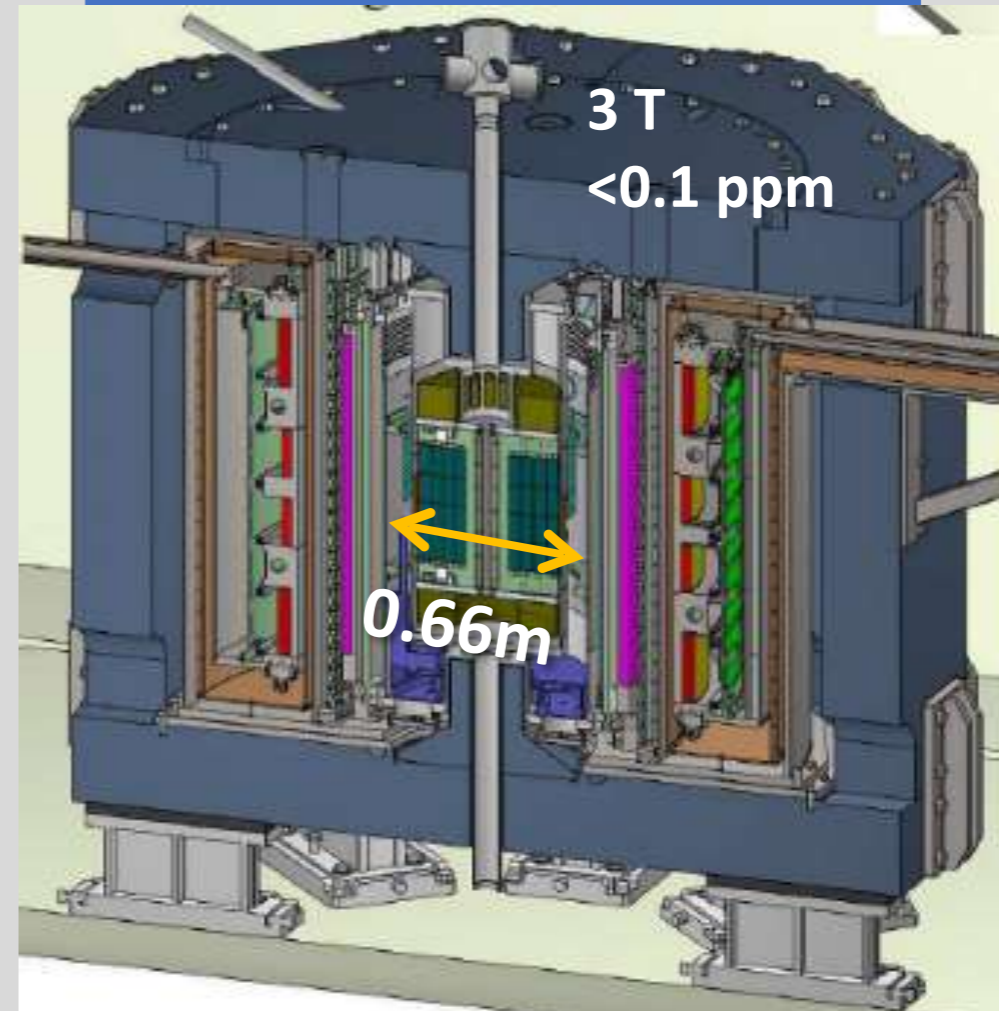
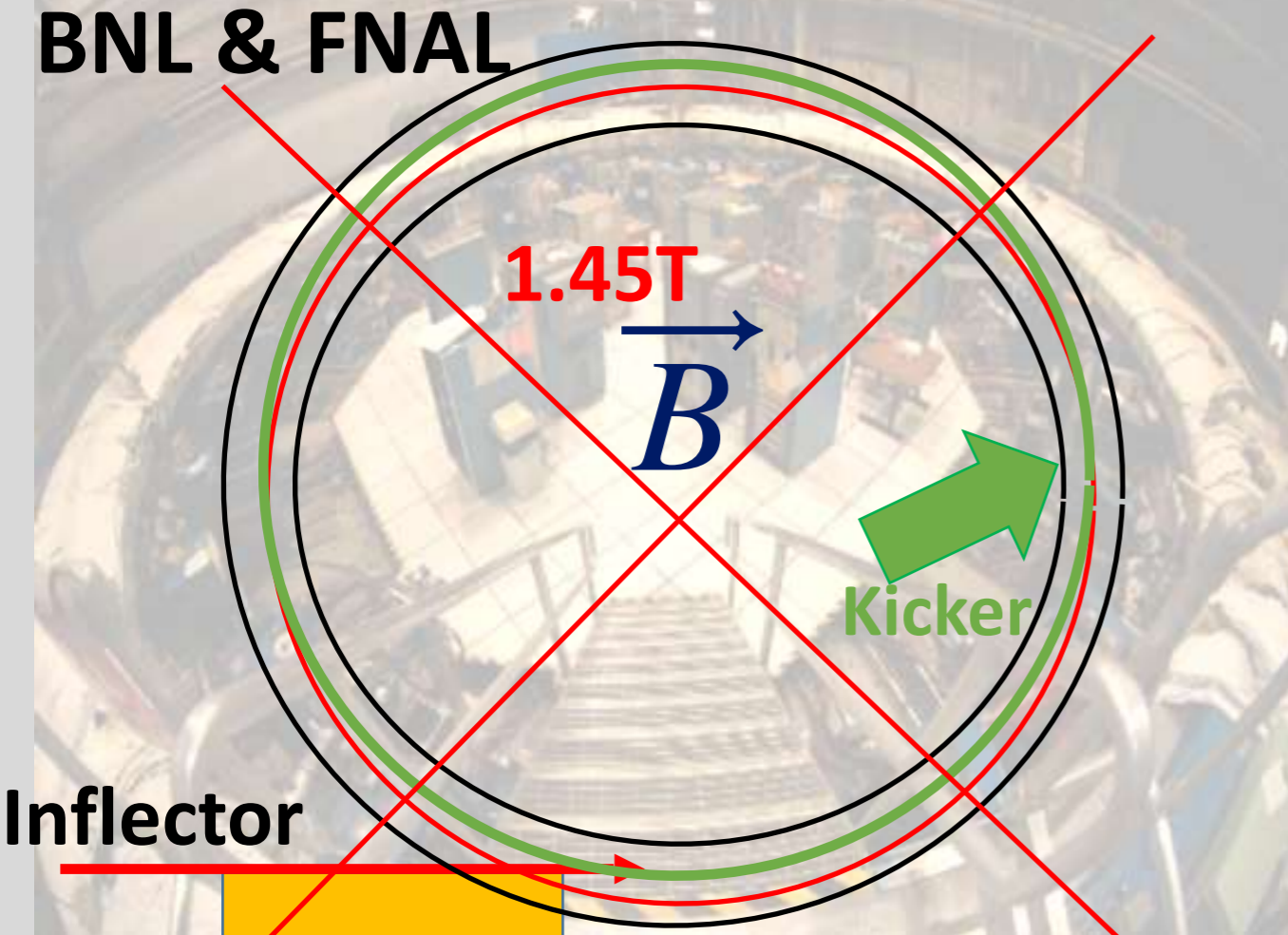


M. Abe et. al., Nuclear Inst. and Methods in Physics Research A 890, 51 (2018)

Why inject beam 3D spirally?

Conventional 2D injection (BNL)

J-PARC E34 storage magnet



14 m orbit, To avoid beam hit at inflector (77 mm), kick angle become **10.8 mrad** within **149 ns**.

Injection efficiency : 3-5%(*)

0.66 m orbit kick angle is **233 mrad** within **7.4 ns**.
3 T is too high to be canceled by inflector.
Impossible by any existent technology.
Need to develop new 3D spiral injection!

Injection efficiency : **~85%**

(*) PRD73,072003 (2006)

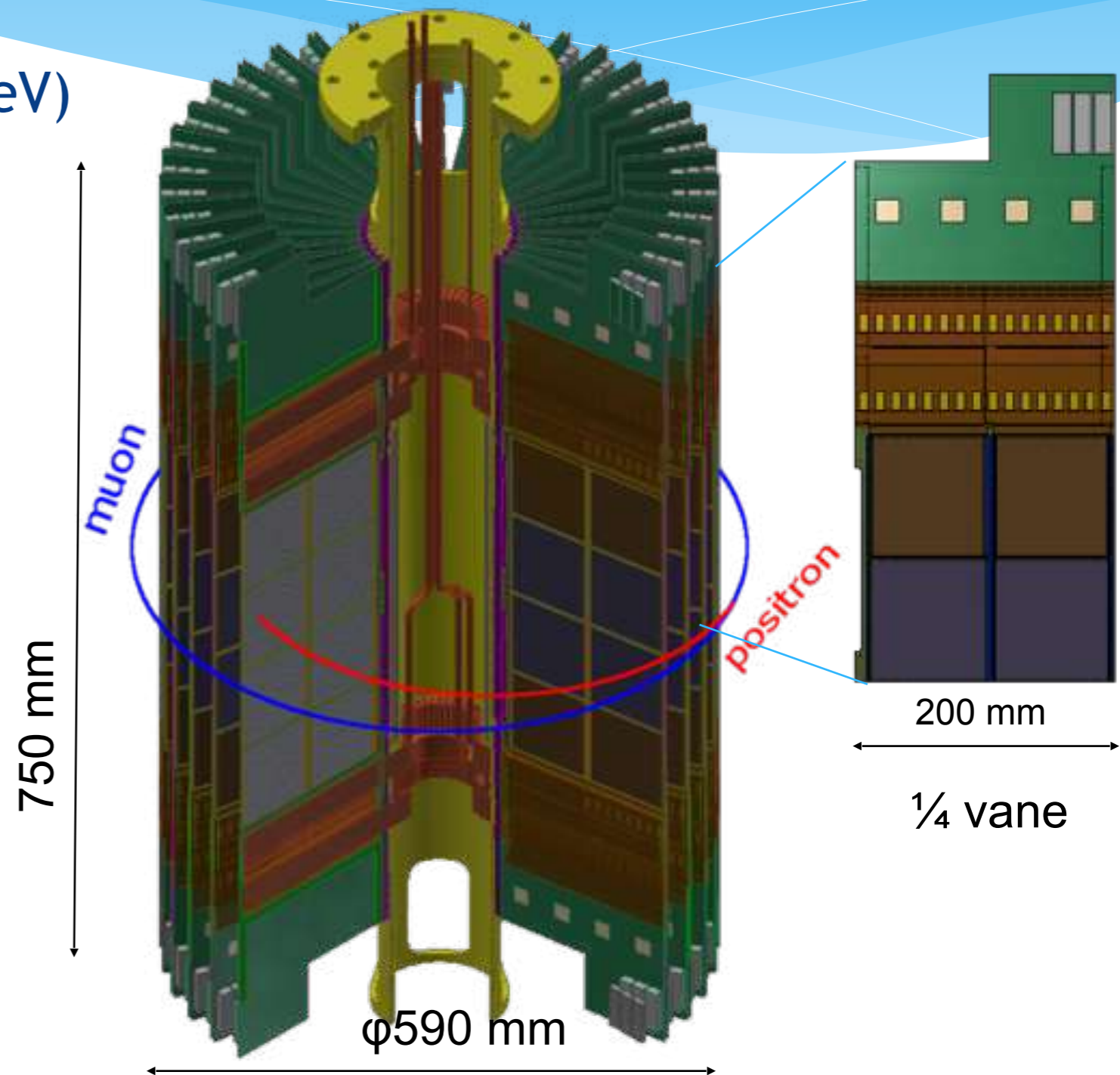
Positron tracking detector

* Requirements

- * Detection of e^+ ($100 < E < 300$ MeV)
- * Reconstruction of momentum vector
- * Stability over rate changes
(1.4 MHz \rightarrow 14 kHz)

* Specifications

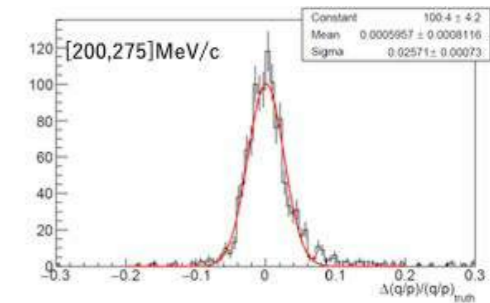
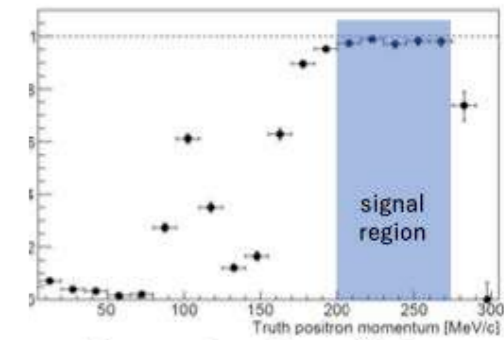
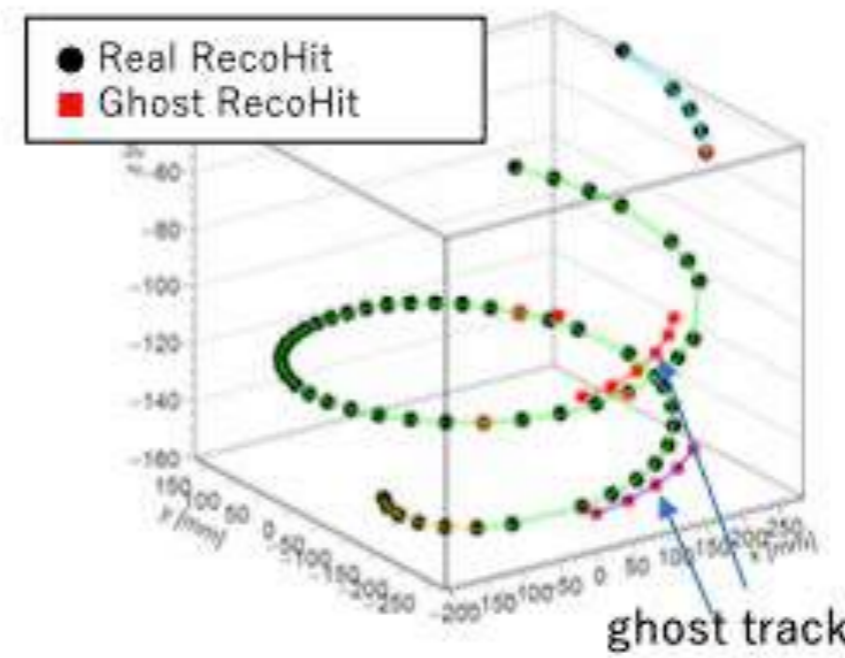
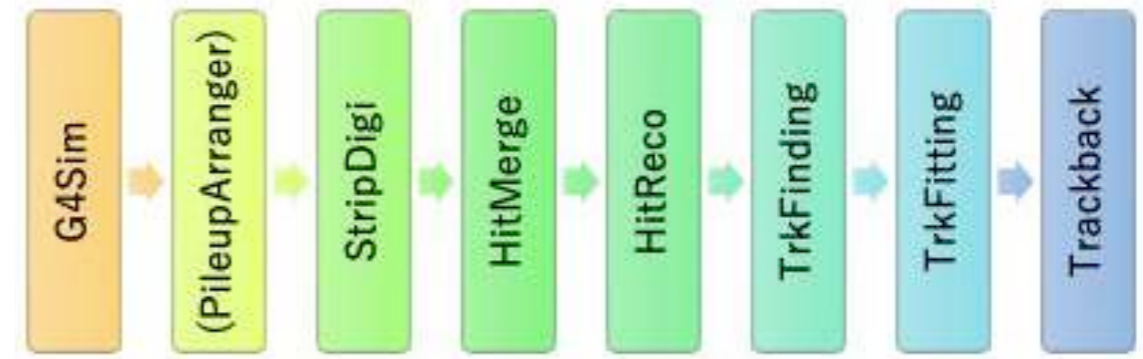
- * Sensor: p-on-n single-sided strip
- * Number of vanes: 40
- * Number of sensors : 640
- * Number of strips : 655,360
- * Area of sensors : 6.24 m²



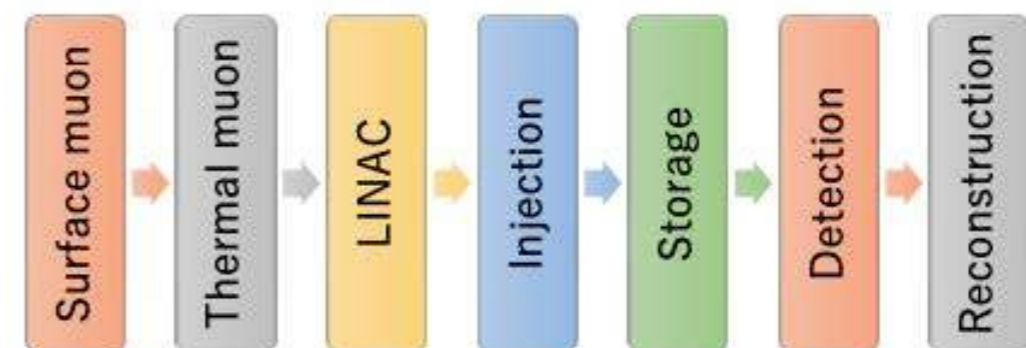
Software

- **Positron track reconstruction**
 - A custom software framework “g2esoft” has been developed.
 - Developed improved tracking performance by connecting disconnected-tracklets, removing ghosts.
 - Evaluating rate-dependent effects in tracking performance.
- **End-to-end simulation**
 - Statistics increased from 16k muons to 1M (x100).
 - Analysis in progress to study various correlations.
 - Optimization of experimental conditions is in progress.

overview of g2esoft



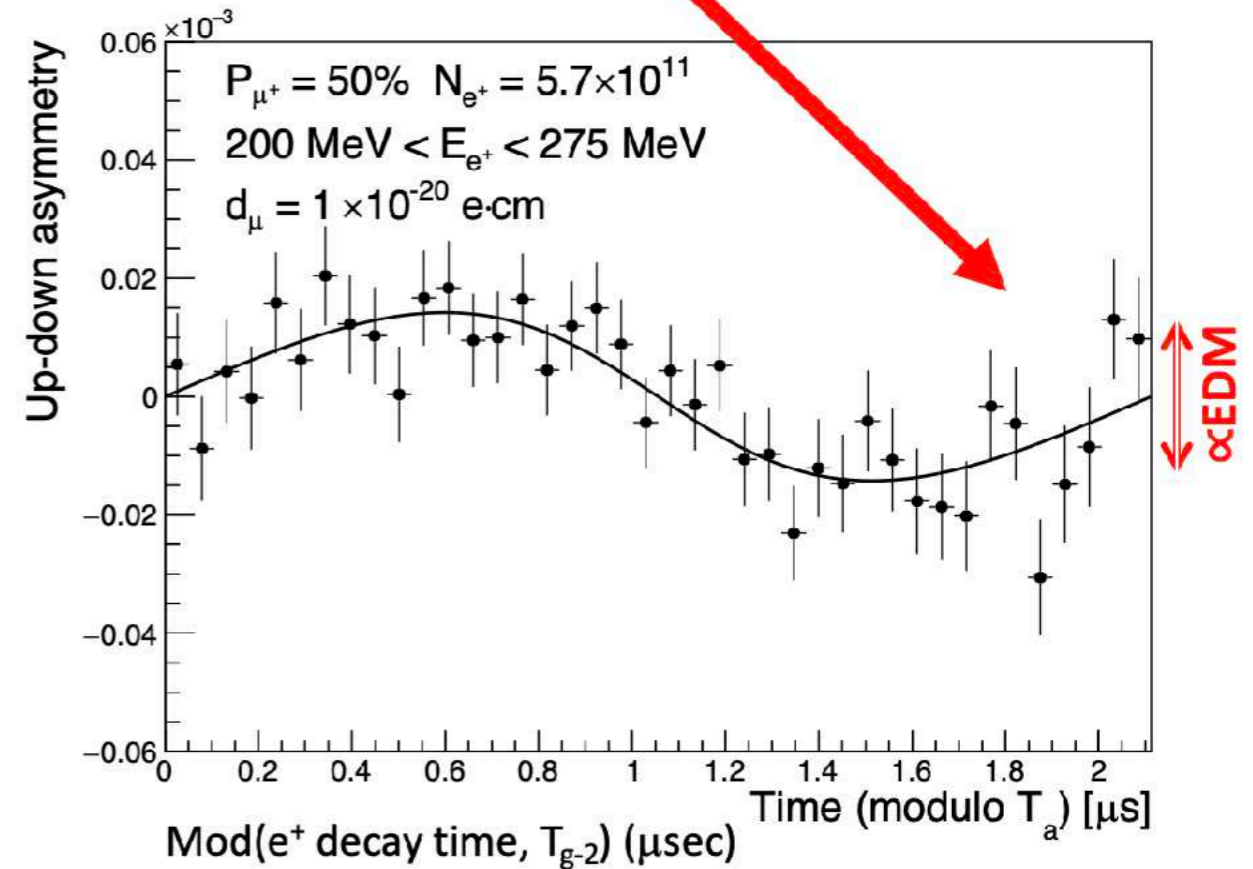
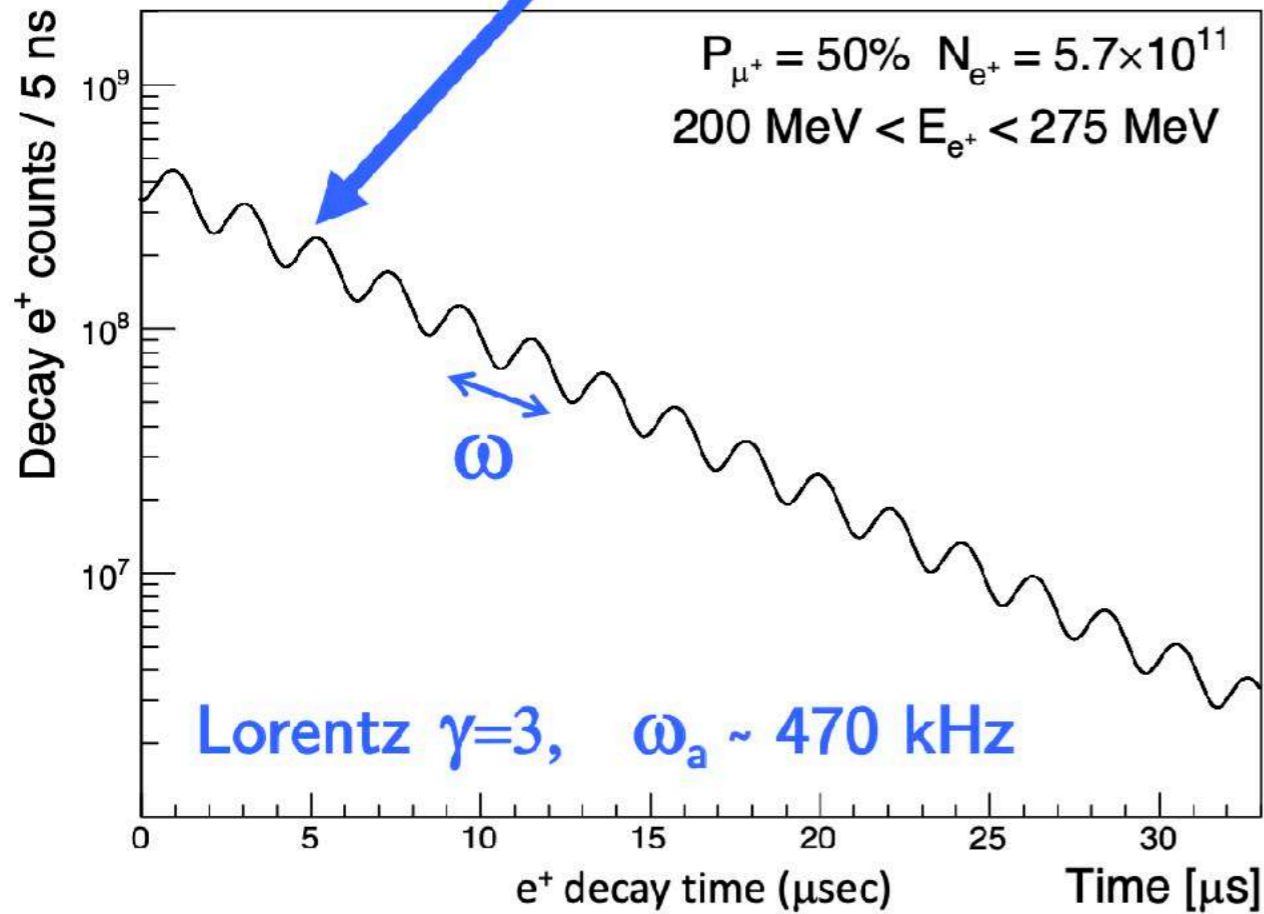
overview of end-to-end simulation



Expected results

$$\vec{\omega} = -\frac{e}{m} \left[a_{\mu} \vec{B} + \frac{\eta}{2} (\vec{\beta} \times \vec{B}) \right]$$

Expected time spectrum of e^+ in $\mu \rightarrow e^+ \nu \bar{\nu}$ decay



Precision estimation & comparison

Completed

Running

In preparation

	BNL-E821	Fermilab-E989	Our Experiment
Muon momentum	3.09 GeV/c		300 MeV/c
Lorentz γ	29.3		3
Polarization	100%		50%
Storage field	$B = 1.45$ T		$B = 3.0$ T
Focusing field	Electric quadrupole		Very weak magnetic
Cyclotron period	149 ns		7.4 ns
Spin precession period	4.37 μ s		2.11 μ s
Number of detected e^+	5.0×10^9	1.6×10^{11}	5.7×10^{11}
Number of detected e^-	3.6×10^9	—	—
a_μ precision (stat.)	460 ppb	100 ppb	450 ppb (Phase-0)
(syst.)	280 ppb	100 ppb	<70 ppb
EDM precision (stat.)	0.2×10^{-19} e · cm	—	1.5×10^{-21} e · cm
(syst.)	0.9×10^{-19} e · cm	—	0.36×10^{-21} e · cm

Corrections, Uncertainties on g-2

Fermilab Run1

PRL 126, 141801 (2021)

Quantity	Correction Terms	Uncertainty	Fermilab goal	J-PARC estimate
	(ppb)	(ppb)		
ω_a^m (statistical)	スピン歳差運動 統計誤差	—	100	450
ω_a^m (systematic)	系統誤差	—		
C_e		489	108	70
C_p	スピン歳差運動補正值・系統誤差	180		
C_{ml}		-11		
C_{pa}		-158		
$f_{\text{calib}} \langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle$		—	56	114
B_k	磁場測定 系統誤差	-27		
B_q		-17		
$\mu'_p(34.7^\circ)/\mu_e$		—		
m_μ/m_e	変換に用いる物理量	—		
$g_e/2$		—		
Total systematic		—	100	<70
Total fundamental factors		—		
Totals		544	140	460

21 x BNL

Towards ultimate test of the muon g-2 anomaly

T.Mibe, inspired by K. Jungmann's slide

$$\omega_a = a_\mu \frac{eB}{m}$$

$$B = \frac{\hbar\omega_p}{2\mu_p}$$

MRI

Fermilab E989

J-PARC E34 (our experiment)

g-2

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p}}$$

120 ppb

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_p}{\mu_e}} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

3 ppb 22 ppb 0.3 ppt

MUSEUM(J-PARC)

Mu HFS

Mu-MASS(PSI), **new exp.(J-PARC)**

Mu 1S-2S

muonium

$$\Delta\nu_{1S-HFS} \simeq \frac{16}{3} \alpha^2 R_\infty \frac{\mu_\mu}{\mu_B} \left(1 + \frac{m_e}{m_\mu}\right)^{-3}$$

$$\nu_{34} - \nu_{12} \propto \frac{\mu_\mu}{\mu_p}$$

m_μ

muonium

$$\Delta\nu_{1S2S} \simeq \frac{3\alpha^2}{8h} m_e c^2 \left(1 + \frac{m_e}{m_\mu}\right)^{-1}$$

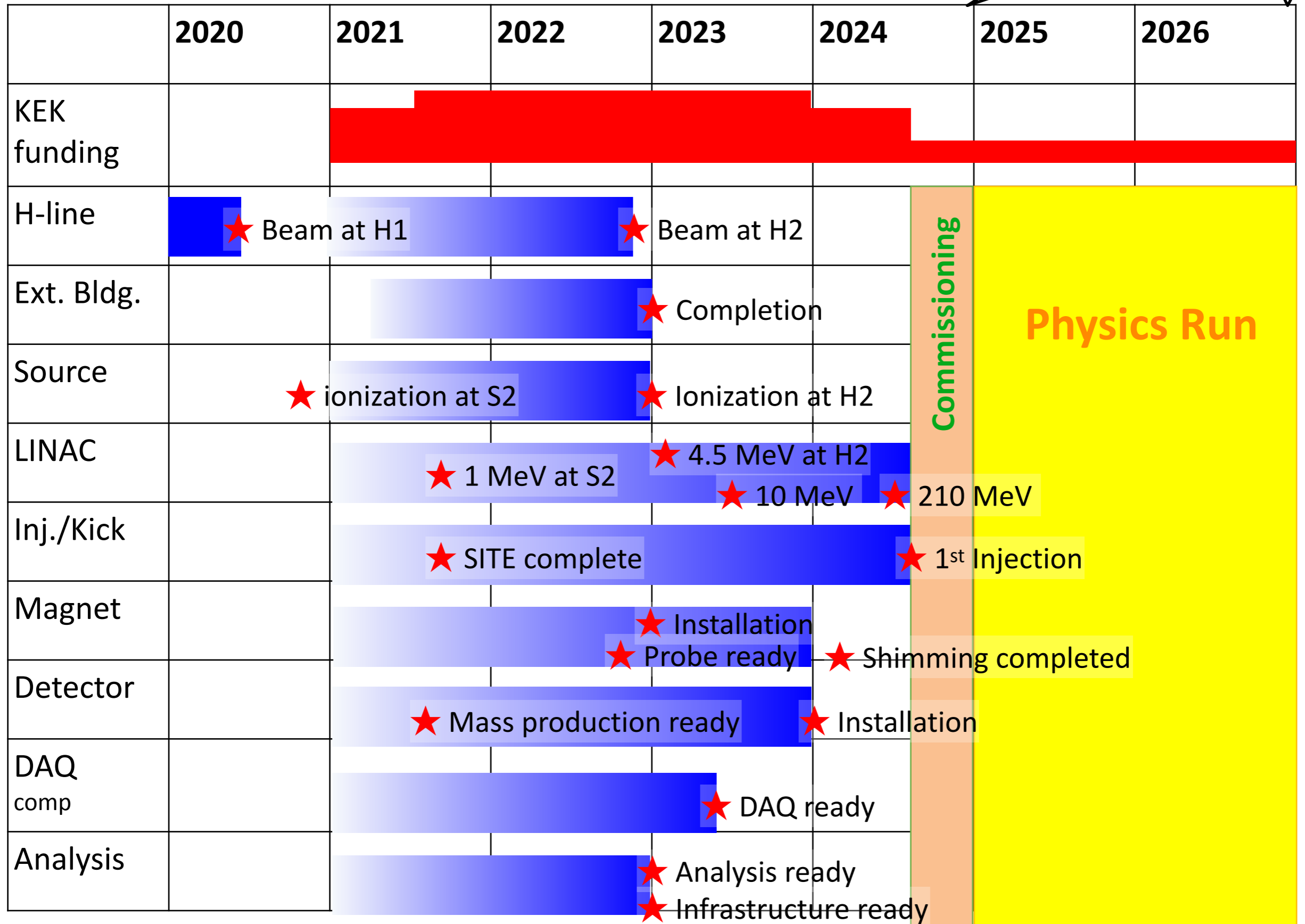
Three quantities are mutually correlated.

Closing a triangle with new experiments will establish ultimate precision.

Schedule and milestones

Physics Run ready

First result



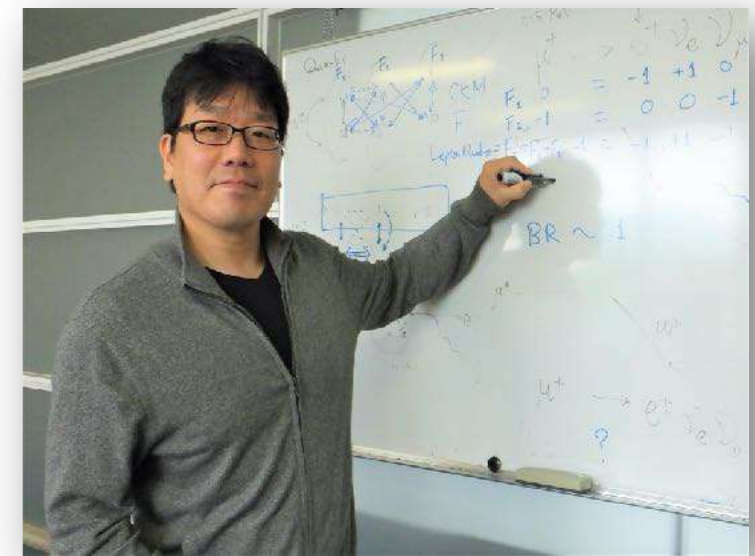
Contact



www.g-2.kek.jp



齊藤 直人 (Naohito Saito)
Initiator of this experiment
naohito.saito@kek.jp



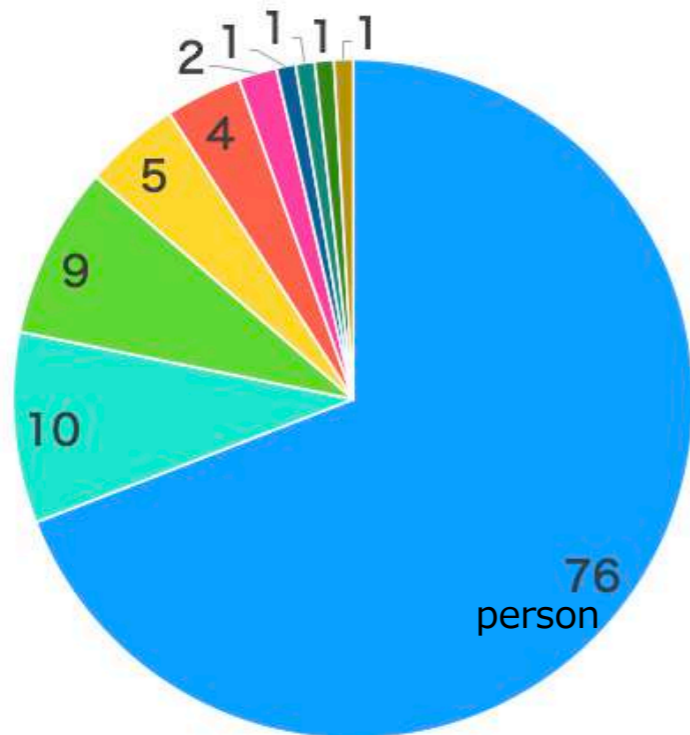
三部 勉 (Tsutomu Mibe)
Current spokesperson
mibe@post.kek.jp

Distribution of human resource

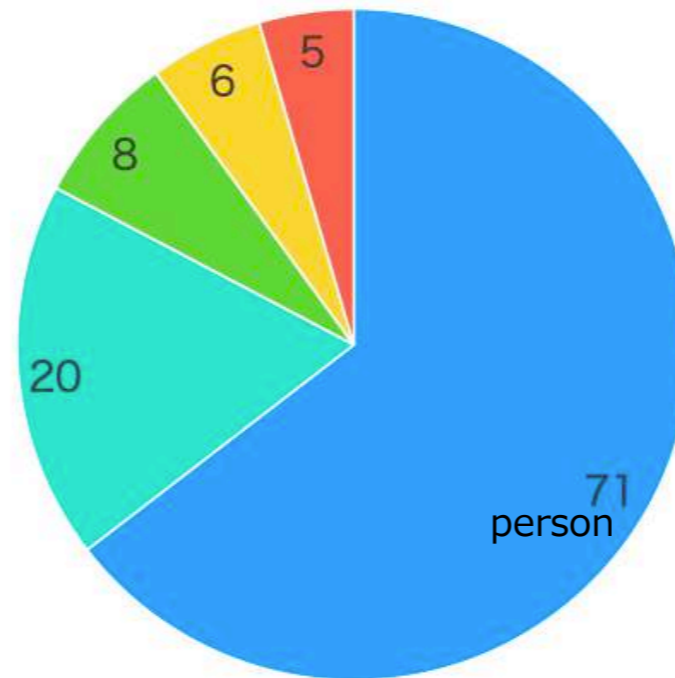
From T. Mibe

110 active collaboration members

Countries



Positions



Area of contribution



- Japan
- Russia
- Canada
- China
- India

- Republic of Korea
- France
- Czech Republic
- Germany
- USA

- Staff scientist
- Emeritus
- Postdoc
- Student
- Engineer

- Detector
- LINAC
- BPM
- Software
- Other
- Muon source
- Beamline
- Injection
- Magnet

Coordinations, field monitors, theory

- 30% from outside of Japan.
- 65% staff scientists, 18% grad. students.
- Distributed over various subsystems

Summary

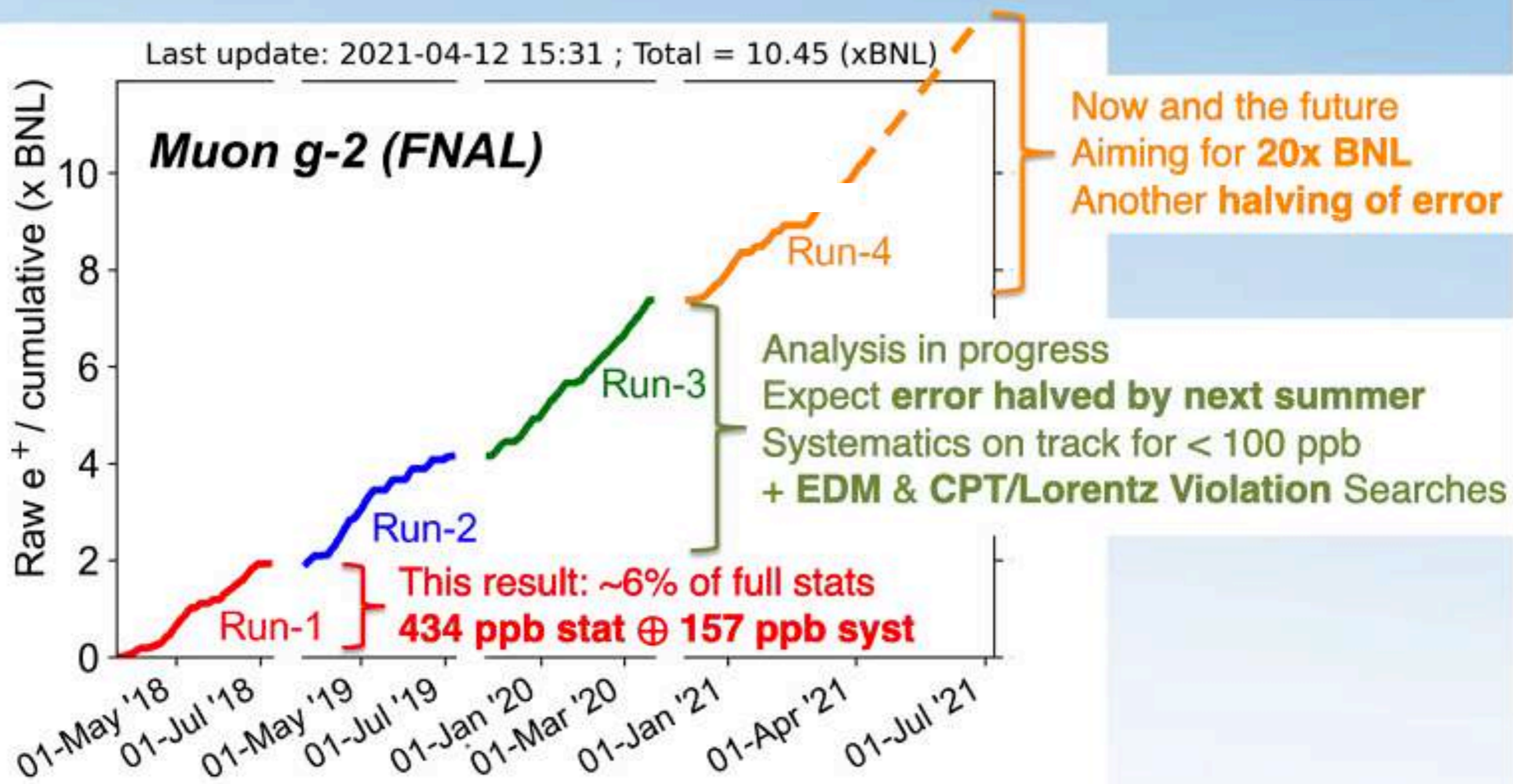
- The recent Muon $g - 2$ result from Fermilab shows 4.2σ deviation between the standard model prediction and the combined measurement.
- A new muon $g-2$ experiment has been proposed at J-PARC with different approach, aiming at final uncertainty of a_μ to be 100 ppb, comparable with FNAL final goal.
- In the new experiment, several novel techniques will be developed, including the re-accelerated thermal muon source, world's first muon RF acceleration, 3D beam injection scheme etc.
- Now, R&D phase is ending and construction phase is starting. Our experiment plan to start the physics run from 2025.

Thank you for your attention!

Backup

Outlook: Muon g – 2

- Future looks bright – there's much more data still to come



实际上的计算公式

$$a_{\mu} = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_{\mu}}{m_e} \frac{g_e}{2}$$

其它更精确的基本常数

$\tilde{\omega}'_p(T)$	Proton Larmor precession frequency in a spherical water sample. Temperature dependence known to < 1 ppb/°C. <i>Metrologia</i> 13 , 179 (1977), <i>Metrologia</i> 51 , 54 (2014), <i>Metrologia</i> 20 , 81 (1984)
$\frac{\mu_e(H)}{\mu_e}$	Measured to 10.5 ppb accuracy at T = 34.7°C
$\frac{\mu'_p(T)}{\mu_e(H)}$	<i>Metrologia</i> 13 , 179 (1977)
$\frac{\mu_e}{\mu_e(H)}$	Bound-state QED (exact)
$\frac{m_{\mu}}{m_e}$	Known to 22 ppb from muonium hyperfine splitting
$\frac{g_e}{2}$	Measured to 0.28 ppt
	<i>Phys. Rev. A</i> 83 , 052122 (2011)

束流动力学修正

$$R'_{\mu} = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{calib} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

Corrections due to beam dynamics

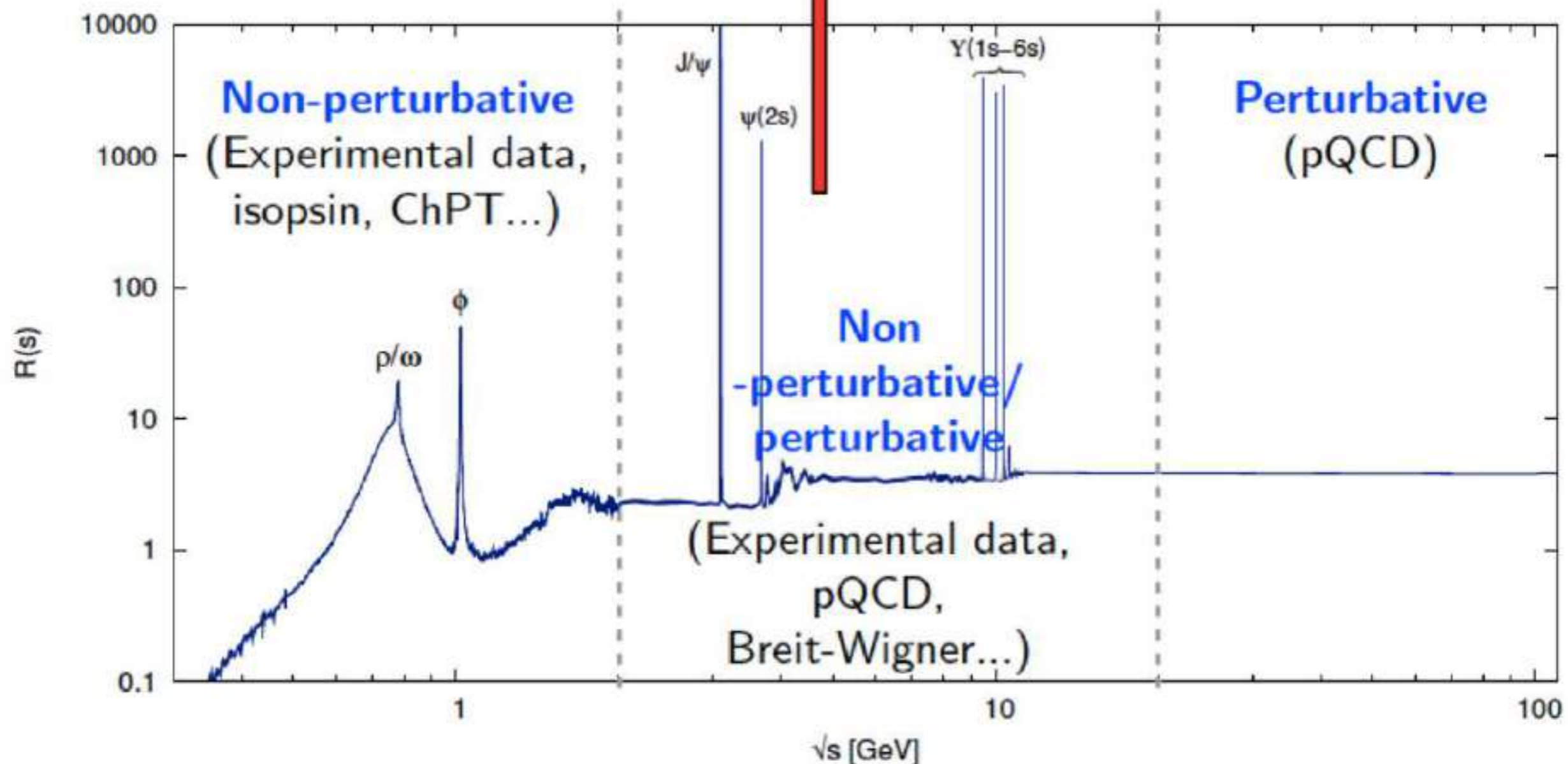
瞬态场修正

Corrections due to transient magnetic fields

$\tilde{\omega}'_p(T)$ = Proton Larmor precession frequency in a spherical water sample weighted for muon distribution

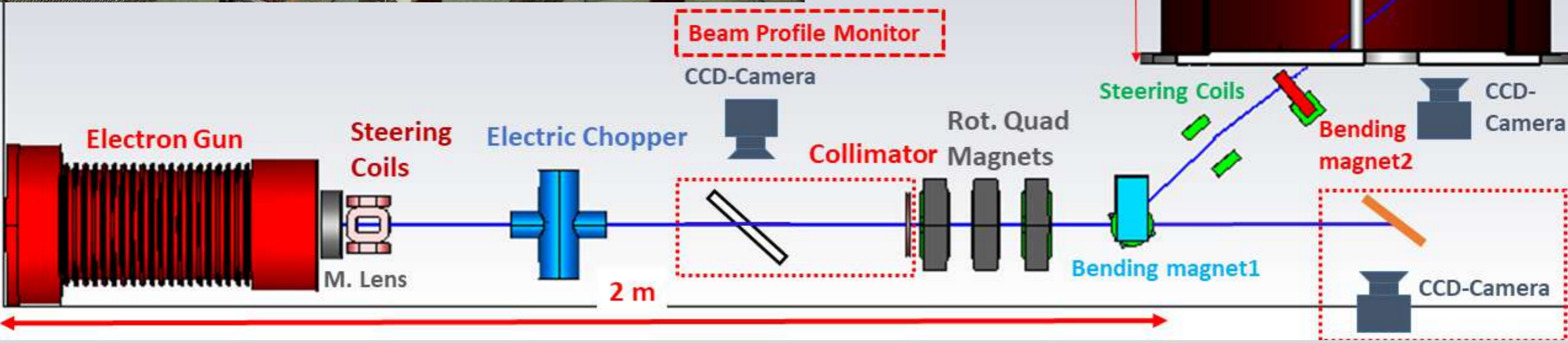
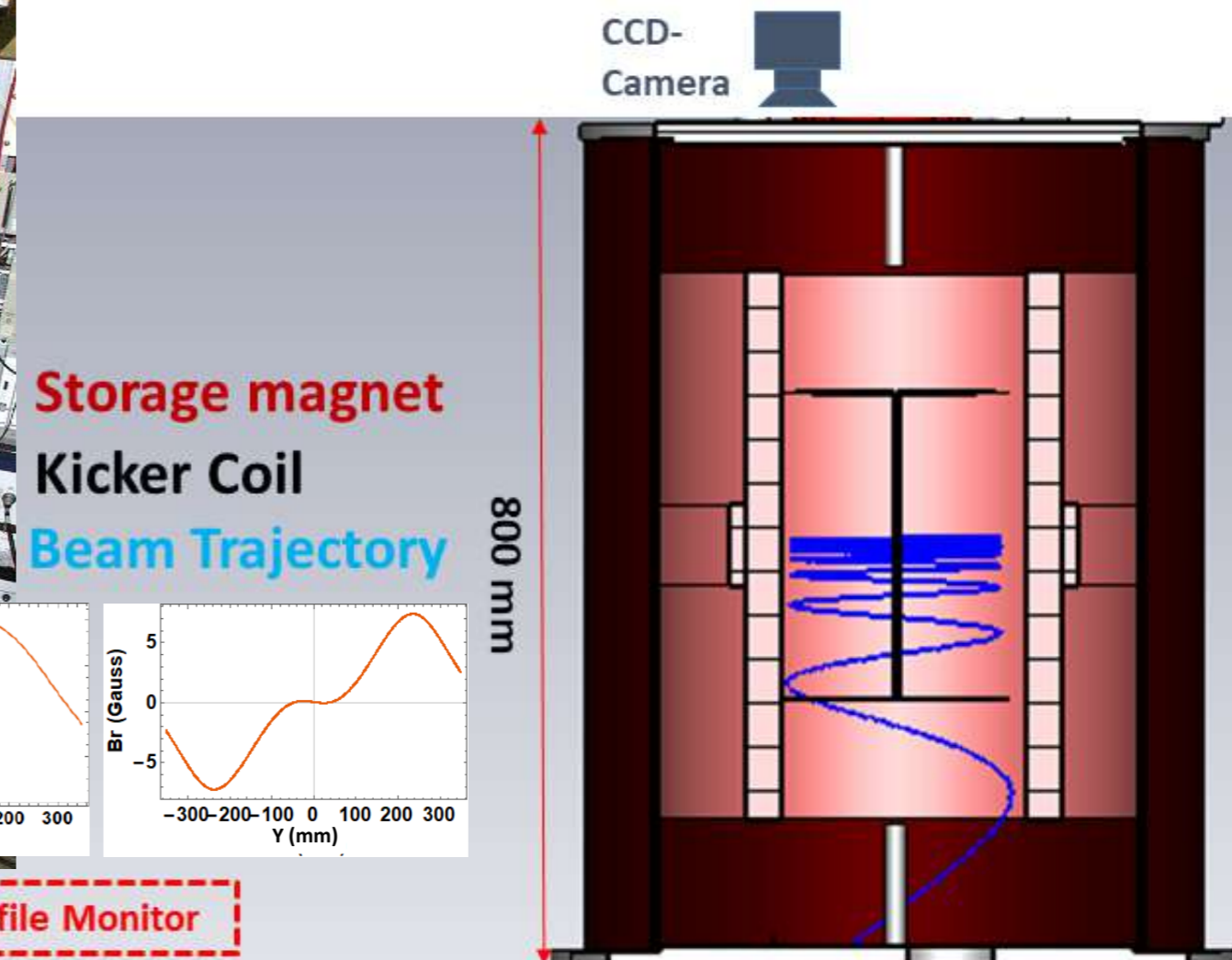
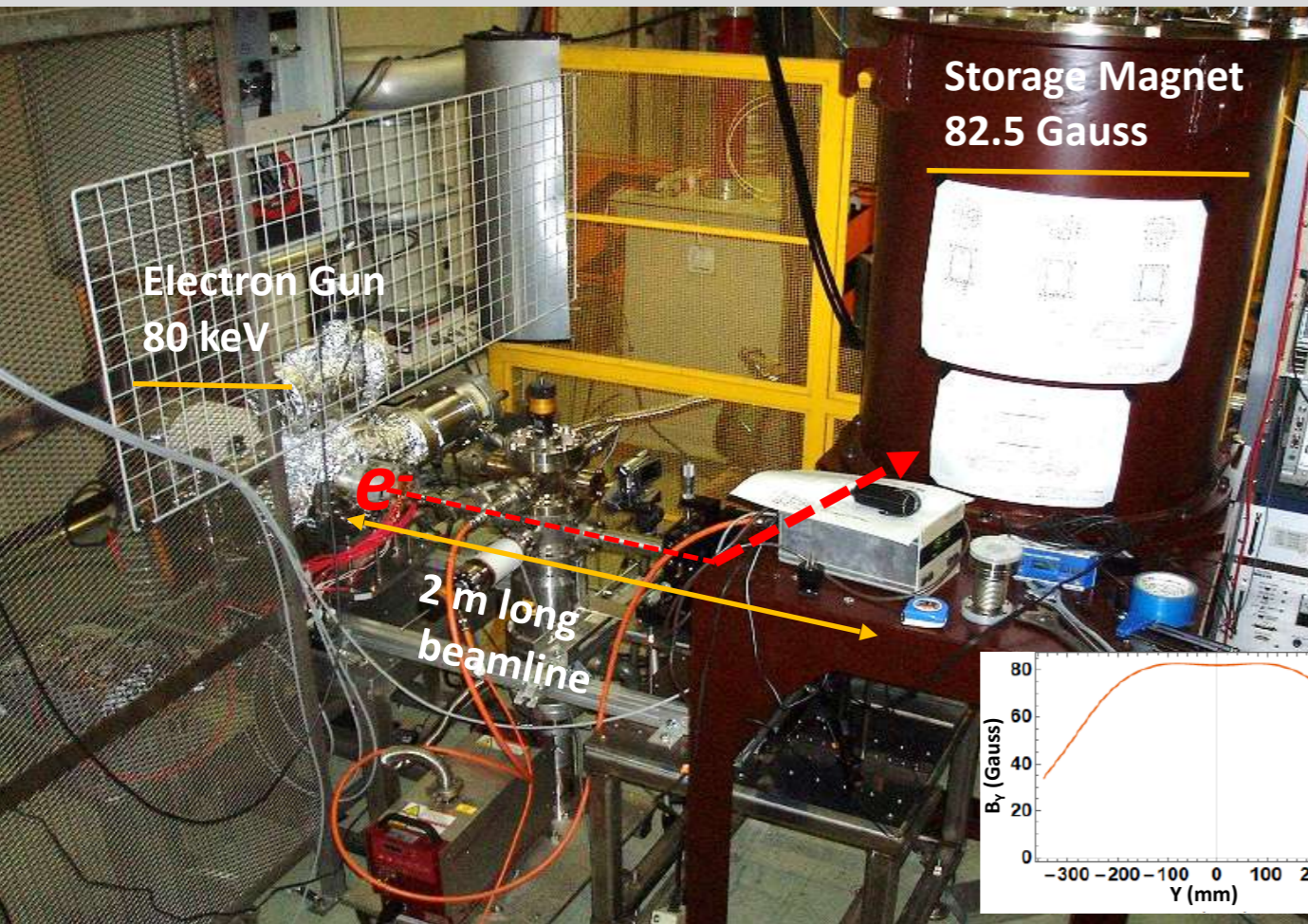
Building the hadronic R -ratio

$$a_{\mu}^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} R(s) K(s), \text{ where } R(s) = \frac{\sigma_{\text{had},\gamma}^0(s)}{4\pi\alpha^2/3s}$$



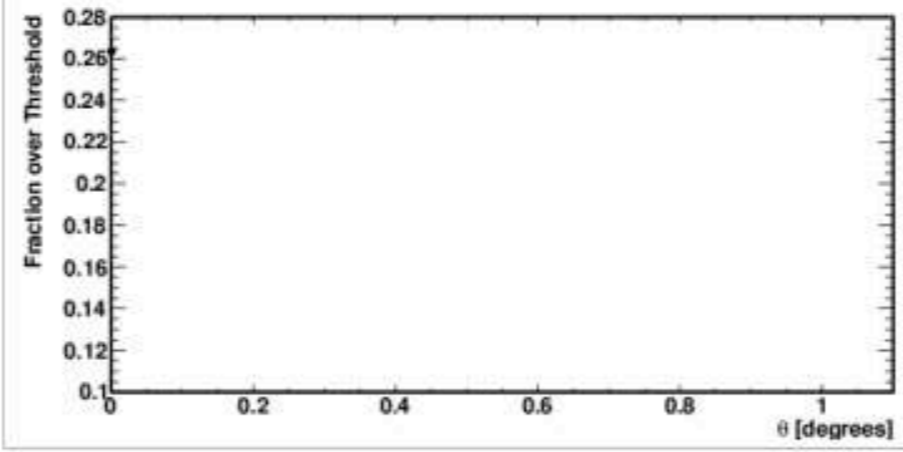
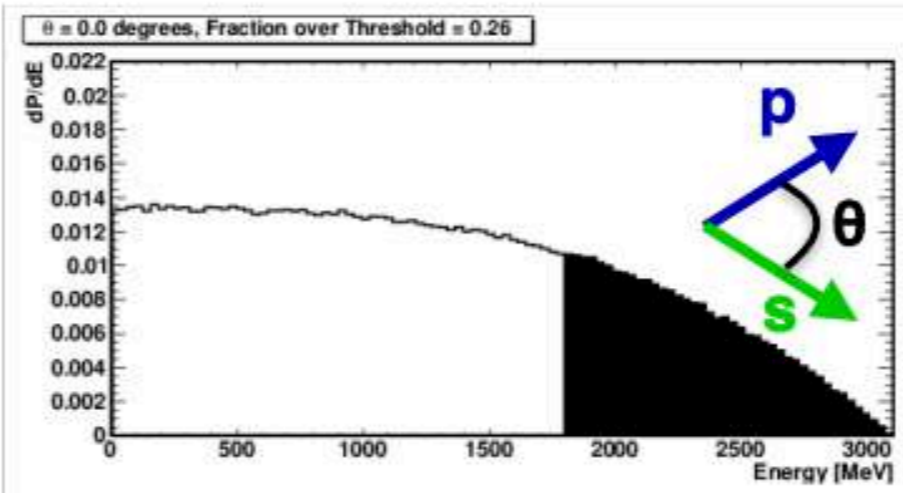
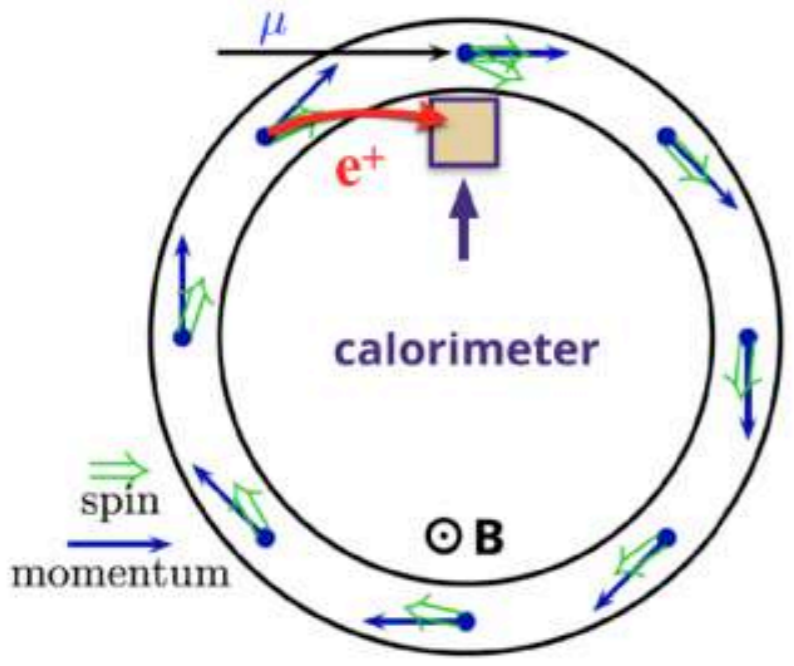
Spiral Injection Test Experiment (SITE)

Spiral Injection Test Experiment Setup at KEK Tsukuba Campus



Frequency extraction: fitting the modulation

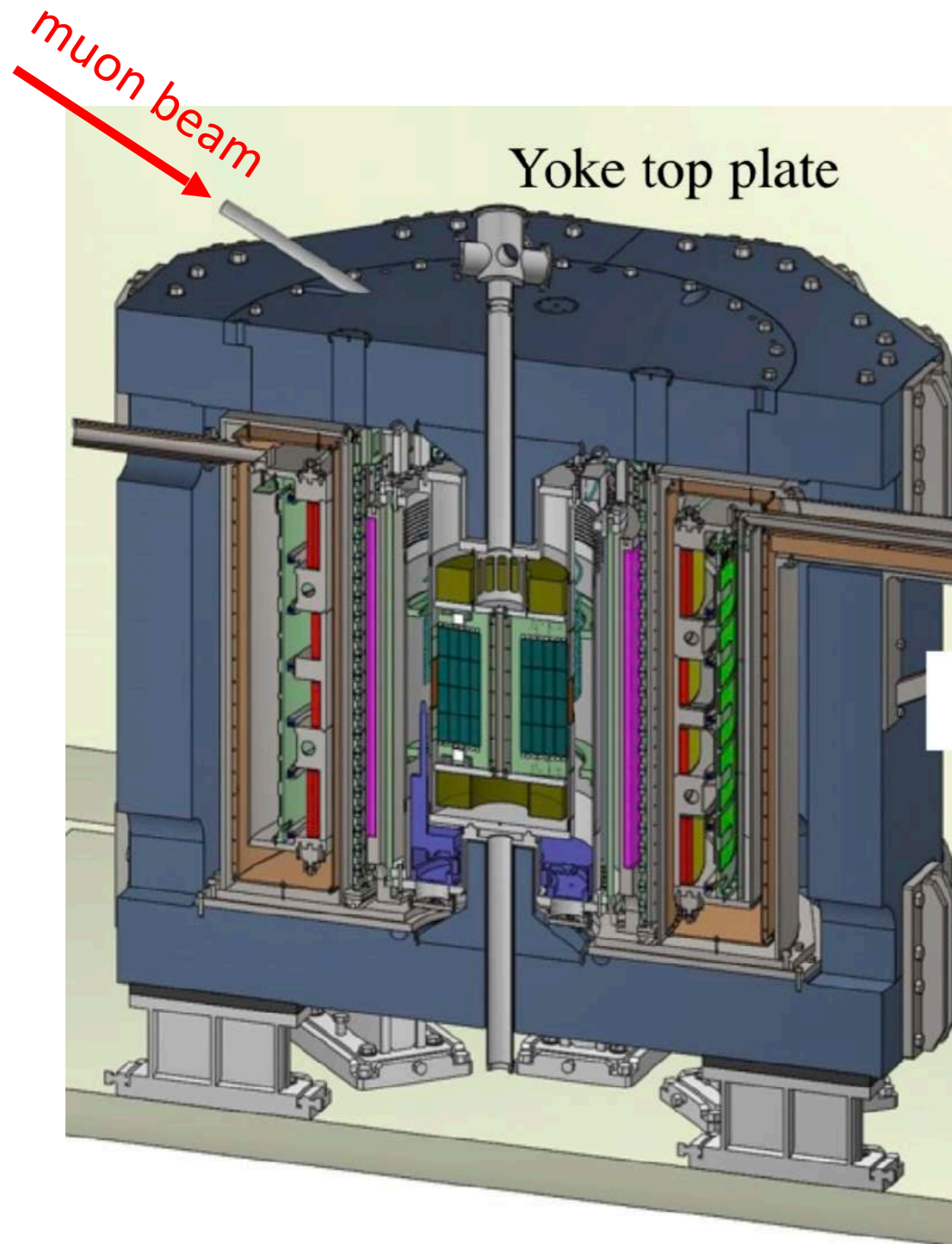
$$\omega_a = a_\mu \frac{eB}{m}$$



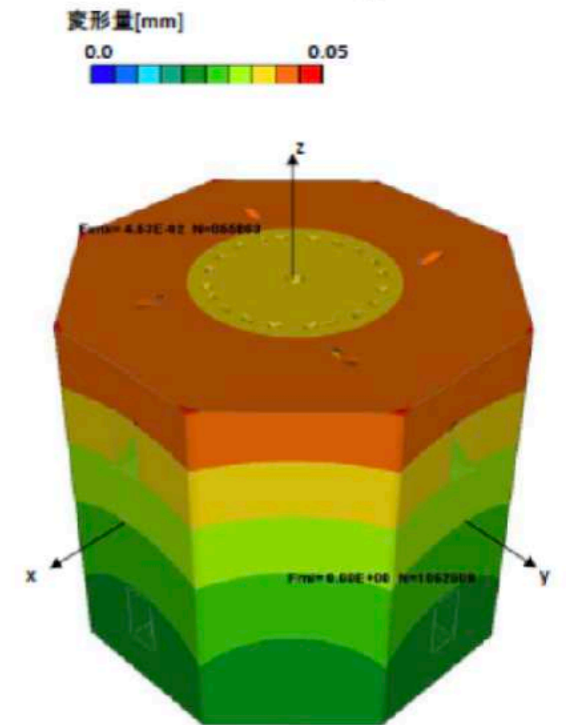
$$N(t) = N_0 e^{-t/\tau} \left[1 + A_\mu \cos(\omega_a t + \phi) \right]$$



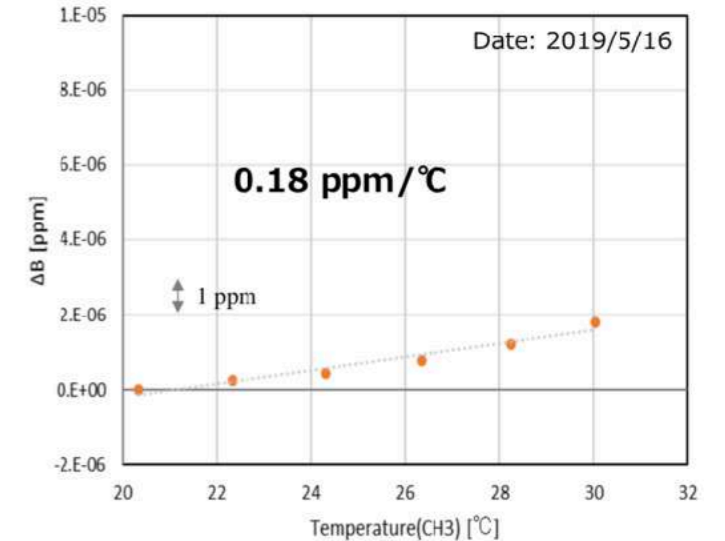
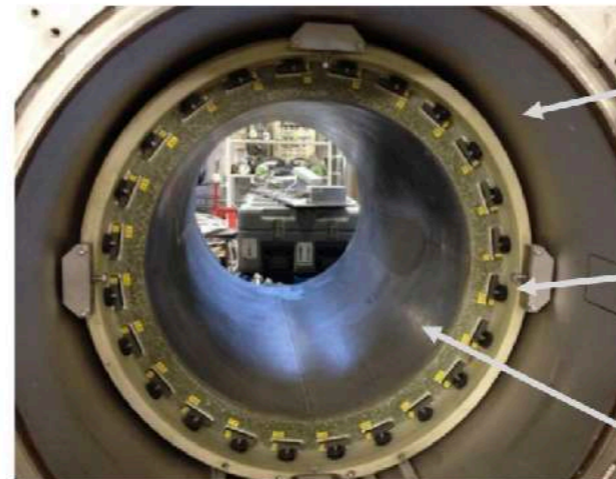
Storage magnet and field measurement



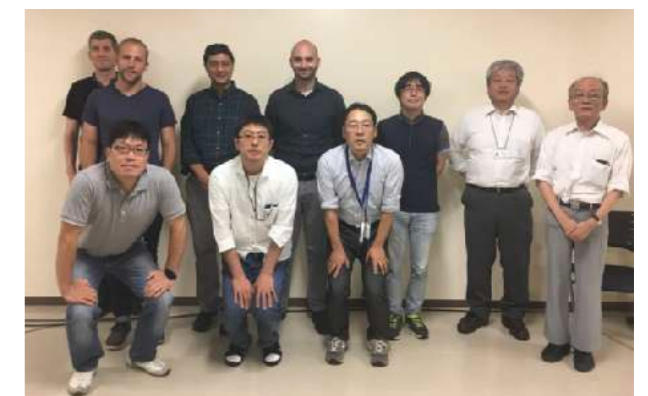
Thermal expansion of yoke has been simulated.



Temperature effect on B-field was evaluated by using a test magnet



US-Japan collaboration on cross calibration of B-field probes. Had a collaboration meeting at J-PARC, Sep 2019.



End-to-end simulations

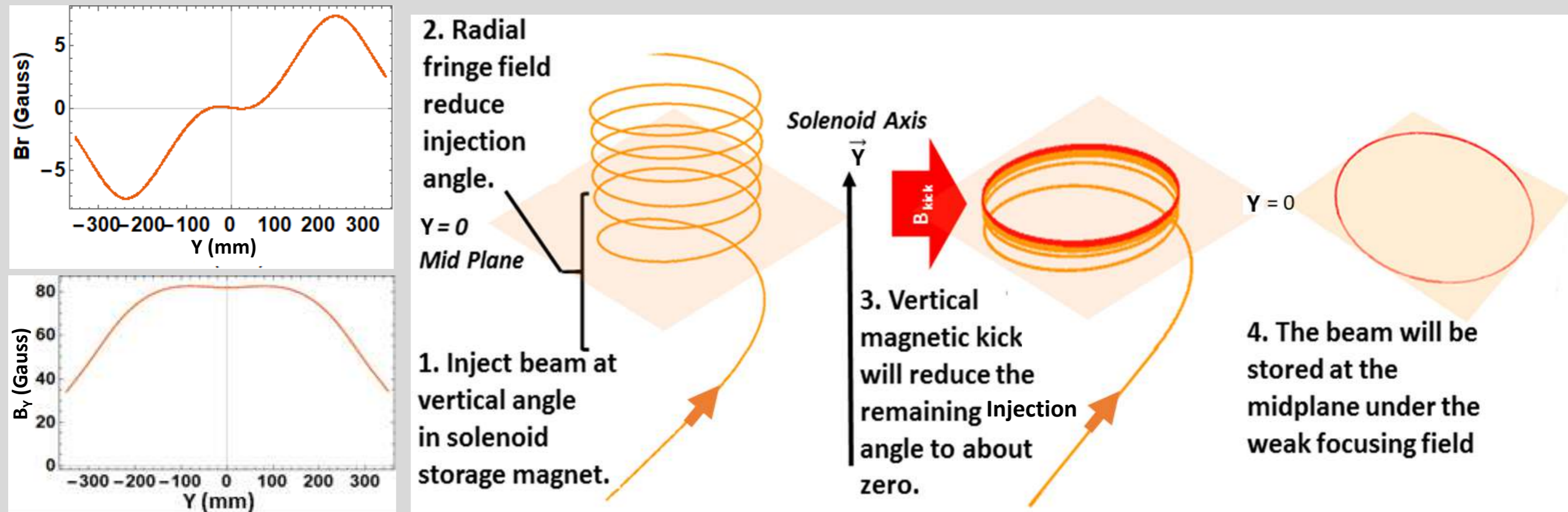
Current TDR (2017)

Table 14.1: Efficiency and beam intensity

Quantity	Reference	Efficiency	Cumulative	Intensity (Hz)
Muon intensity at production target	[2]			1.99E+09
H-line transmission	[2]	1.62E-01	1.62E-01	3.22E+08
Mu emission	[3]	3.82E-03	6.17E-04	1.23E+06
Laser ionization	[4]	7.30E-01	4.50E-04	8.97E+05
Metal mesh	[5]	7.76E-01	3.49E-04	6.96E+05
Init.Acc.trans.+decay	[5]	7.18E-01	2.51E-04	5.00E+05
RFQ transmission	[6]	9.45E-01	2.37E-04	4.72E+05
RFQ decay	[6]	8.13E-01	1.93E-04	3.84E+05
IH transmission	[7]	9.87E-01	1.90E-04	3.79E+05
IH decay	[7]	9.89E-01	1.88E-04	3.75E+05
DAW transmission	[8]	9.95E-01	1.87E-04	3.73E+05
DAW decay	[8]	9.61E-01	1.80E-04	3.58E+05
High beta transmission	[9]	1.00E+00	1.80E-04	3.58E+05
High beta decay	[9]	9.88E-01	1.78E-04	3.54E+05
Injection transmission	[10]	8.5E-01	1.42E-04	2.83E+05
Injection decay	[10]	9.9E-01	1.41E-04	2.80E+05
Detector start time	[10]	9.27E-01	1.30E-04	2.60E+05
Muon at storage				2.60E+05

How to inject beam spirally?

To resolve technical challenges a new 3D Spiral Injection scheme has been invented



The Elegance and Advantages

- Smooth connection between injection and storage sections: No need of Inflector
- All in one storage magnet, which reduce source of error fields: No Quad
- No need to kick within a single turn: Simple kicker

However

- Unprecedented

Therefore, it is indispensable to prove the feasibility of this new scheme.

Experimental determination of $a_\mu = (g-2)/2$

$$a_\mu = (g-2)/2$$

ω_a : anomalous spin precession frequency

ω_p : proton's Lamor precession frequency

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

Present : 0.54ppm (BNL) \rightarrow 140ppb (FNAL/J-PARC goal)

(\rightarrow 400ppb (J-PARC phase-I))

Present uncertainty \sim 0.12ppm (LANL) \rightarrow 60ppb (MuSEUM goal)