On determination of
the fine-structure constant:
Electron g-2 and Atom interferometers

Makiko Nio (RIKEN)

December 9, 2020
Muon g-2/EDM CM21

QED part w/ T. Aoyama(KEK), M. Hayakawa (Nagoya U),
A. Hirayama(Saitama U), T. Kinoshita(Cornell U, UMass Amherst)

New value of α

Article | Published: 02 December 2020

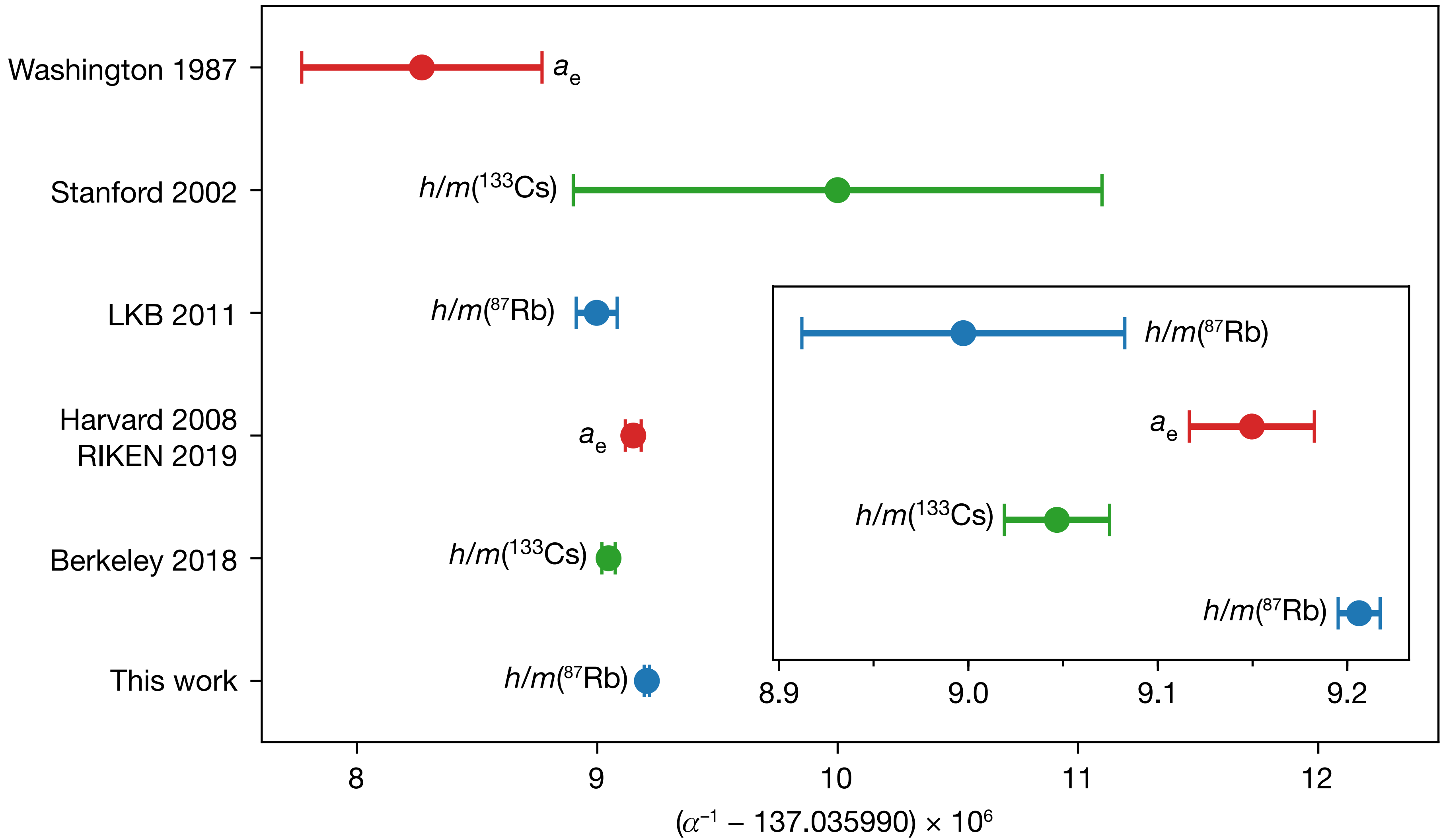
Determination of the fine-structure constant with an accuracy of 81 parts per trillion

Léo Morel, Zhibin Yao, Pierre Cladé & Saïda Guellati-Khélifa

Nature **588**, 61–65(2020) | Cite this article

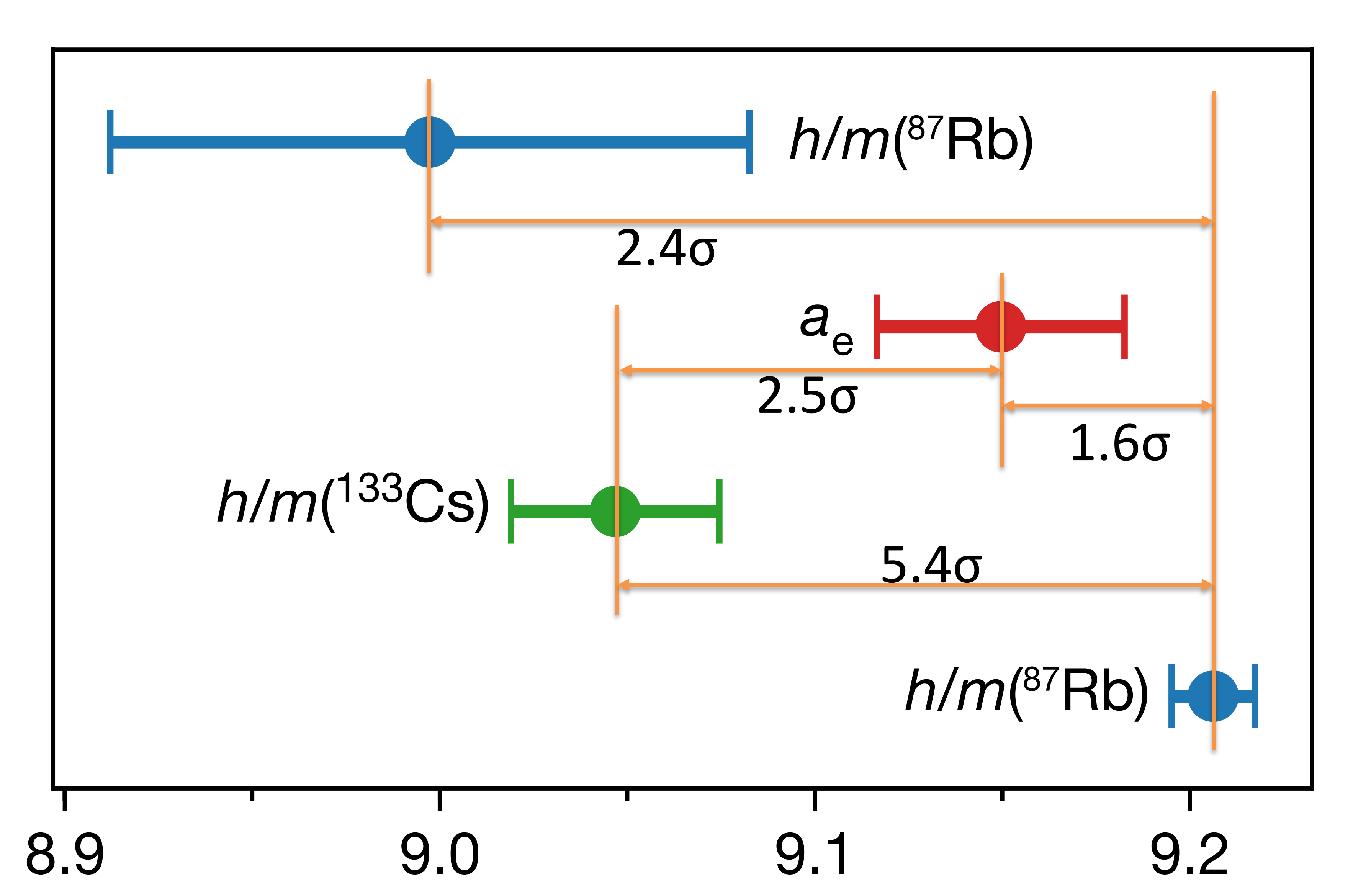
Three ways to determine α

L. Morel et al. 2020



Best values of α

L. Morel et al. 2020



Plan of my talk

- What is the fine-structure constant α?

- Atom interferometers and derived α

  Berkeley Cs and LKB-in-Paris Rb

- Electron g-2 and derived α

  Possible improvements in experiment and theory

- Comparison of α and electron g-2

The fine-structure constant α =1/137.03 …

Explanation

strength of electro-magnetic interaction

a dimensionless constant

named after the fine structure of the hydrogen atom spectral lines

by A. Sommerfeld in 1916

The definition in SI units:

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$$

In natural units

$$\alpha = \frac{e^2}{4\pi}$$

		Old SI before 2019	New SI after 2019
e	elementary charge	derived	$1.602\ 176\ 634 \times 10^{-19}\mathrm{C}$ exact
c	sleep of light in vacuum	$299\ 792\ 458\mathrm{m/s}$ exact	exact, unchanged
ϵ_0	electric constant	$1/(\mu_0 c^2),\ \mu_0 = 4\pi \times 10^{-7}\mathrm{N/A}^2$	derived
$h = 2\pi\hbar$	Planck constant	derived	$6.626\ 070\ 15 \times 10^{-34}\mathrm{J}\cdot\mathrm{s}$

determination of α from the H-atom

- Binding energy of Bohr Model

$$E_n = -m_e c^2 \frac{\alpha^2}{2} \frac{1}{n} = -R_\infty hc \frac{1}{n}$$

R_∞ Rydberg constant

- Energy or frequency can be precisely determined
- To obtain α, need to know the precise value of the electron-mass m_e or m_e/h
- precise value of α cannot be directly obtained from the atom spectroscopy only

東北大学物理斎藤研
https://flex.phys.tohoku.ac.jp/~rsaito/spectrum/
撮影は、電気通信大　伊東敏雄先生

Quantum Hall Effect

VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

and

G. Dorda
Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper
Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom
(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.

Experimental discovery of Integer Quantum Hall effect
The authors seem happy with the nice value of α

Quantum Hall resistance R_K

von Klitzing constant

$$R_K = \frac{h}{e^2} = \frac{1}{2\alpha\epsilon_0 c}$$

In old SI, $\epsilon_0, \; c$ are exact.

$$R_K = 25812.68 \pm 0.028 \; \Omega$$
$$\alpha^{-1} = 137.0353 \pm 0.004$$

K. v. Klitzing et al PRL 1980

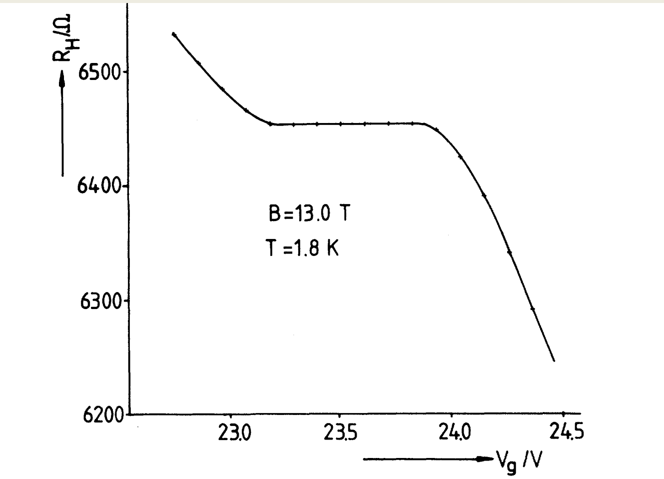


FIG. 2. Hall resistance R_H, and device resistance, R_{pp}, between the potential probes as a function of the gate voltage V_g in a region of gate voltage corresponding to a fully occupied, lowest ($n=0$) Landau level. The plateau in R_H has a value of 6453.3 ± 0.1 Ω. The geometry of the device was $L = 400$ μm, $W = 50$ μm, and $L_{pp} = 130$ μm; $B = 13$ T.

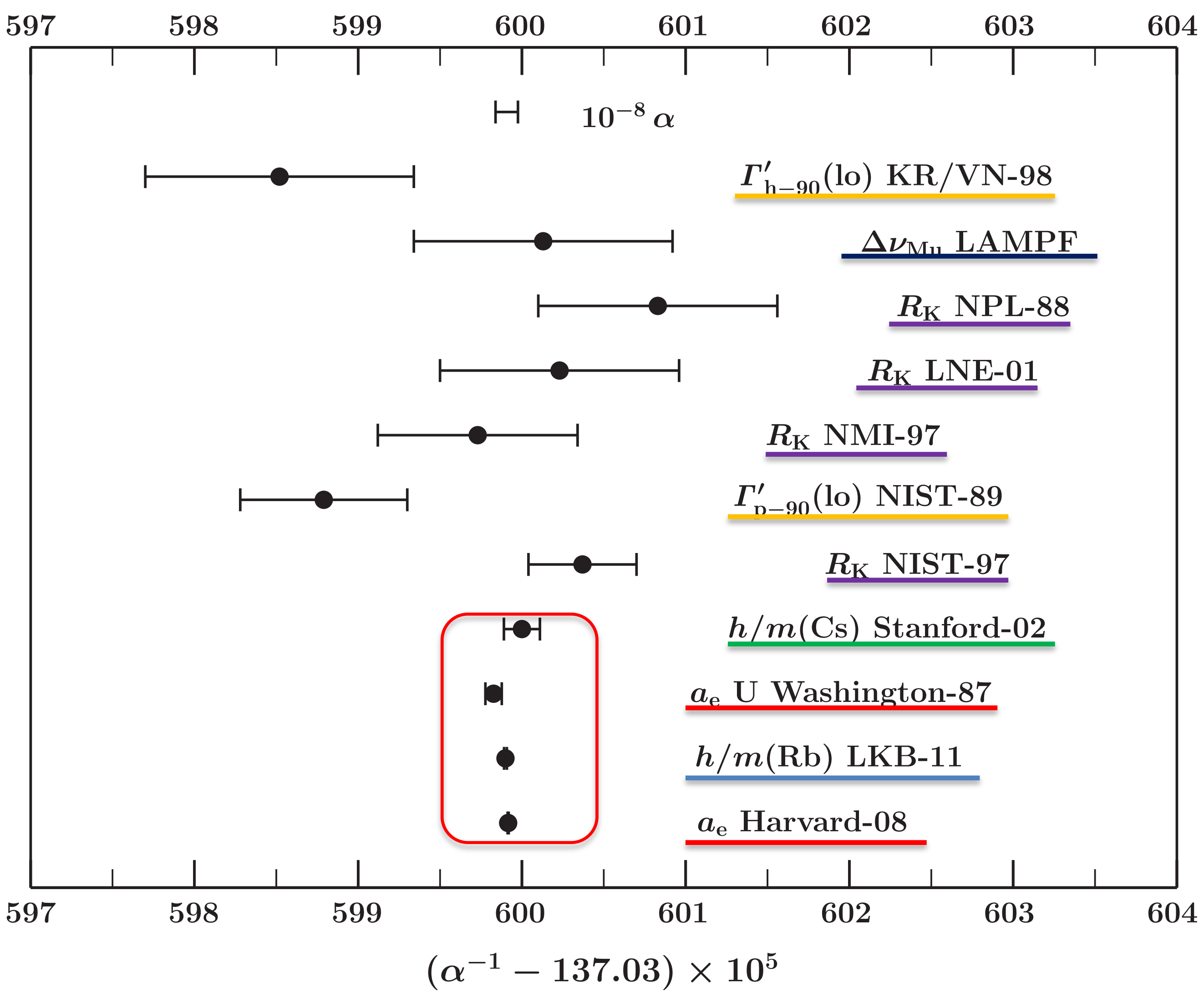
This method is no longer available.

R_K is exact and the standard of resistance in new SI.

$$R_K = 25812.807\;45 \; \Omega$$

Values of α in 2014

CODATA2014, 2016



Proton gyromagnetic ratio

Muonium HFS

Quantum Hall R_K

Cs Interferometer

Rb Interferometer

Electron g-2

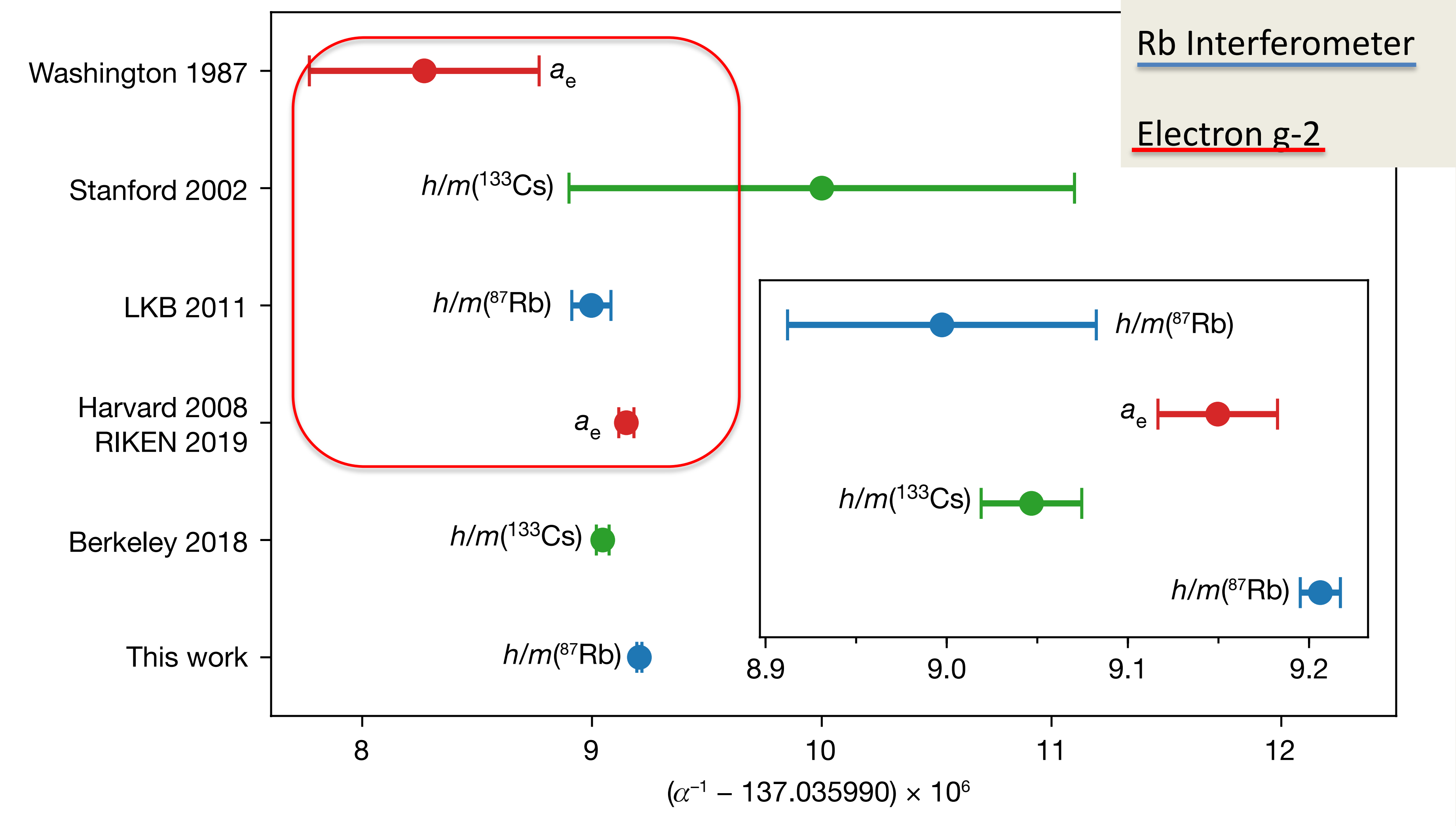
Values of α in 2020

Cs Interferometer

Rb Interferometer

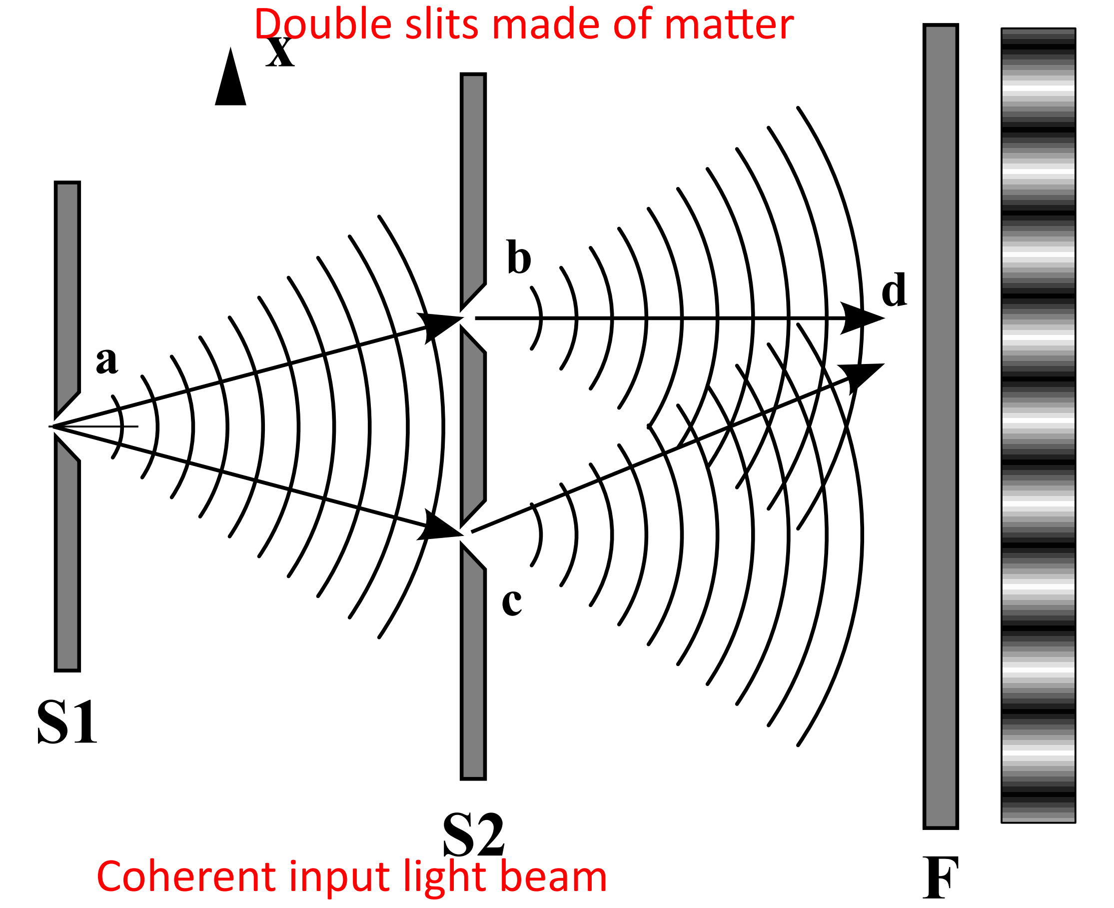
Electron g-2

L. Morel et al. 2020



Light Interferometer

Young's-double slit interferometer of light



Difference in paths makes an interference pattern

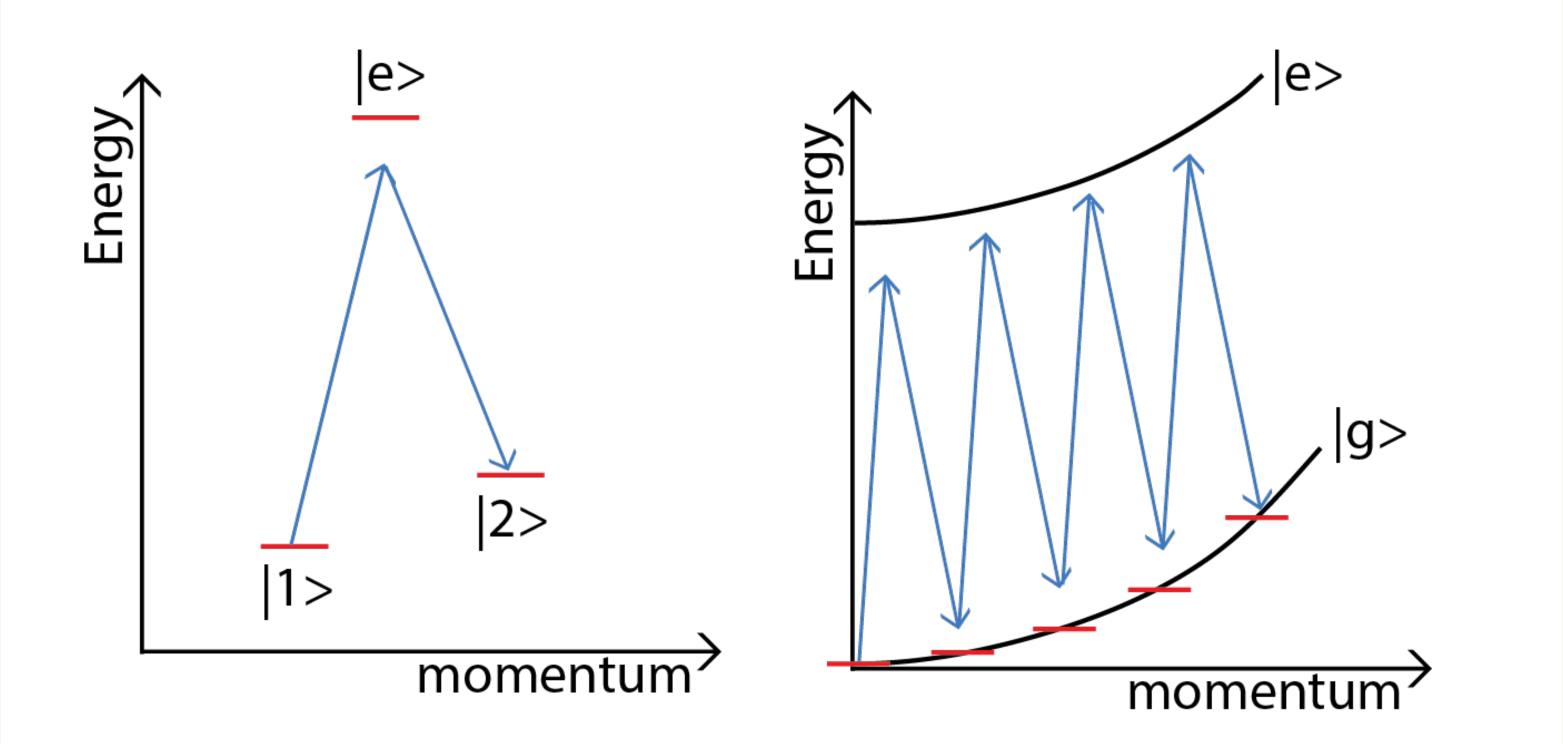
Atom interferometer
light → atom
slits → light

An atom must travel two different paths

Diagram for the double-slit experiment
https://en.wikipedia.org/wiki/Double-slit_experiment#/media/File:Doubleslit.svg

Manipulation of an atom w/ light

Suppose that an atom has two states |1> and |2>

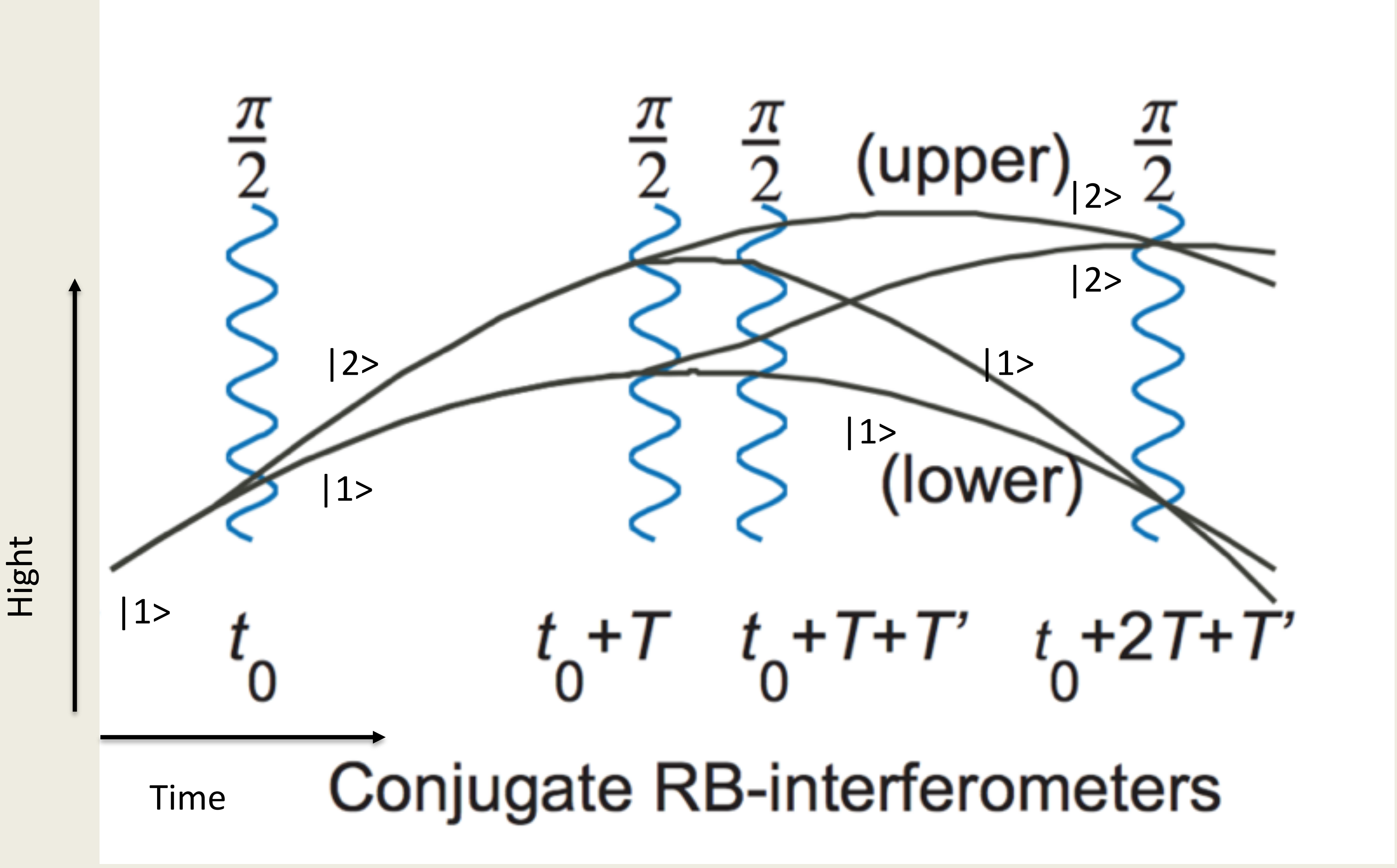


Raman transition Pi/2 pulse laser
Used as a beam splitter

Bloch oscillation of optical lattice
Used to accelerate an atom

Ramsey-Bordé Interferometer

H. Mueller arXiv:1312.6449



Recoil or Doppler shift

An atom goes up from |1> to |2> absorbing a photon (ω).
Then, it comes back from |2> to |1> emitting a photon (ω').

Energy conservation:

$$\omega - \omega' = (\vec{k} + \vec{k}') \cdot \vec{v} + \frac{\hbar}{2M}(\vec{k} + \vec{k}')^2$$

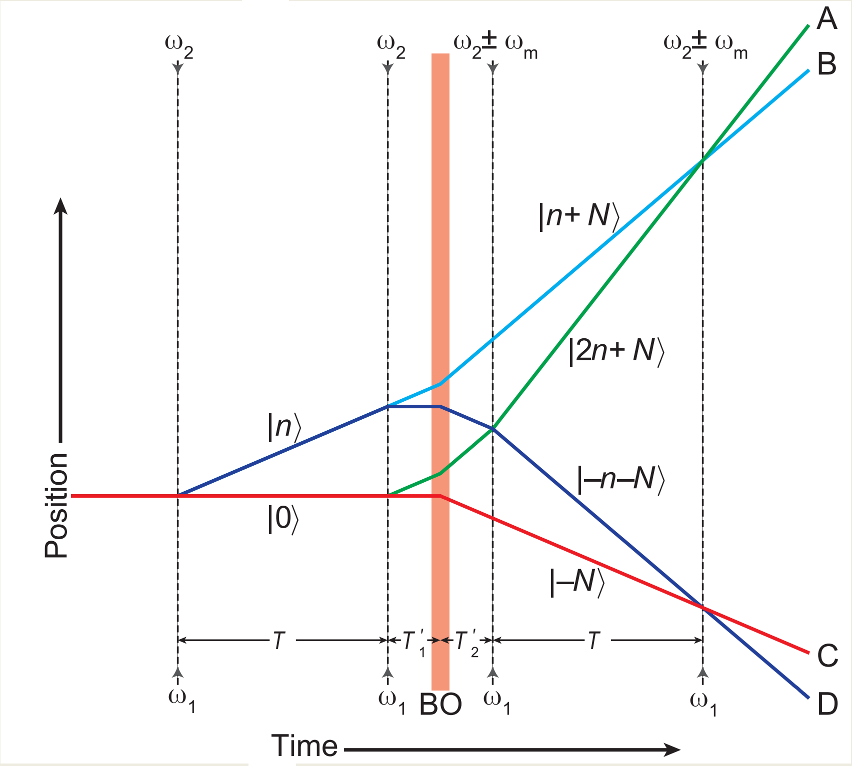
Photon frequencies can be precisely determined.
So, if the velocity of an atom is determined,
h/M can be determined.

S. Chu 2001
Nobel Symposia

Hyperfine level F=3 and F=4 of Cs atom
Hyperfine level F=1 and F=2 of Rb atom will be used.

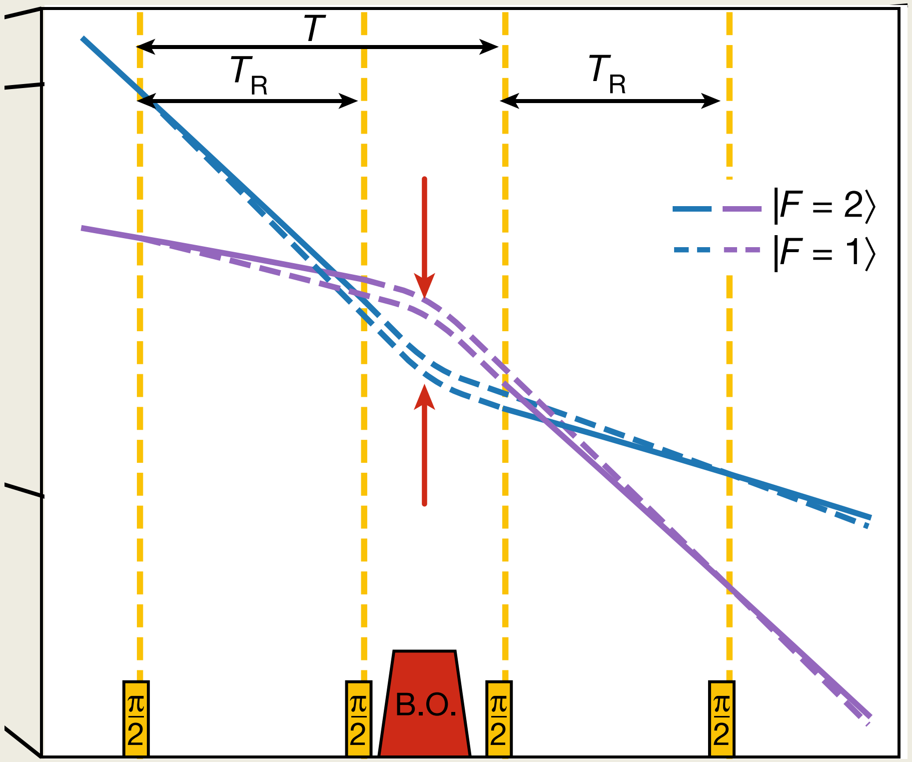
Atom interferometers

h/m(Cs) 2018, Science
Parker et al.



$$\Phi = T[8n(n+N)\frac{\hbar k^2}{m_{\rm Cs}} - 2n\omega_m]$$

h/m(Rb) 2020, Nature
Morel et al.

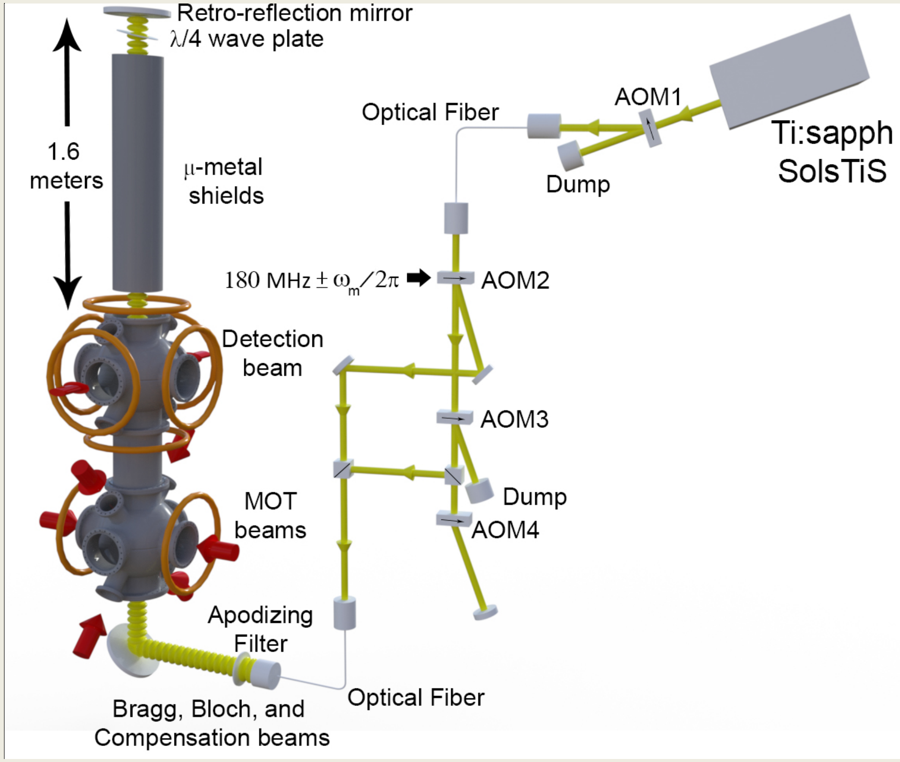


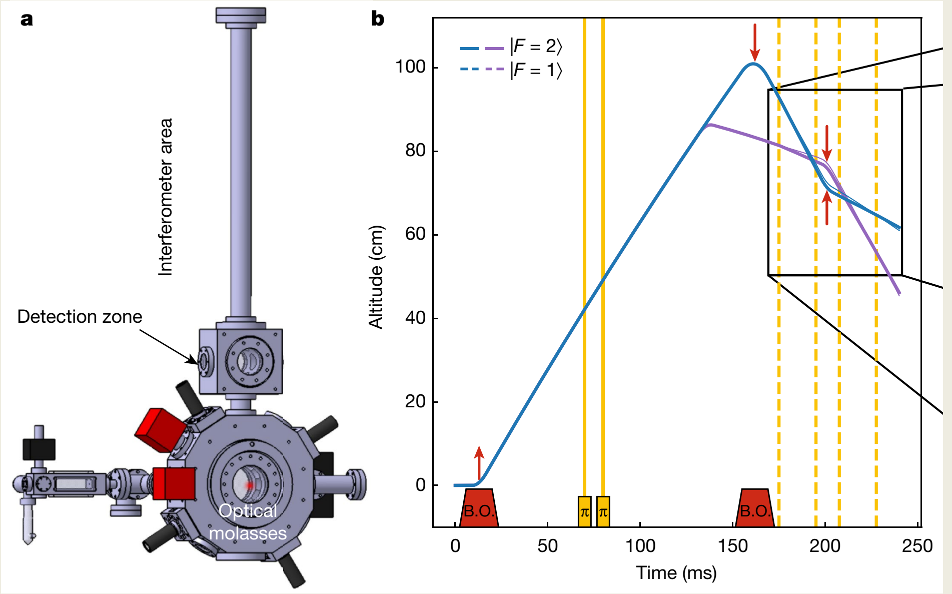
$$\Phi = T_R[4N_B\frac{\hbar k_R k_B}{m_{\rm Rb}} - \delta\omega_R]$$

Find difference in Raman frequencies ω_m or $\delta\omega_R$ s. t. $\Phi = 0$

Apparatus

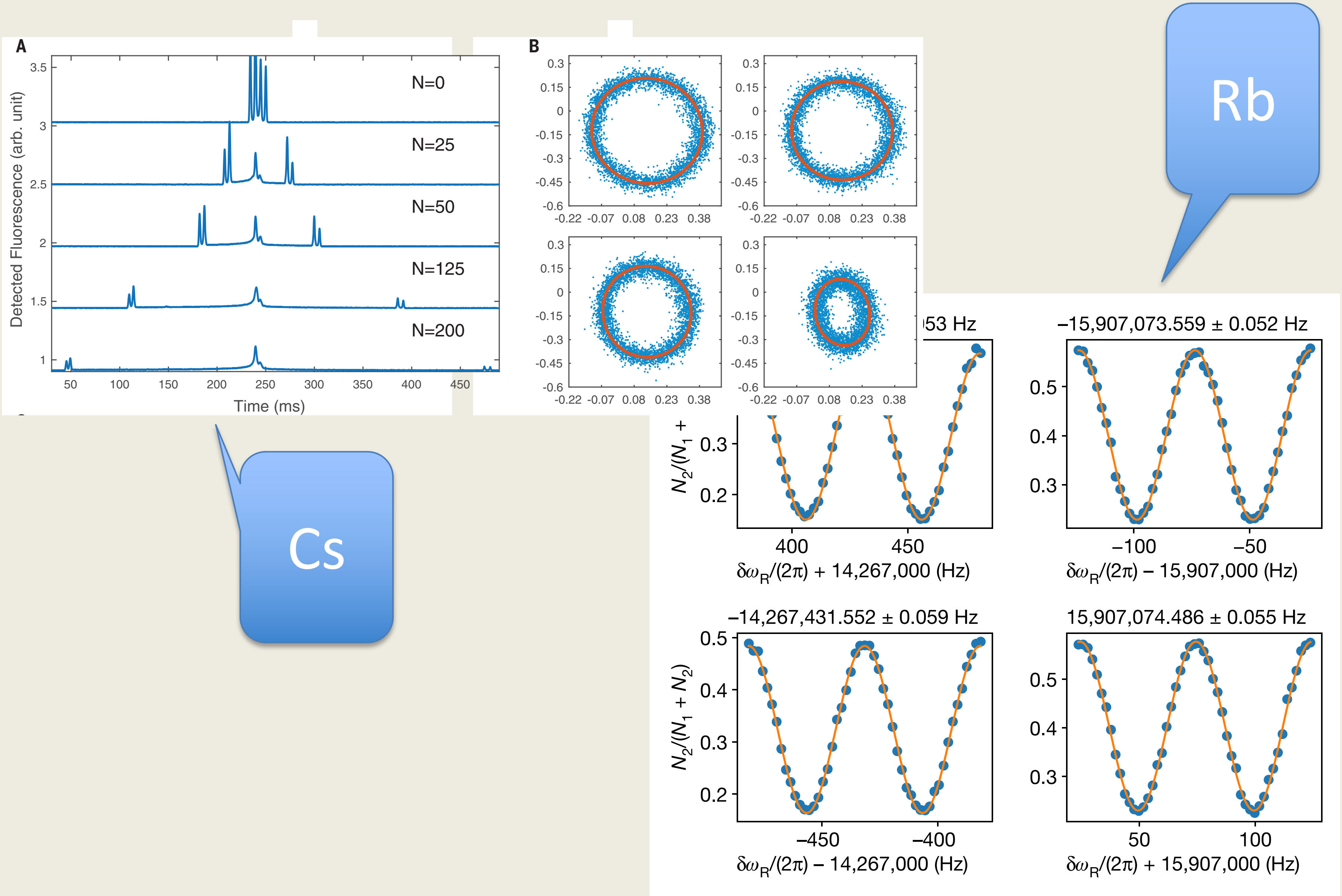
Cs
2018





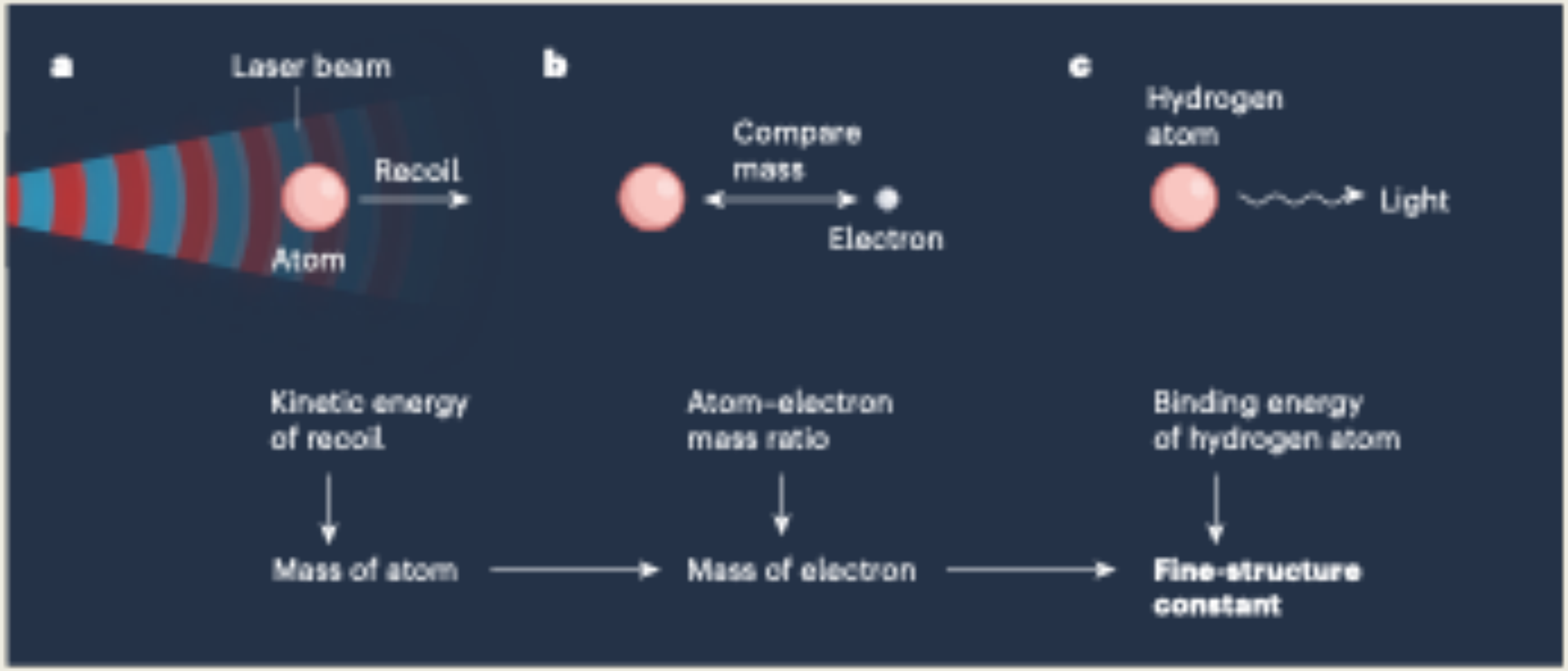
Rb
2020

Interference patterns



α from h/M

H. Mueller, Nature 2020



$$\alpha = \left[\frac{h}{M} \times \frac{A_r(M)}{A_r(m_e)} \times \frac{2R_\infty}{c} \right]^{1/2}$$

Constants determination

CODATA2014

- Rydberg constant R_∞

  Hydrogen atom spectroscopy + QED calculation

- Relative atomic mass $A_r(M)$, $A_r(m_e)$

  $A_r(M)$ Cs^+ or Rb^+ ion in Penning trap

  an ion mass is converted to an atom mass

  adding an electron mass and ionization energy.

  $A_r(m_e)$ $^{12}C^{+5}$ ion in Penning trap, bound-g factor

  Cyclotron frequency $\sim 1/m_C$ and Zeeman splitting $\sim 1/m_e$

Values of R_∞

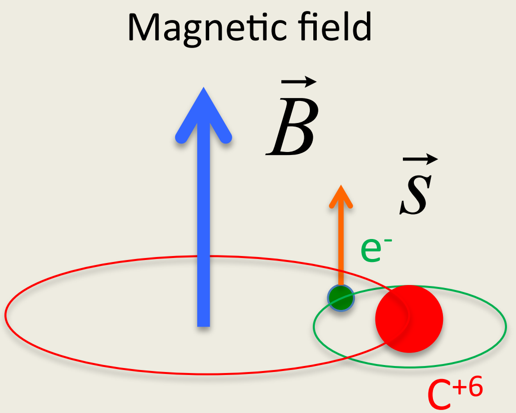
To clarify proton charge radius puzzle,
two new experiments on H-atom were performed

CODATA2018	10 973 731.568 160 (21) m^{-1}	Announced in 2020
1S-3S	10 973 731.568 53 (14) m^{-1}	Fleurbaey et al. 2018
2S-4P	10 973 731.568 076 (96) m^{-1}	Beyer et al. 2017
CODATA2014	10 973 731.568 508 (65) m^{-1}	Announced in 2016

2.7σ difference b.w. 2017 and 2018 measurements

h/m(Rb) uses CODATA2018 R_∞ 1.9 ppt
h/m(Cs) uses CODATA2014 R_∞ 5.9 ppt
Both are sufficiently accurate for α w/ 81 ppt

Bound g-factor of an electron



Cyclotron frequency of C^{+5} ion

$$\omega_c = \frac{(6-1)eB}{M_{C^{+5}}}$$

Zeeman spin-flip frequency of e-

$$\omega_s = |g(C^{+5})|\frac{eB}{2m_e}$$

$g(C^{+5})$ is the bound g-factor of the electron, calculated w/ QED

The ratio between two frequencies gives the relative atomic mass $A_r(m_e)$

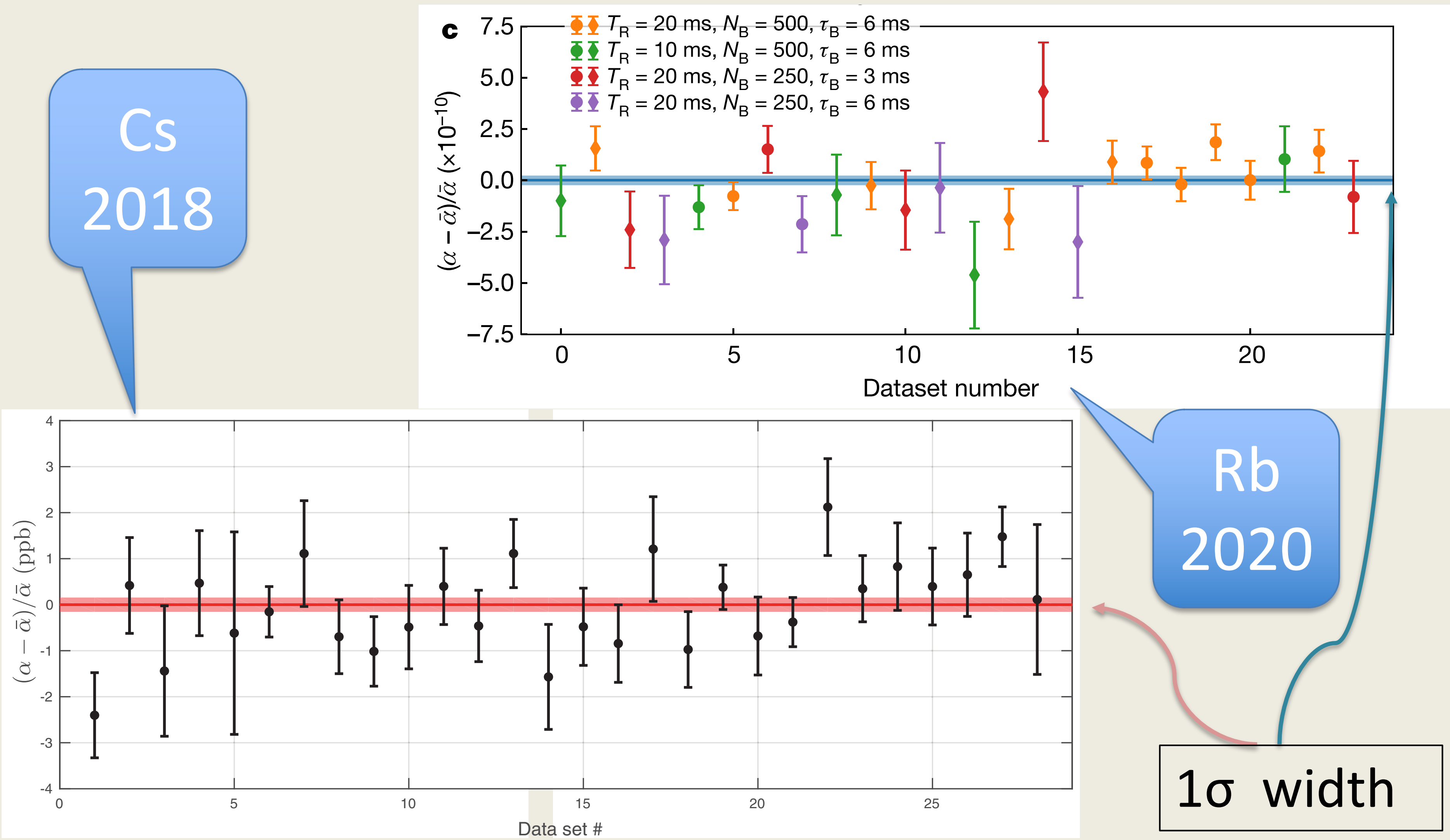
$$A_r(m_e) = 5.485\ 799\ 090\ 65\ (16) \times 10^{-4}$$ 29 ppt

$$A_r(M_{^{133}\mathrm{Cs}}) = 132.905\ 451\ 9615\ (86)$$ 65 ppt

$$A_r(M_{^{87}\mathrm{Rb}}) = 86.909\ 180\ 5310\ (60)$$ 69 ppt

Need to be improved

Data sets analyzed



Error Budgets

Cs

200ppt

Rb

81 ppt

Table 1. Error budget. For each systematic effect, more discussion can be found in the listed section of the supplementary materials. N/A, not applicable.

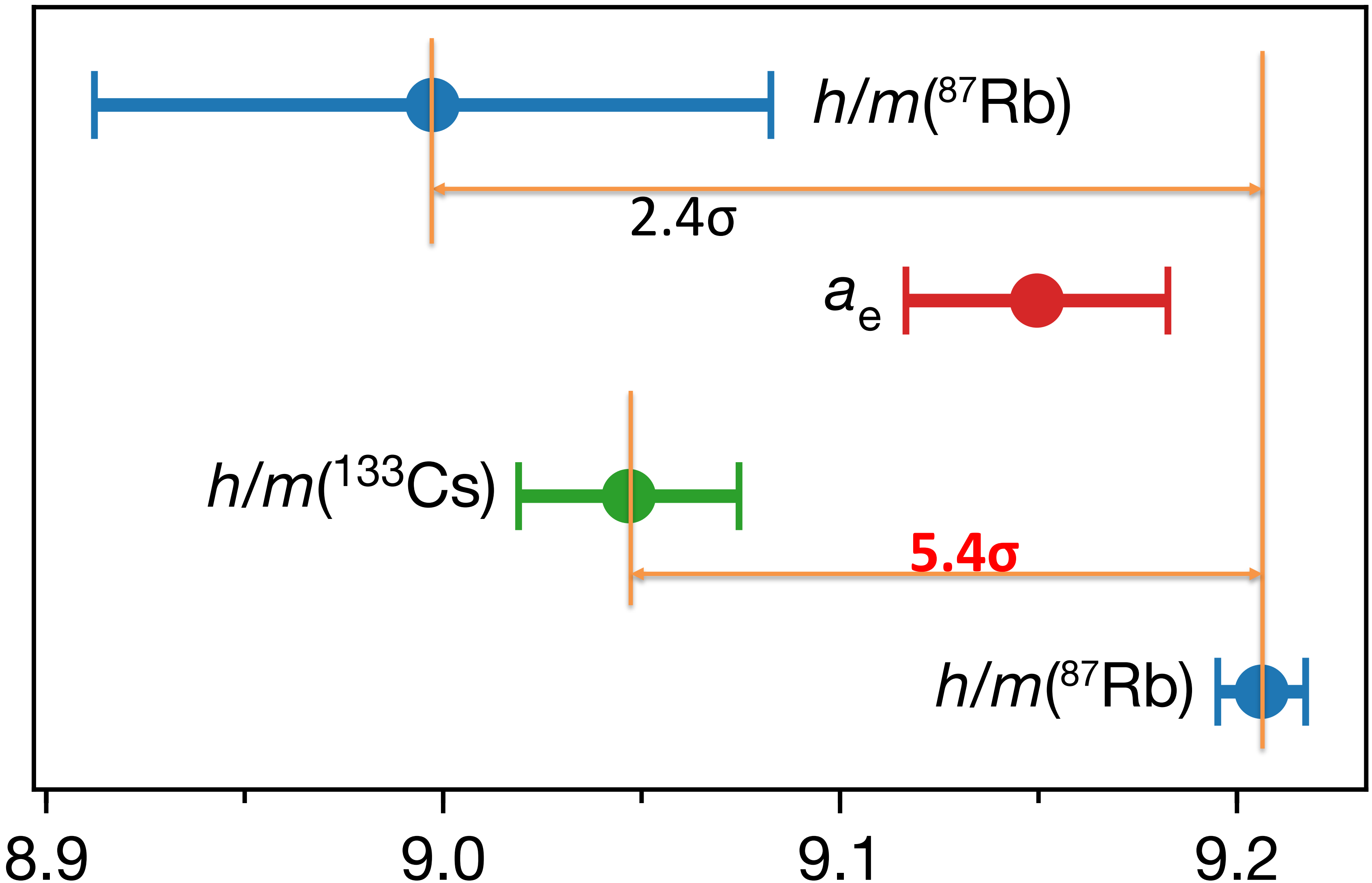
Effect	Section	δα/α (ppb)
This study		
Laser frequency	1	−0.24 ± 0.03
Acceleration gradient	4A	−1.79 ± 0.02
Gouy phase	3	−2.60 ± 0.03
Beam alignment	5	0.05 ± 0.03
Bloch oscillation light shift	6	0 ± 0.002
Density shift	7	0 ± 0.003
Index of refraction	8	0 ± 0.03
Speckle phase shift	4B	0 ± 0.04
Sagnac effect	9	0 ± 0.001
Modulation frequency wave number	10	0 ± 0.001
Thermal motion of atoms	11	0 ± 0.08
Non-Gaussian waveform	13	0 ± 0.03
Parasitic interferometers	14	0 ± 0.03
Total systematic error	All previous	−4.58 ± 0.12
Statistical error	N/A	±0.16
Other studies		
Electron mass *(16)*	N/A	±0.02
Cesium mass *(6, 15)*	N/A	±0.03
Rydberg constant *(6)*	N/A	±0.003
Combined result		
Total uncertainty in α	N/A	±0.20

Table 1 | Error budget on α

Source	Correction (×10^{-11})	Relative uncertainty (×10^{-11})
Gravity gradient	−0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave-front curvature	0.6	0.3
Wave-front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman light shift	2.3	2.3
Index of refraction	0	<0.1
Internal interaction	0	<0.1
Light shift (two-photon transition)	−11.0	2.3
Second-order Zeeman effect		0.1
Phase shifts in Raman phase-lock loop	−39.8	0.6
Global systematic effects	**64.2**	**6.8**
Statistical uncertainty		**2.4**
Relative mass of ^{87}Rb[a]: 86.9091805310(60)		3.5
Relative mass of the electron[b]: 5.48579909065(16) × 10^{-4}		1.5
Rydberg constant[b]: 10,973,731.568160(21) m^{-1}		0.1
Total: α^{-1} = 137.035999206(11)		**8.1**

Consistency of α within h/m expt.

L. Morel et al. 2020



Suspected reasons

H. Mueller 2020

- Speckle?

  small-scale spatial variations of the laser intensity

- a phase shift arising in electronic-signal processing?

- Laser beam profile ? – largest error source –

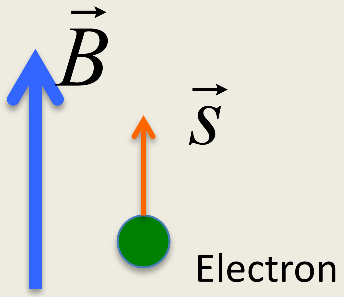
  Cs overcorrected ?

  Rb under-corrected ?

- Further study needed

  Cs **``20 folds improvement ! ”** H. Mueller 2019

Electron g-2



Electron in the static magnetic field

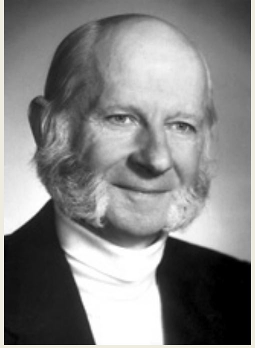
$$H = -\vec{\mu}_e \cdot \vec{B} \qquad \vec{\mu}_e = g_e \frac{e\hbar}{2m_e} \frac{\vec{s}}{\hbar}$$

Deviation from 2 is the anomalous magnetic moment

$$a_e = \frac{g_e - 2}{2}$$

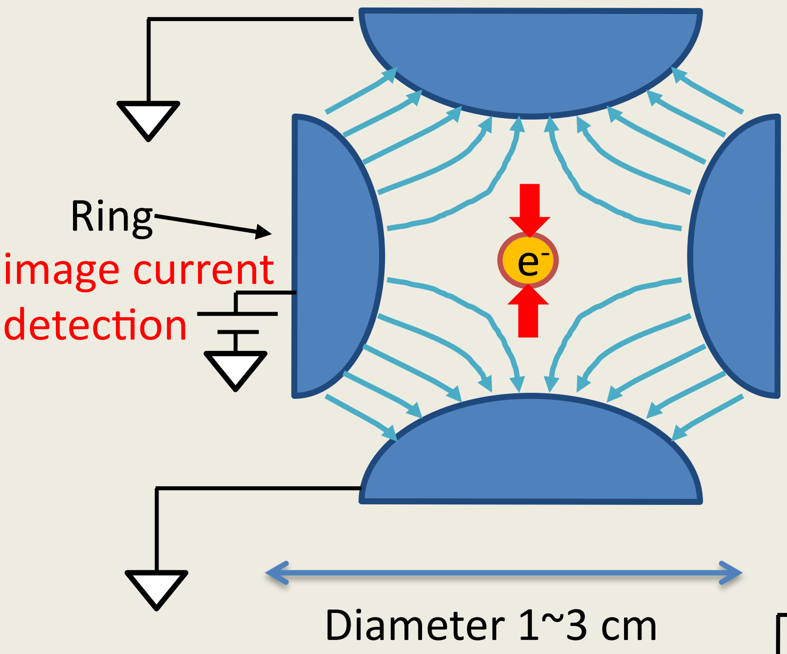
Very precisely measured and calculable within the SM

Single electron Penning Trap



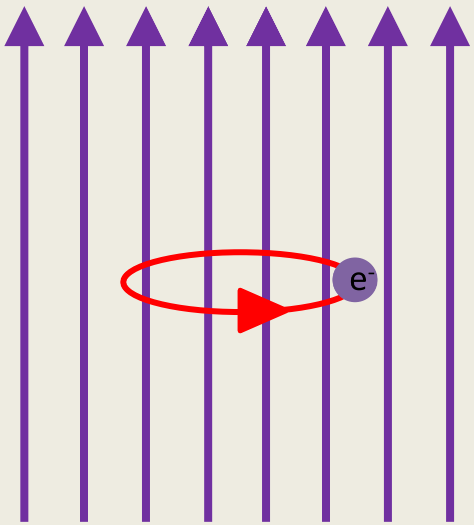
H. Dehmelt

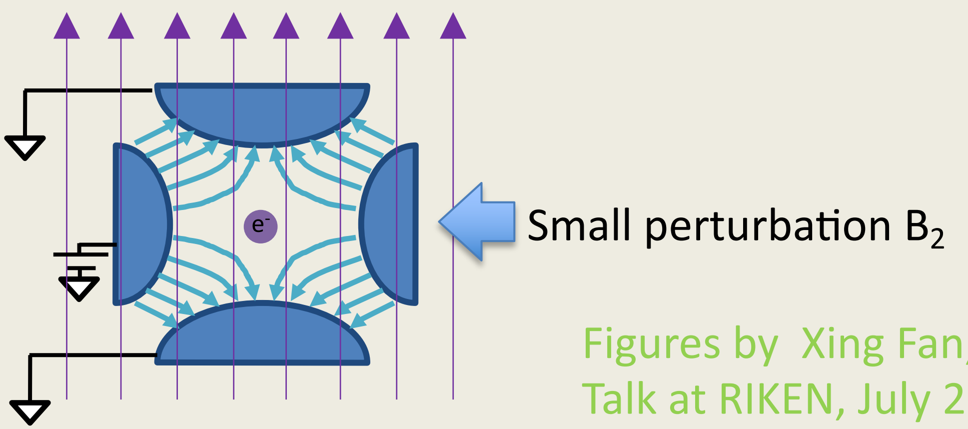
Electric field



+

Magnetic field



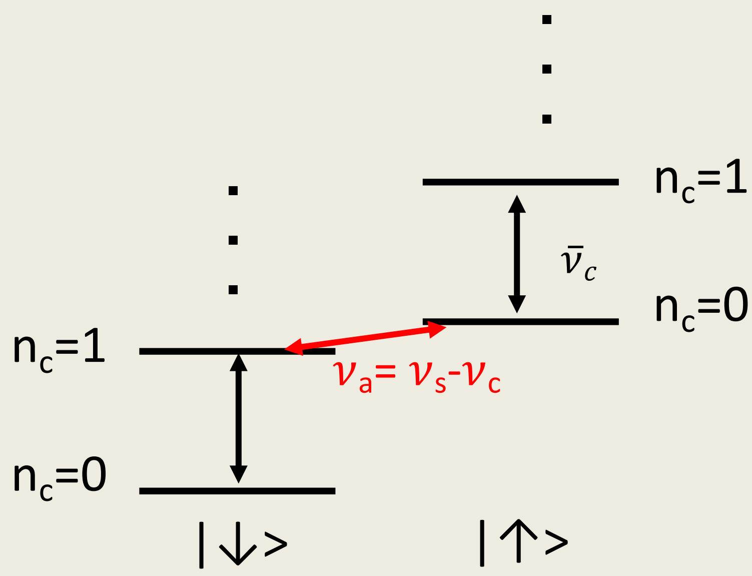


Widely used to
trap charged particles:
Mass-spectroscopy
Quantum computing

Figures by Xing Fan,
Talk at RIKEN, July 2019

Signal detection

$$H = \hbar\omega_c\left(a_c^\dagger a_c + \frac{1}{2}\right) + \hbar\omega_z\left(a_z^\dagger a_z + \frac{1}{2}\right) + \hbar\omega_s\frac{\sigma_z}{2}$$



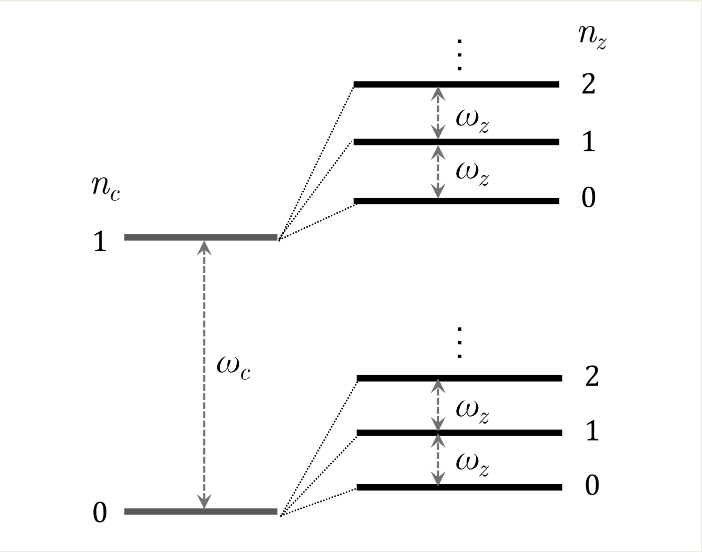
Add a magnetic perturbation B_2

Two oscillators couples

$$V = \hbar\delta_c\left(a_c^\dagger a_c + \frac{1}{2}\right)\left(a_z^\dagger a_z + \frac{1}{2}\right)$$

change in n_c is detected through change in n_z

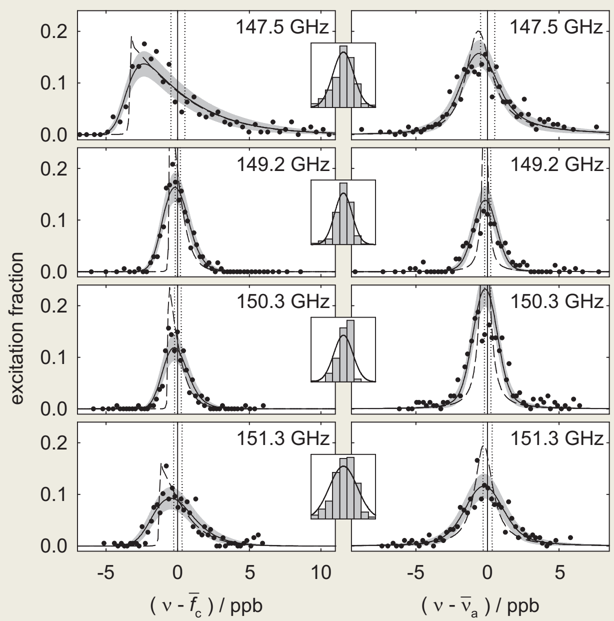
→ image current



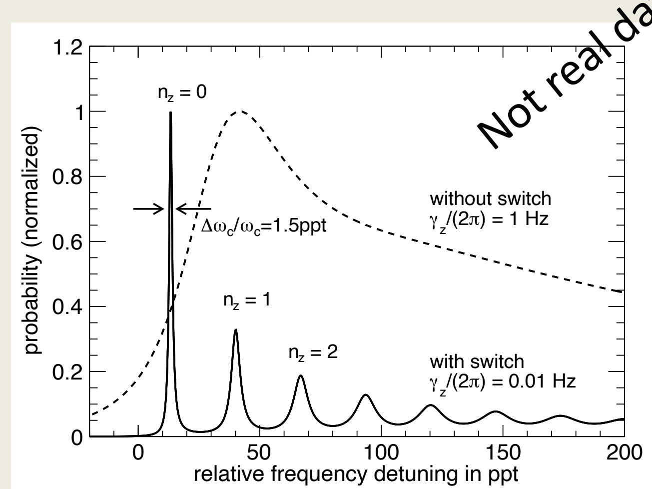
X. Fan and G. Gabrielse 2020

Error source of electron g-2

Backaction from the axial motion detection to the electron



2008 Measurement
Line shapes of cyclotron and anomaly transitions Hanneke et al.



Not real data!

Theoretical analysis
+ new electoorical circuit X. Fan et al.2020

``20 folds improvement in a year "

Gabrielse 2019

Theory of electron g-2

SM contribution

$$a_e = a_e(\mathrm{QED}) + a_e(\mathrm{hadron}) + a_e(\mathrm{weak})$$

1.5ppb 0.025ppb

mass-dependence

$$a_e(\mathrm{QED}) = A_1 + A_2\left(\frac{m_e}{m_\mu}\right) + A_2\left(\frac{m_e}{m_\tau}\right) + A_3\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right)$$

2.4ppb

Universal for any point-like spin ½ particles

Perturbation in α

Numerically calculated

$$A_1 = A_1^{(2)}\left(\frac{\alpha}{\pi}\right) + A_1^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_1^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_1^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + A_1^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

All terms are analytically known, double checked

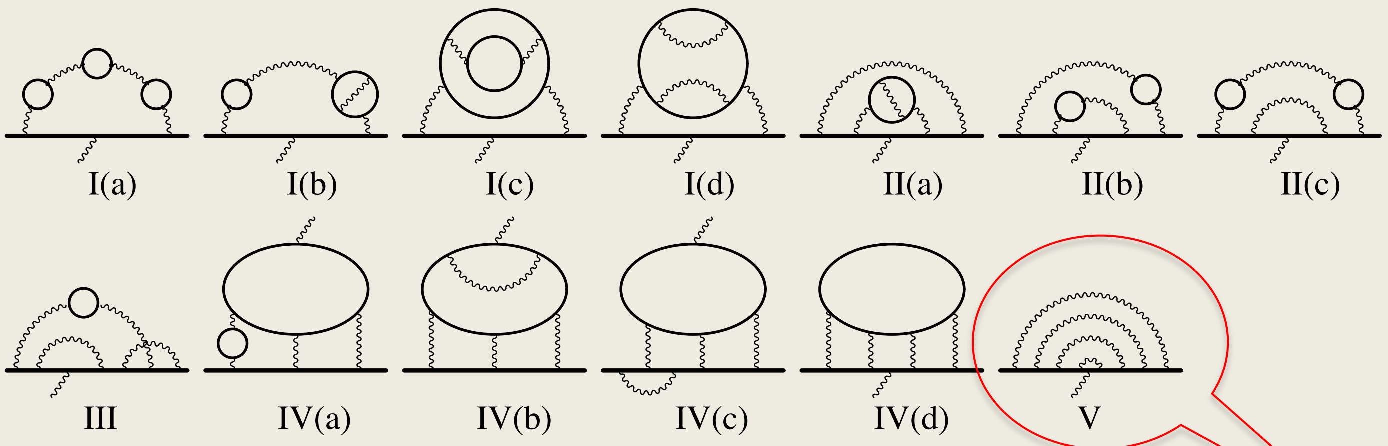
QED results up to the 10$^{\text{th}}$-order

Coefficient $A_i^{(2n)}$	Value (Error)
$A_1^{(2)}$	0.5
$A_2^{(2)}(m_e/m_\mu)$	0
$A_2^{(2)}(m_e/m_\tau)$	0
$A_3^{(2)}(m_e/m_\mu, m_e/m_\tau)$	0
$A_1^{(4)}$	$-0.328\,478\,965\,579\,193\cdots$
$A_2^{(4)}(m_e/m_\mu)$	$0.519\,738\,676\,(24)\times10^{-6}$
$A_2^{(4)}(m_e/m_\tau)$	$0.183\,790\,(25)\times10^{-8}$
$A_3^{(4)}(m_e/m_\mu, m_e/m_\tau)$	0
$A_1^{(6)}$	$1.181\,241\,456\,587\cdots$
$A_2^{(6)}(m_e/m_\mu)$	$-0.737\,394\,164\,(24)\times10^{-5}$
$A_2^{(6)}(m_e/m_\tau)$	$-0.658\,273\,(79)\times10^{-7}$
$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau)$	$0.1909\,(1)\times10^{-12}$
$A_1^{(8)}$	$-1.912\,245\,764\cdots$
$A_2^{(8)}(m_e/m_\mu)$	$0.916\,197\,070\,(37)\times10^{-3}$
$A_2^{(8)}(m_e/m_\tau)$	$0.742\,92\,(12)\times10^{-5}$
$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau)$	$0.746\,87\,(28)\times10^{-6}$
$A_1^{(10)}$	$6.737\,(159)$
$A_2^{(10)}(m_e/m_\mu)$	$-0.003\,82\,(39)$
$A_2^{(10)}(m_e/m_\tau)$	$\mathcal{O}(10^{-5})$
$A_3^{(10)}(m_e/m_\mu, m_e/m_\tau)$	$\mathcal{O}(10^{-5})$

Uncertainty comes from
muon-electron mass ratio
tau-electron mass ratio

Uncertainty comes from
numerical integration

8th-order calculation $A_1^{(8)}$



Hardest to compute

All 891 diagrams

Laporta	(2017)	$-1.912\ 245\ 764 \cdots$
AHKN	(2015)	$-1.912\ 98\ (84)$
Marquard et al.	(2017)	$-1.87\ (12)$

518 diagrams of Set V

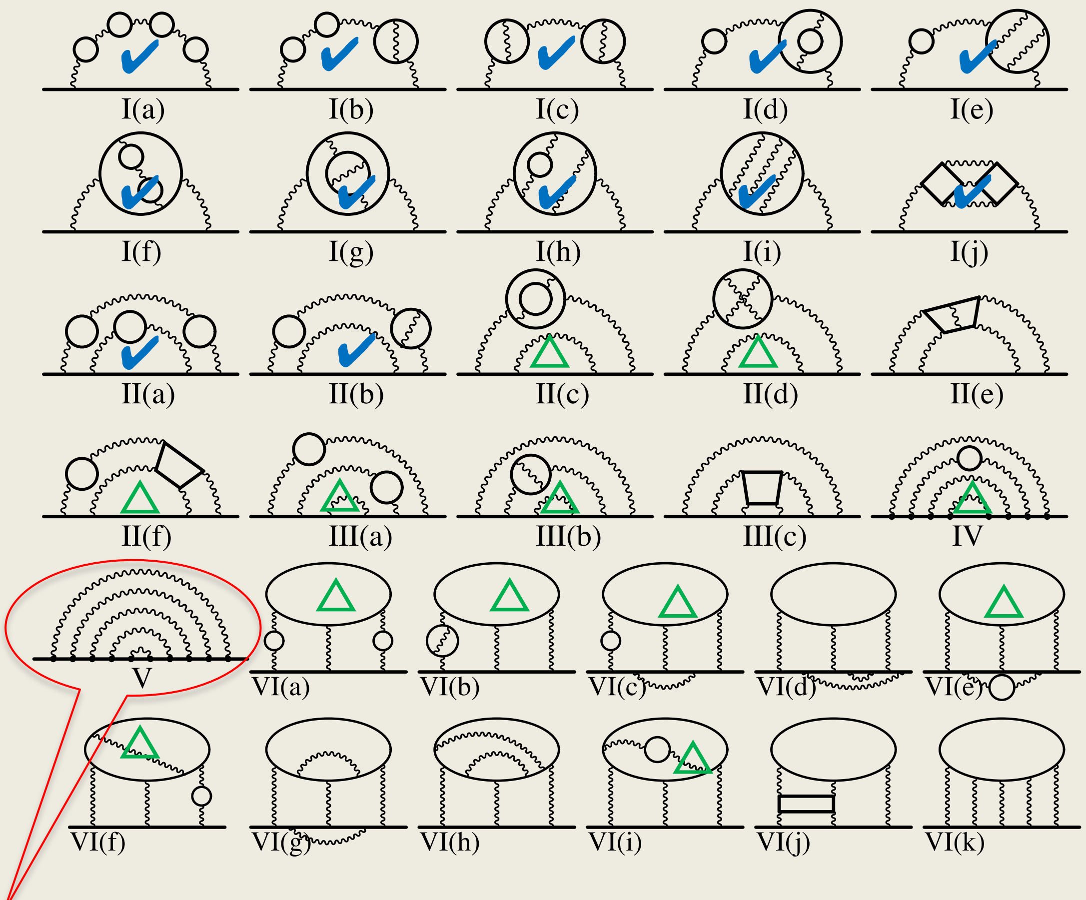
Laporta	(2017)	$-2.176\ 886\ 02 \cdots$
AHKN	(2015)	$-2.177\ 33\ (82)$
Volkov	(2018)	$-2.1790\ (22)$

- 8th-order is established
- The numerical calculation methods are confirmed.

10^{th}-order calculation $A_1^{(10)}$

12,672 vertex diagrams
Some of them are doubly checked

Baikov et al. 2013
Laporta et al. 1994



✔ Independent check confirms the result

△ Easy extension from the computer programs for the 8^{th}-order diagrams

6354 vertex diagrams of this type are the hardest ones to evaluate

Two results of $A_1^{(10)}$[Set V]

Home Alone

AHKN	(2018)	7.668 (159)
Volkov	(2019)	6.793 (90)
diff.		0.875 (183)

4.8σ tension!

Is a meaningful difference?

$$0.875 \left(\frac{\alpha}{\pi}\right)^5 = 0.059 \times 10^{-12}$$

No. The uncertainty of the current experiment:

$$\delta a_e(\mathrm{HV2008}) = 0.28 \times 10^{-12}$$

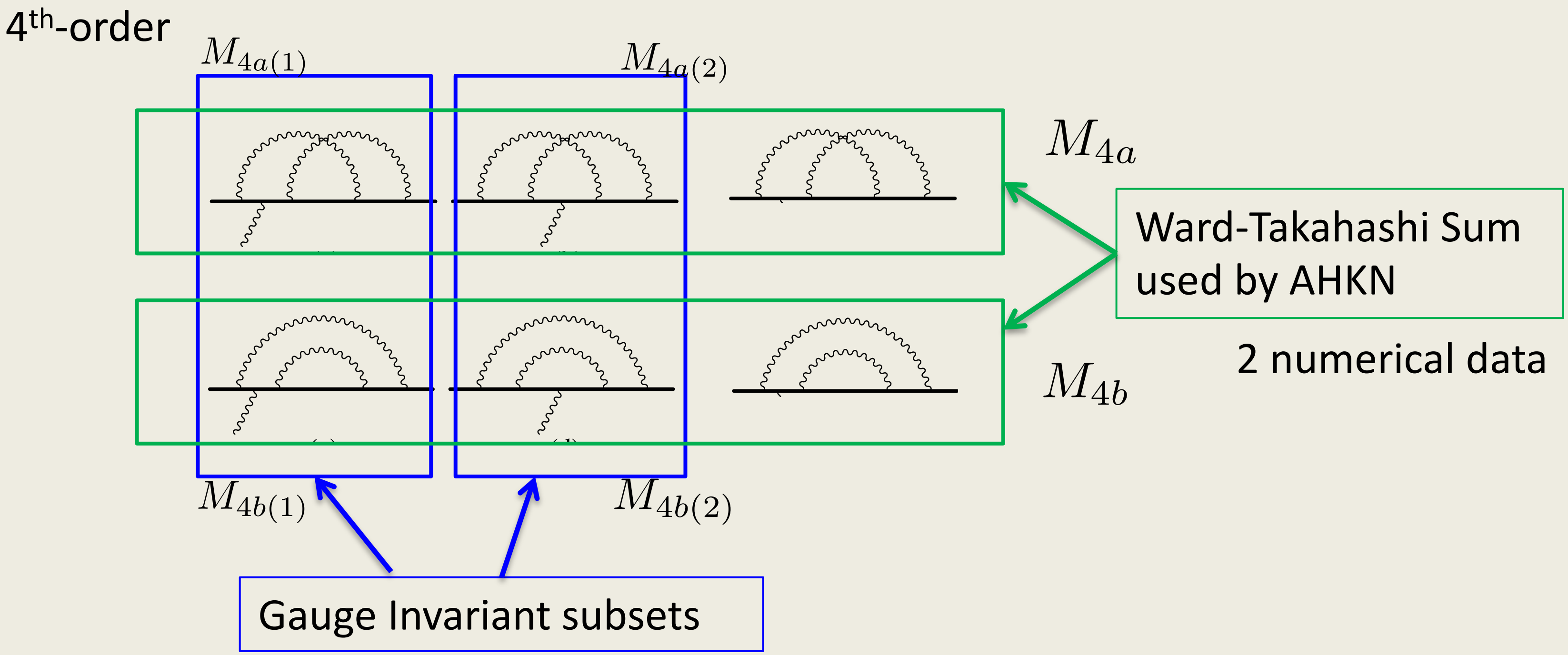
Yes. Soon, the NW team will reduce the uncertainty to

$$\delta a_e(\mathrm{NW202x}) = 0.02 \times 10^{-12}$$

Vertex sum v.s. Ward-Takahashi sum

Volkov directly calculated 3,213 vertex diagrams

AHKN calculated the Ward-Takahashi 389 sum



Every vertices are calculated by Volkov
4 numerical data

Different renom. constants

On-shell renormalization constants for a self-energy diagram *G*:

$L_{G(i)}$ for vertex renormalization

B_G for wave-function renormalization

Volkov used IR-free and gauge-invariant:

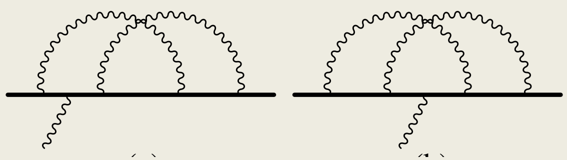
$$BV_G + \sum_{i=1}^{2n-1} LV_{G(i)} = 0$$

We used IR free, easy-determined, but not gauge-invariant:

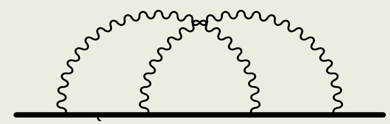
$$BK_G + \sum_{i=1}^{2n-1} LK_{G(i)} + \Delta LB_G = 0$$

Connection b.w. Volkov and AHKN

Volkov



AHKN



$$a_{4a} = M_{4a(1)} + 2M_{4a(2)} - 2\ L_2\ M_2$$
$$= \Delta M_{4a(1)} + 2\Delta M_{4a(2)} - 2(L_2 - LV_2)\ M_2$$

$$a_{4a} = M_{4a} - 2\ L_2\ M_2$$
$$= \Delta M_{4a} - 2(L_2 - LK_2)\ M_2$$

The same physical contribution. This is IR divergent.

Numerical and finite numbers

$$\Delta M_{4a} - (\Delta M_{4a(1)} + 2\Delta M_{4a(2)}) = 2\ \delta L_2\ M_2$$

where $$\delta L_2 = LV_2 - LK_2$$

finite!

New calculation of $\delta L_{n(i)}$

Difference of renormalization constants $\delta L_{n(i)}$

are newly calculated for n=2,4,6,8 . No 10th-order.

(#) …# of independent diagrams, time-reversal symmetry

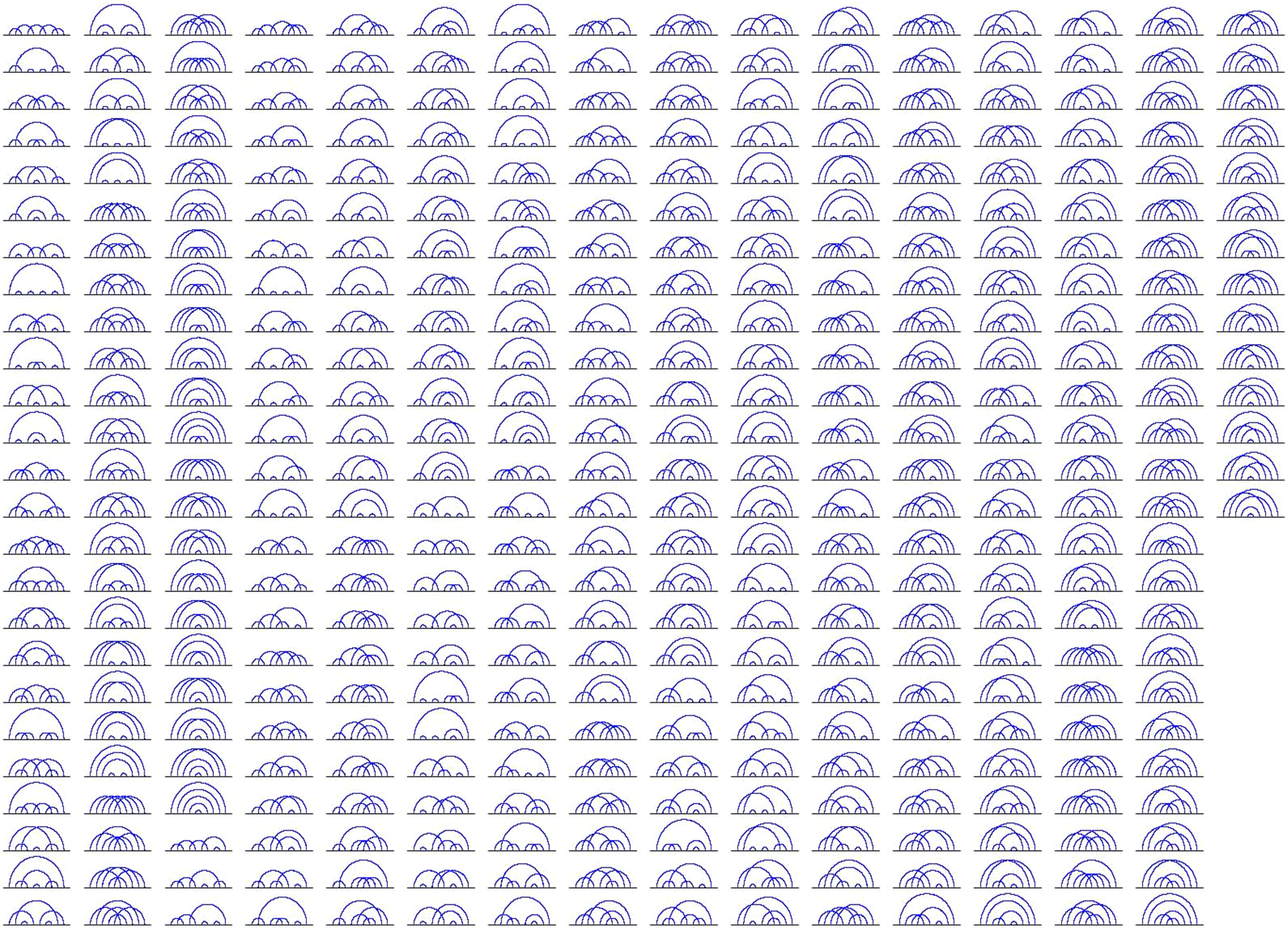
Order n	2	4	6	8
# of vertex diagrams	1	6 (4)	50 (28)	518 (269)
# of diagrams calculated so far	✔ 1	✔ 6 (4)	✔ 50 (28)	on-going (132)

132 x 1 hour x 40 core = 5,280 core x hours, 1 night at RIKEN's HOKUSAI-BW

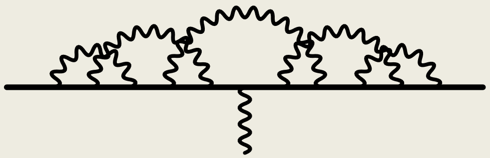
very small calculation compared to the 10th-order g-2 calculation

Ref. One diagram evaluation of 10th-order g-2 requires O(10^5) core x hours

6354 (3213) vertex diagrams represented by 706 (389) self-energy diagrams



X001 as an example



$$\Delta M_{X001} - \sum_{i=1}^{9} \Delta M_{X001(i)} = \Delta M_2 \ (-3(\delta L_{4a1})^2 - 6\delta L_2 \delta L_{6f1} + 12(\delta L_2)^2 \delta L_{4a1} - 5(\delta L_2)^4 + 2\delta L_{01v1}) + \Delta M_{01} \ (2\delta L_2) + \Delta M_{6f} \ (2\delta L_{4a1} - 3(\delta L_2)^2) + \Delta M_{4a} \ (2\delta L_{6f1} - 6\delta L_2 \delta L_{4a1} + 4(\delta L_2)^3)$$

$$\begin{aligned} \text{l.h.s} &= -0.16083\ (334) - 0.58095\ (534) \\ &= -0.74178\ (630) \\ \text{r.h.s} &= -0.73854\ldots \\ \text{l.h.s} - \text{r.h.s} &\quad -0.00324\ (630) \end{aligned}$$

Consistently 0 !

X001 safely passes the numerical check.

135 of 389 have been checked. All are consistent.

Electron g-2 Experiment v.s. Theory

Best 3 values of α

$$\begin{aligned} \alpha^{-1}(a_e) &= 137.035\ 999\ 150\ (33) && 240\ \mathrm{ppt} \\ \alpha^{-1}(\mathrm{Cs18}) &= 137.035\ 999\ 046\ (27) && 200\ \mathrm{ppt} \\ \alpha^{-1}(\mathrm{Rb20}) &= 137.035\ 999\ 206\ (11) && 81\ \mathrm{ppt} \end{aligned}$$

Experiment

$$a_e(\mathrm{HV08}) = 1\ 159\ 652\ 180.73\ (28)$$

10th-order
AKHN QED

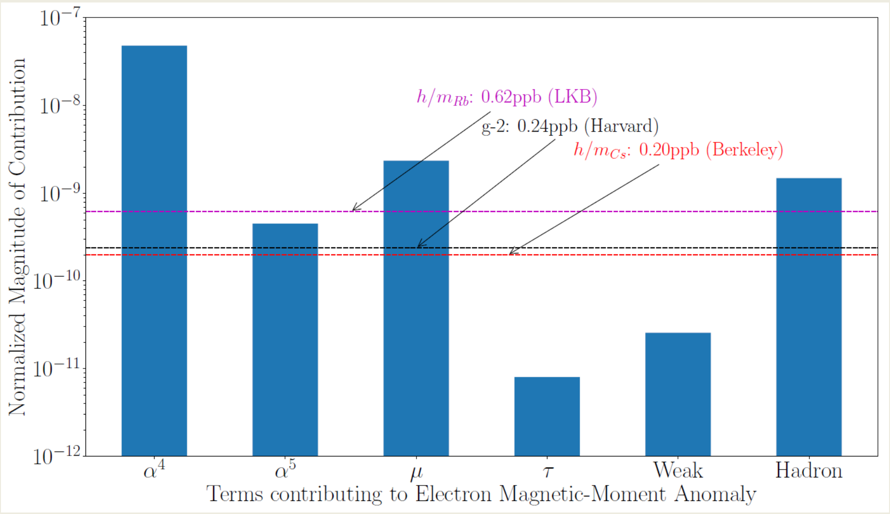
Theory

Hadron
A. Keshavarzi et al. 2019

$$\begin{aligned} a_e(\alpha(\mathrm{Cs})) &= 1\ 159\ 652\ 181.616\ (229)(11)(9)\ [229] \\ a_e(\alpha(\mathrm{Rb})) &= 1\ 159\ 652\ 180.265\ (93)(11)(9)\ [94] \end{aligned}$$

Come from α solely!

Contributions to Th. Electron g-2

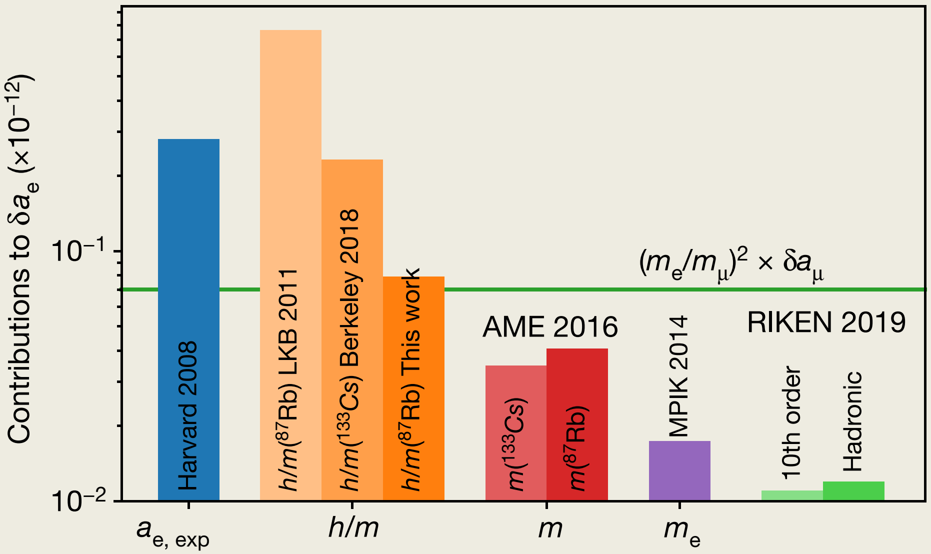


Normalized absolute contribution

Cs 2018

Rb 2020

Error budget of each contribution

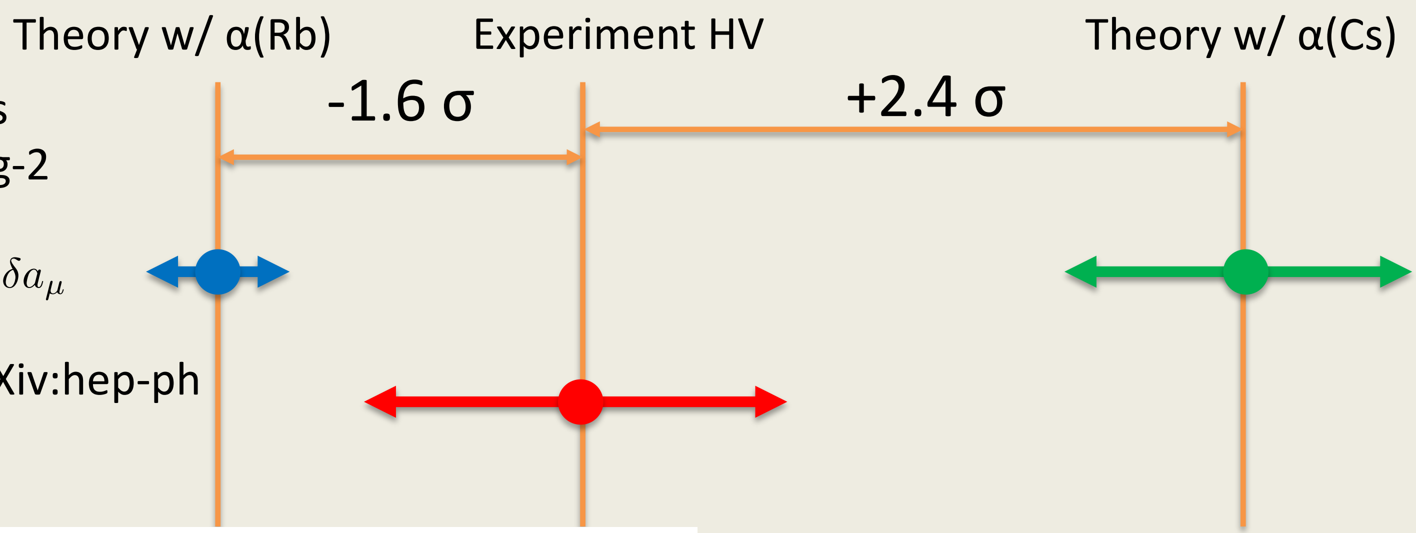


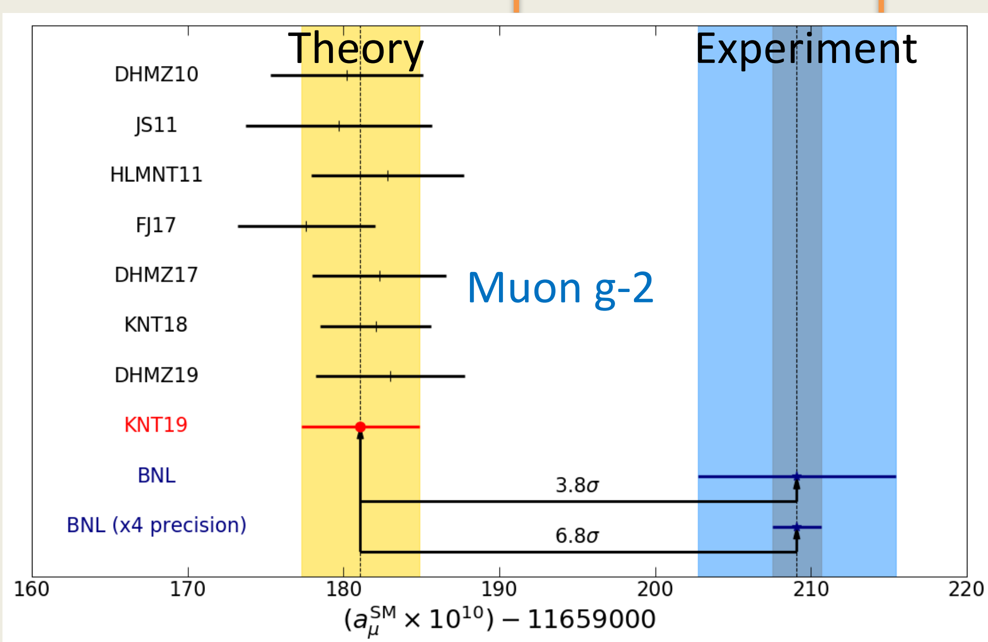
Electron g-2 Experiment v.s. Theory

Same new physics can explain both g-2

$$\delta a_e \sim \left(\frac{m_e}{m_\mu}\right)^2 \delta a_\mu$$

"muon g-2" in arXiv:hep-ph
419 papers





Difficult to explain both from same new physics

arXiv hep-ph papers
2011.05083,2006.07929,2006.01934,
2003.09781,2003.07638,2003.06633,
2003.03386,2002.10230,1910.10734,
1908.03607,1906.08768,1806.10252
…

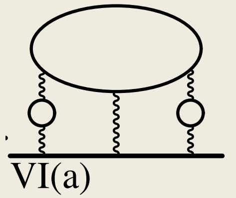
A. Keshavarzi et al. 2019

QED contribution to muon g-2

$$
\begin{aligned}
a_\mu(\mathrm{QED}; \alpha(a_e)) &= 116\ 584\ 718.842\ (7)(17)(6)(28) \quad [34] \times 10^{-11} \\
a_\mu(\mathrm{QED}; \alpha(\mathrm{Cs})) &= 116\ 584\ 718.931\ (7)(17)(6)(23) \quad [33] \times 10^{-11} \\
a_\mu(\mathrm{QED}; \alpha(\mathrm{Rb})) &= 116\ 584\ 718.793\ (7)(17)(6)(9) \quad [22] \times 10^{-11}
\end{aligned}
$$

Uncertainties
tau-lepton mass, 8th-order QED, 10th-order QED, α, combined

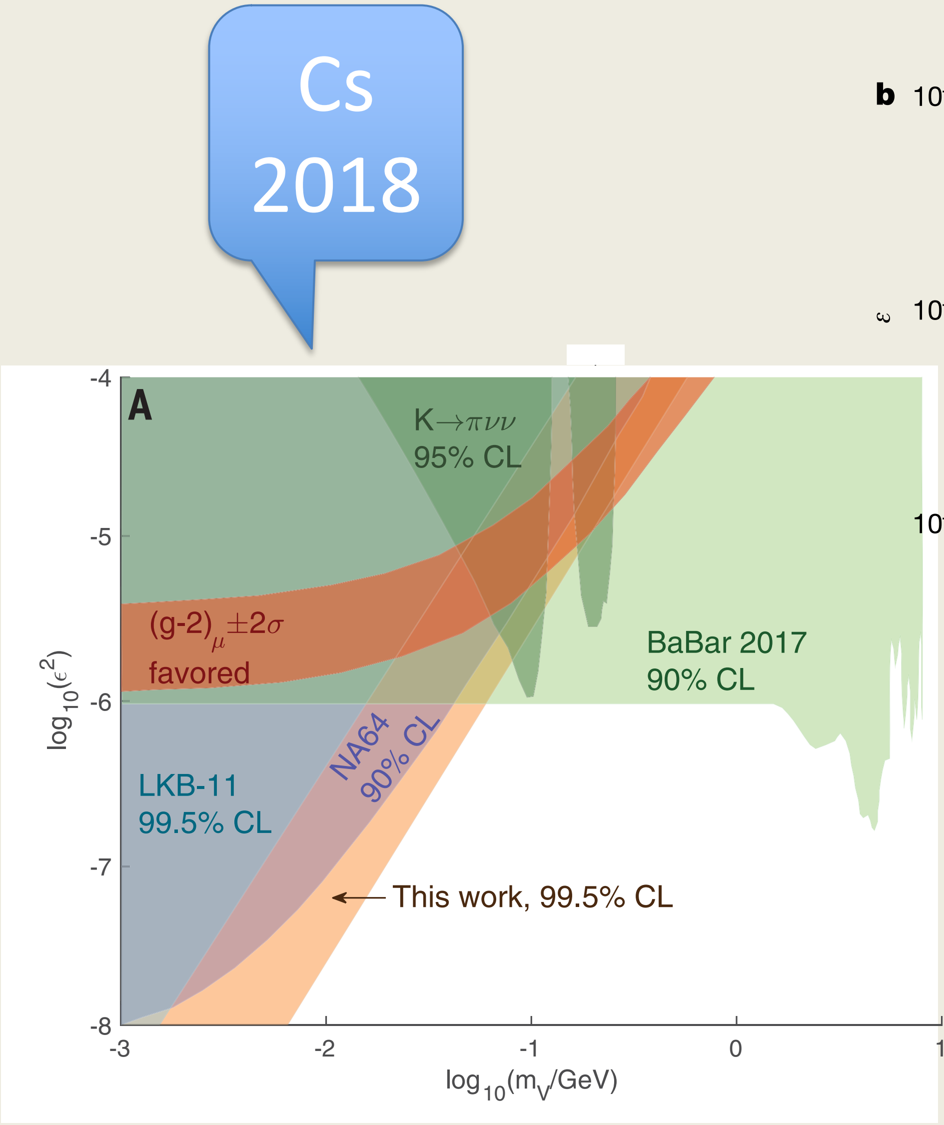
Estimated 12th-order contribution is $\pm 0.100 \times 10^{-11}$

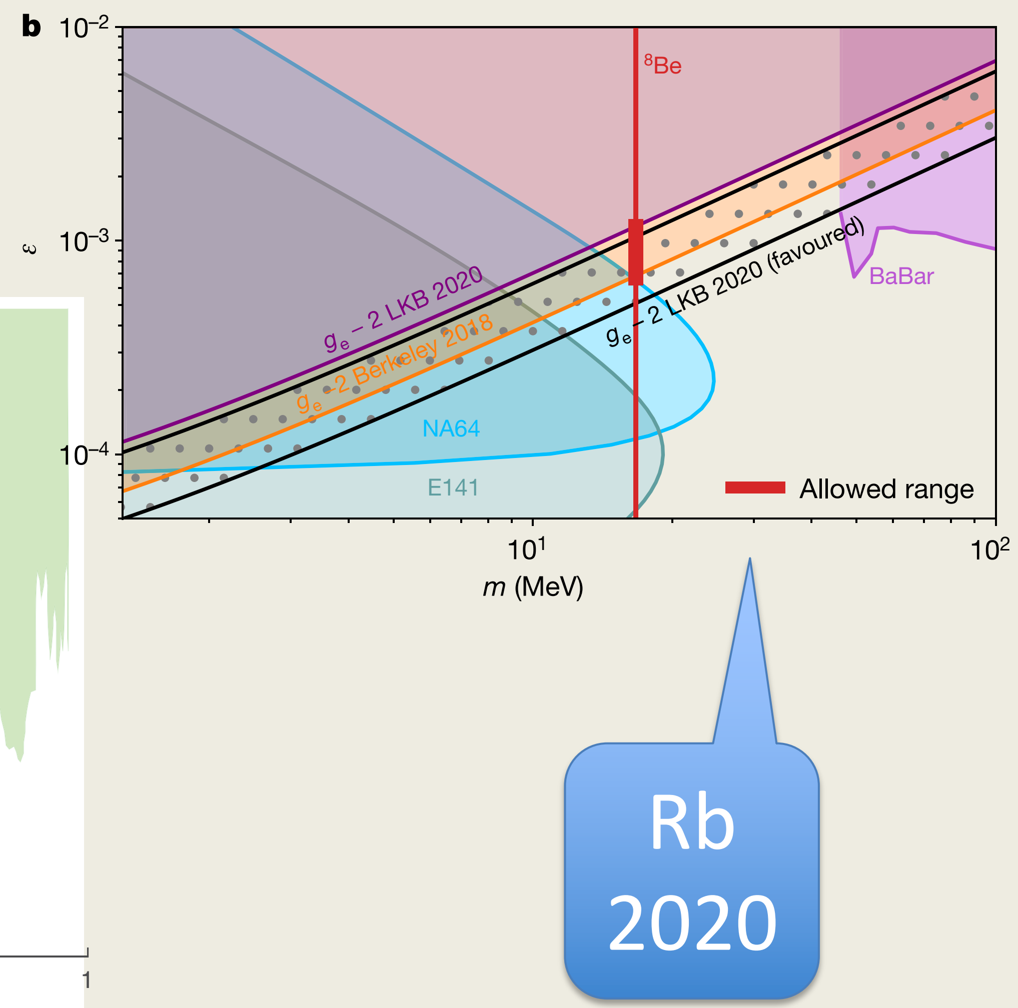


Add one more electron vacuum polarization bubble

Truly dominant at the 8th-order QED

Limits on dark vector boson





Be more careful

Searching

new physics interactable with a photon via

$$a_l(\text{expt.}) \overset{?}{=} a_l(\text{theory} : \alpha(h/M, R_\infty, A_r(e), A_r(M)))$$

- new physics appears in free muon and electron (g-2)'s

$$a_\mu(\text{expt.}) \ , \quad a_e(\text{expt.})$$

- new physics appears in Coulomb binding atoms

R_∞ Rydberg constant

How much?

- new physics appears in the magnetic cyclotron binding

QED derived

$A_r(e)$, $A_r(\text{Cs})$, $A_r(\text{Rb})$ masses of particles

- h/M is probably insensitive to new physics

kinematical determination, Cs and Rb

Summary

- New α from the atom interferometer is explained.
- Progress in electron g-2, both expt. and theory, is explained.
- ``Comparison” is discussed.
  three α’s , electron g-2 expt. and theory
- In near future, ``comparison” will be performed at a few ppt level.