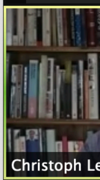
Consistency of the hadronic vacuum polarization between lattice QCD and the R-ratio

Christoph Lehner (Regensburg & BNL)

May 19, 2021 - KEK-PH

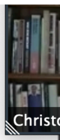


Akimas

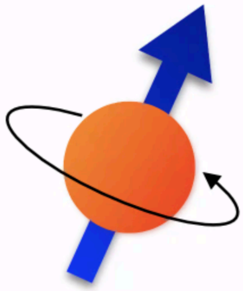
Tsut

nobu

The role of the hadronic vacuum polarization for the muon g-2



What is a muon?



- Elementary point-like particle
- Same electric charge as an electron
- Approximately **200** times heavier than an electron
- Like the electron, behaves as if it was intrinsically spinning about a vector $\vec{S}$

These properties combine to give it a magnetic moment

$$\vec{\mu} = g \left(\frac{e}{2m}\right) \vec{S}$$

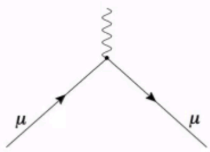
such that when put in a magnetic field, it exhibits precession similar to a spinning top.

We can measure this precession **very** precisely.

The magnetic moment and quantum corrections

The g-factor in $\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$ describes the strength of coupling to a magnetic field, which can be computed from theory also **very** precisely.

Dirac: $g = 2$

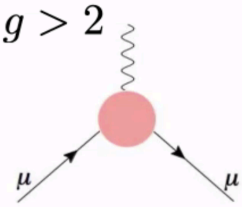


Anomalous magnetic moment

$a = (g-2)/2$

quantum effects

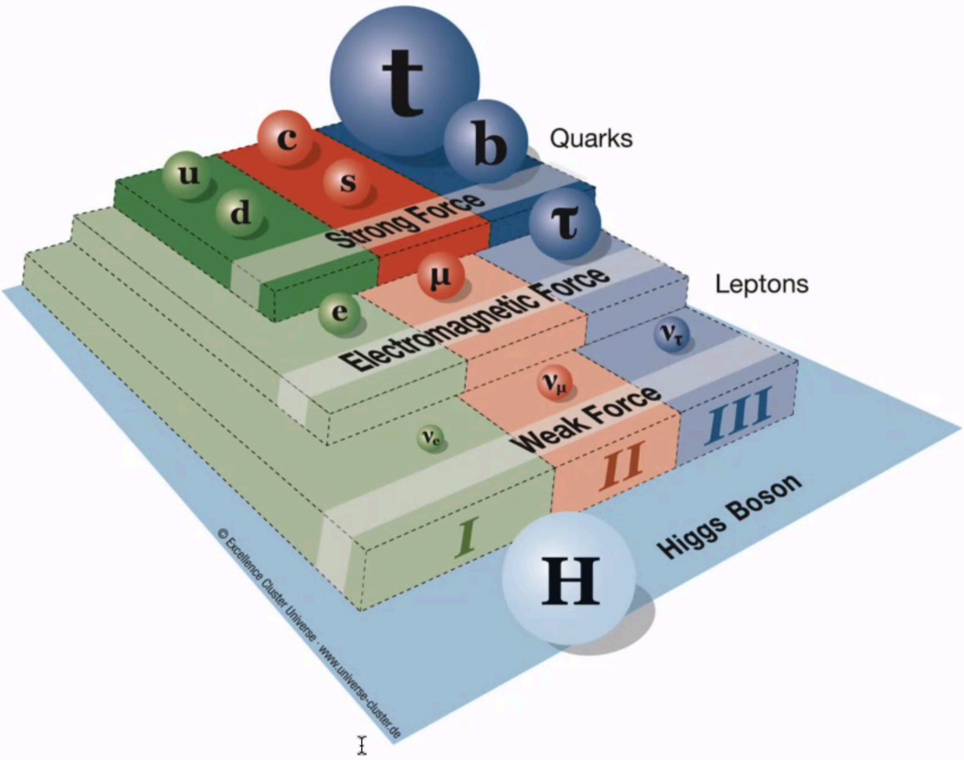
$g > 2$



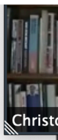
The quantum effects arise from virtual particle contributions from all known **and unknown** particles.

By comparing high-precision experiments and theory, we have the potential to learn about such contributions of new particles.

Contributions from known particles: The Standard Model



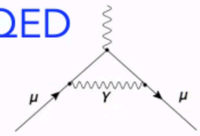
Open questions: dark matter, size of matter-antimatter asymmetry, origin of neutrino masses, . . . ⇒ Standard Model is incomplete



Contributions from known particles: The Standard Model

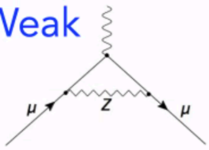
$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

QED



$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak

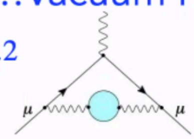


$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

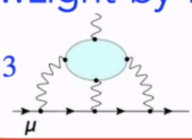
α^2



$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

…Light-by-Light (HLbL)

α^3

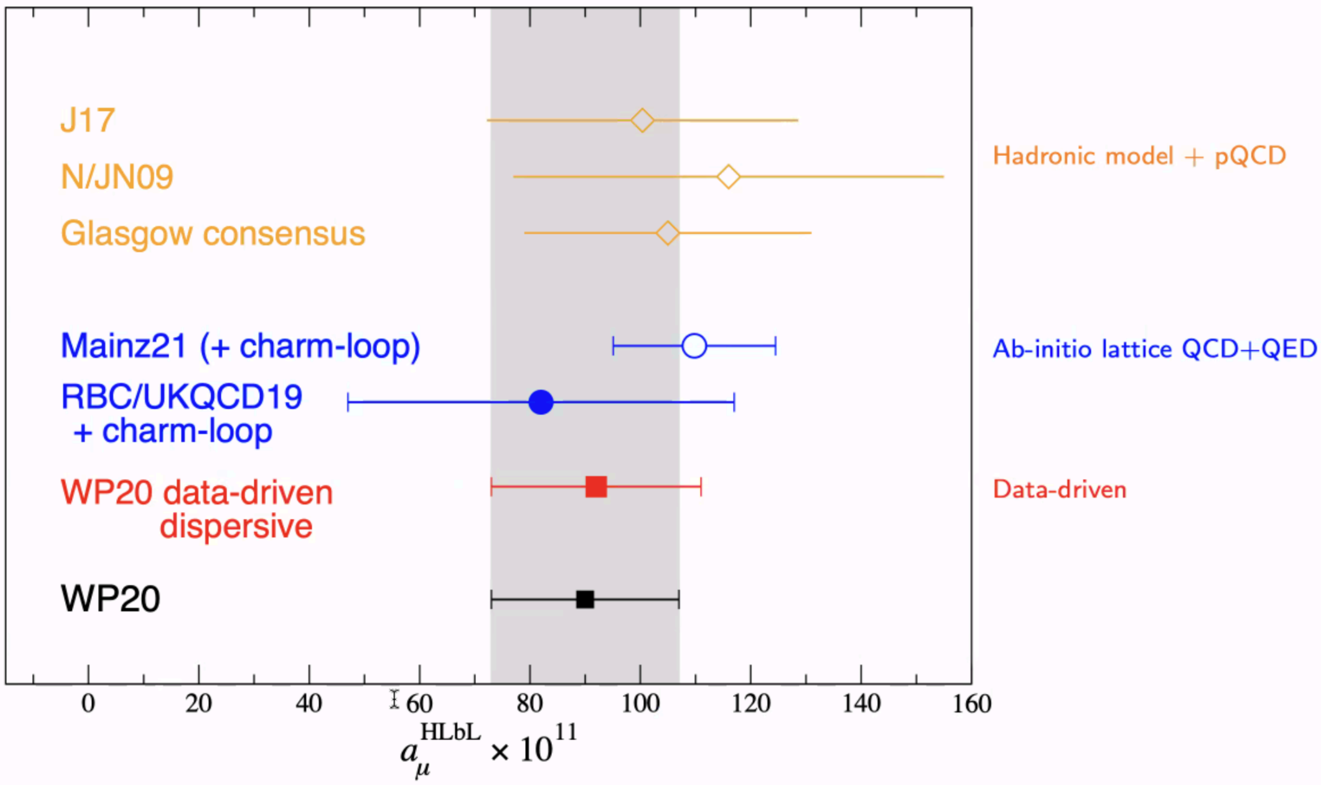


$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

Numbers from Theory Initiative Whitepaper

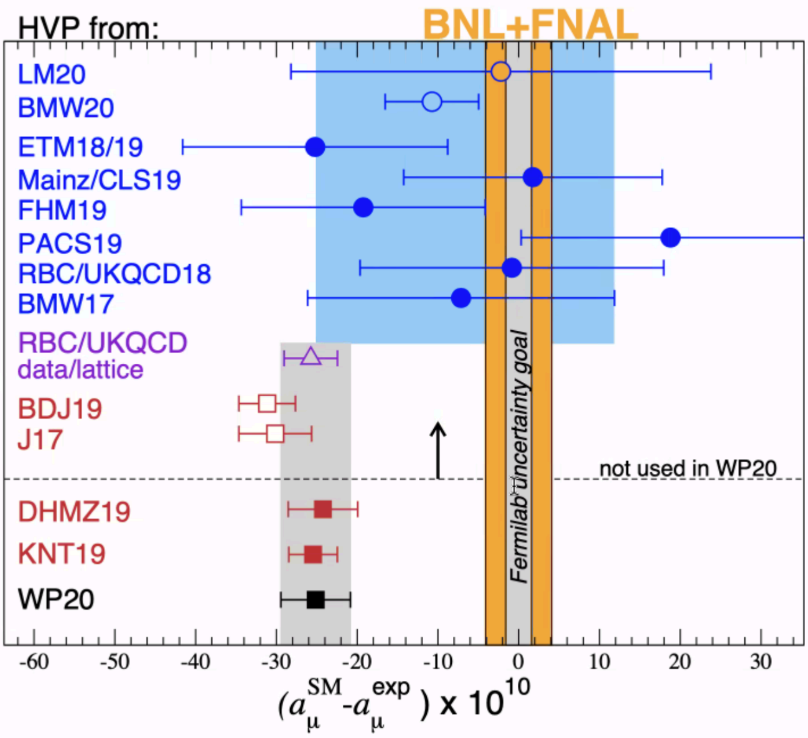
Uncertainty dominated by hadronic contributions

Status of hadronic light-by-light contribution



Systematically improvable methods are maturing; uncertainty to a_μ controlled at 0.15ppm; cross-checks detailed in Theory Initiative whitepaper

Status and impact of hadronic vacuum polarization contribution



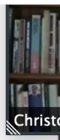
Ab-initio lattice QCD(+QED) calculations are maturing

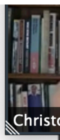
Difficult problem: scales from $2m_\pi$ to several GeV enter; cross-checks needed at high precision

Hybrid window method restricts scales that enter from lattice/dispersive data

Dispersive, $e^+e^- \to$ hadrons (20+ years of experiments)

Now first published lattice result with sub-percent precision available (BMW20), cross-checks are crucial to establish or refute high-precision lattice methodology (same situation as for HLbL)





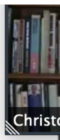
Summary of HVP status:

- Decades of e^+e^- dispersive results suggest a strong tension (4.2σ)
- A single lattice result (BMW20) suggests only minimal tension (1.5σ)

How can we move forward in our understanding? Main topic of this talk.

Two main questions:

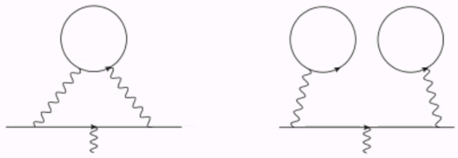
- Consistency of BMW20 lattice result with previously know lattice results
- Consistency of lattice results with R-ratio



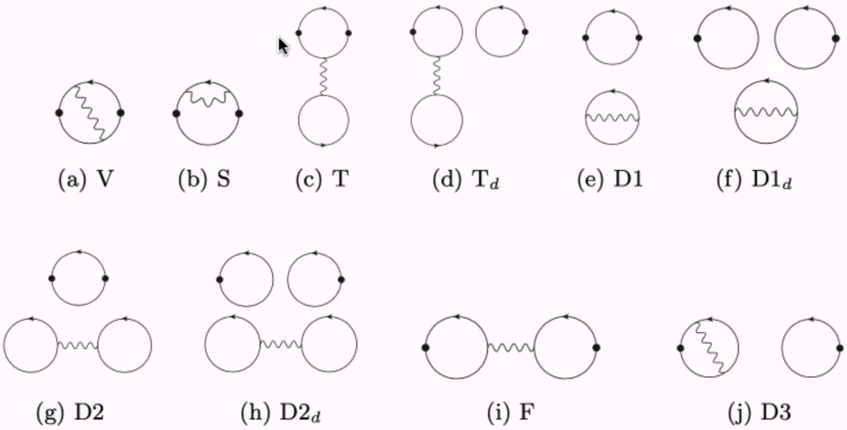
Consistency of BMW20 lattice result with previously know lattice results

Diagrams

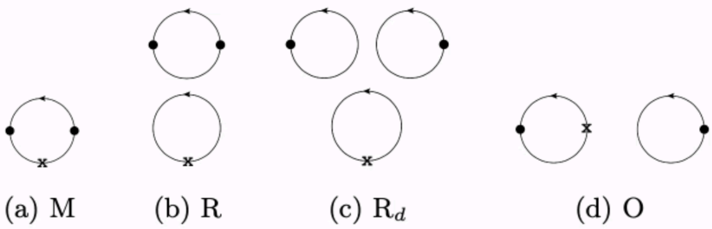
Isospin limit

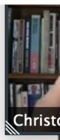


QED corrections



Strong isospin breaking





Diagrams – Isospin limit

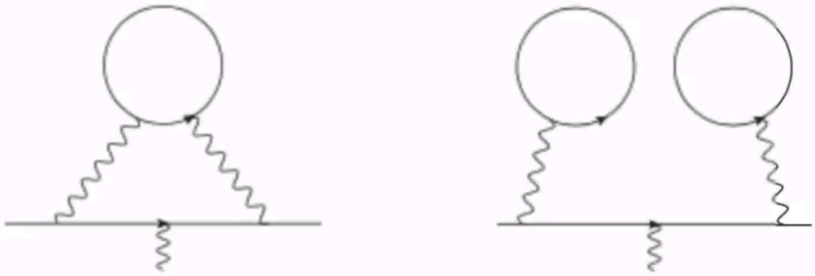
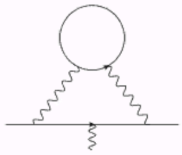
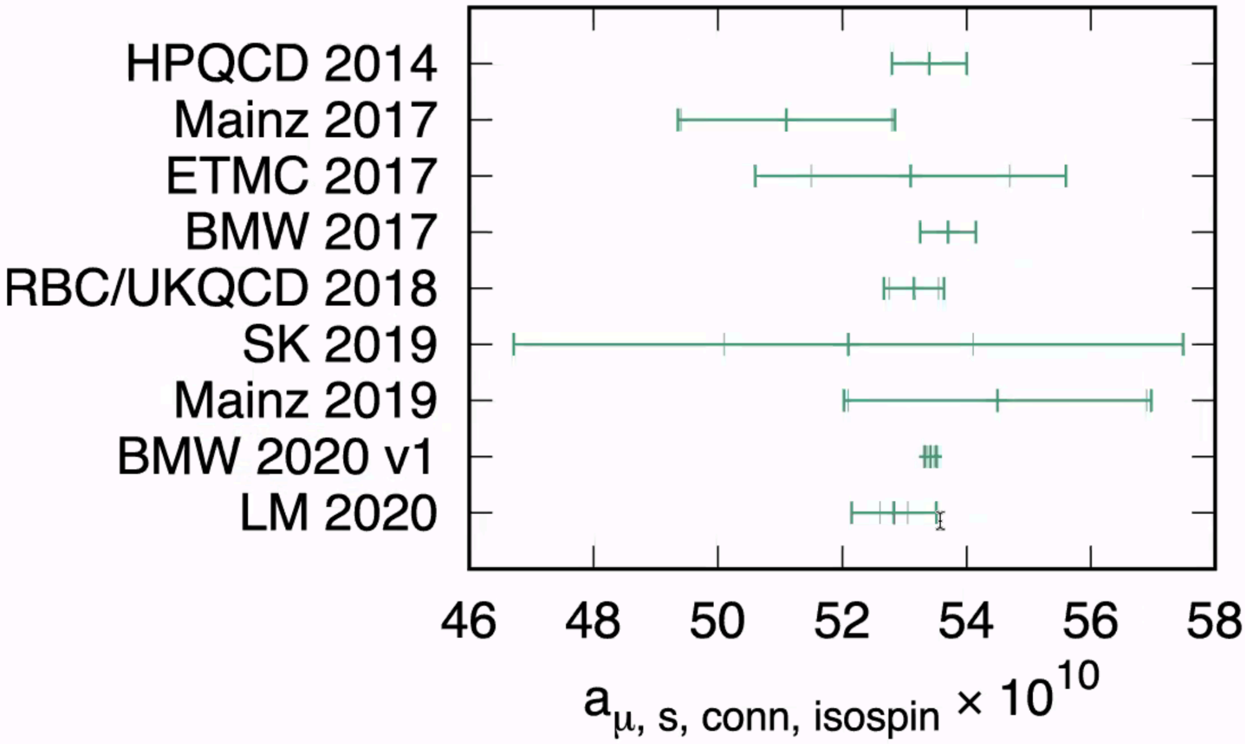


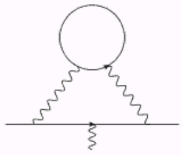
FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.

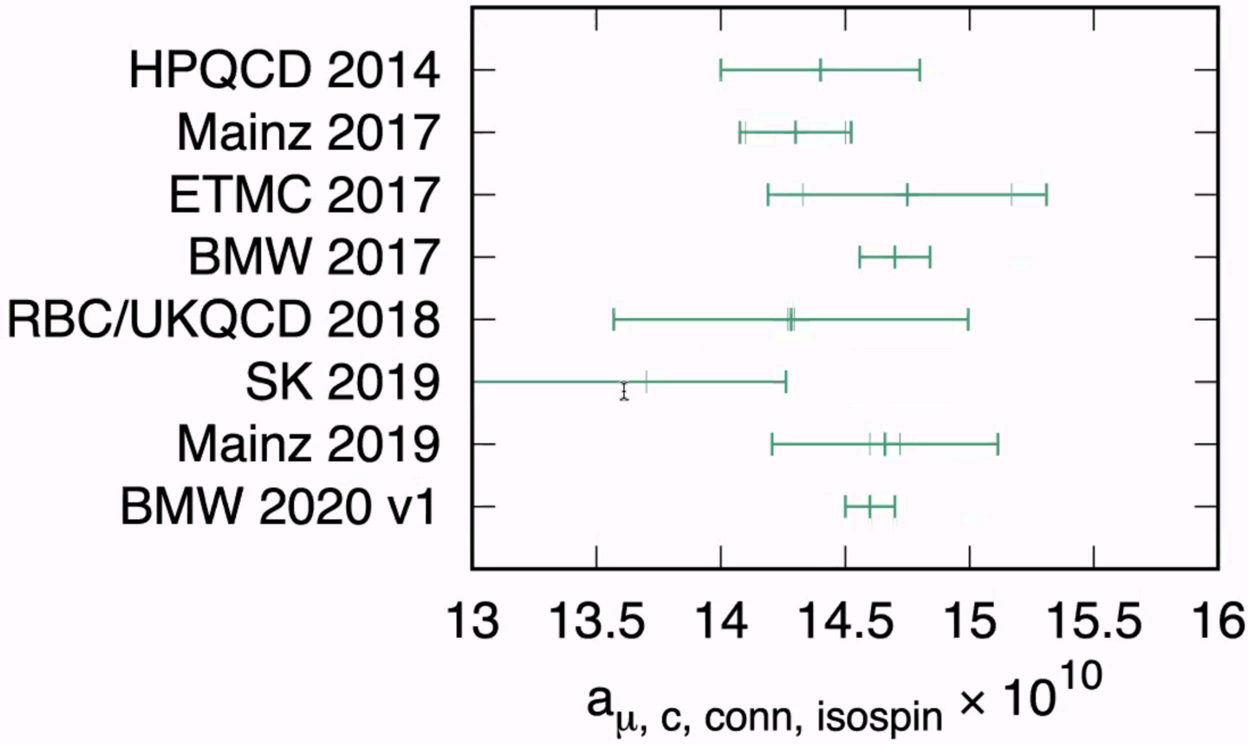
Strange



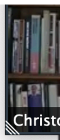

HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
BMW 2020 v1
LM 2020
46
48
50
52
54
56
58
$a_{\mu, s, conn, isospin} \times 10^{10}$

Charm



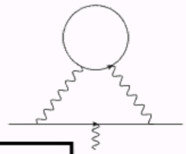


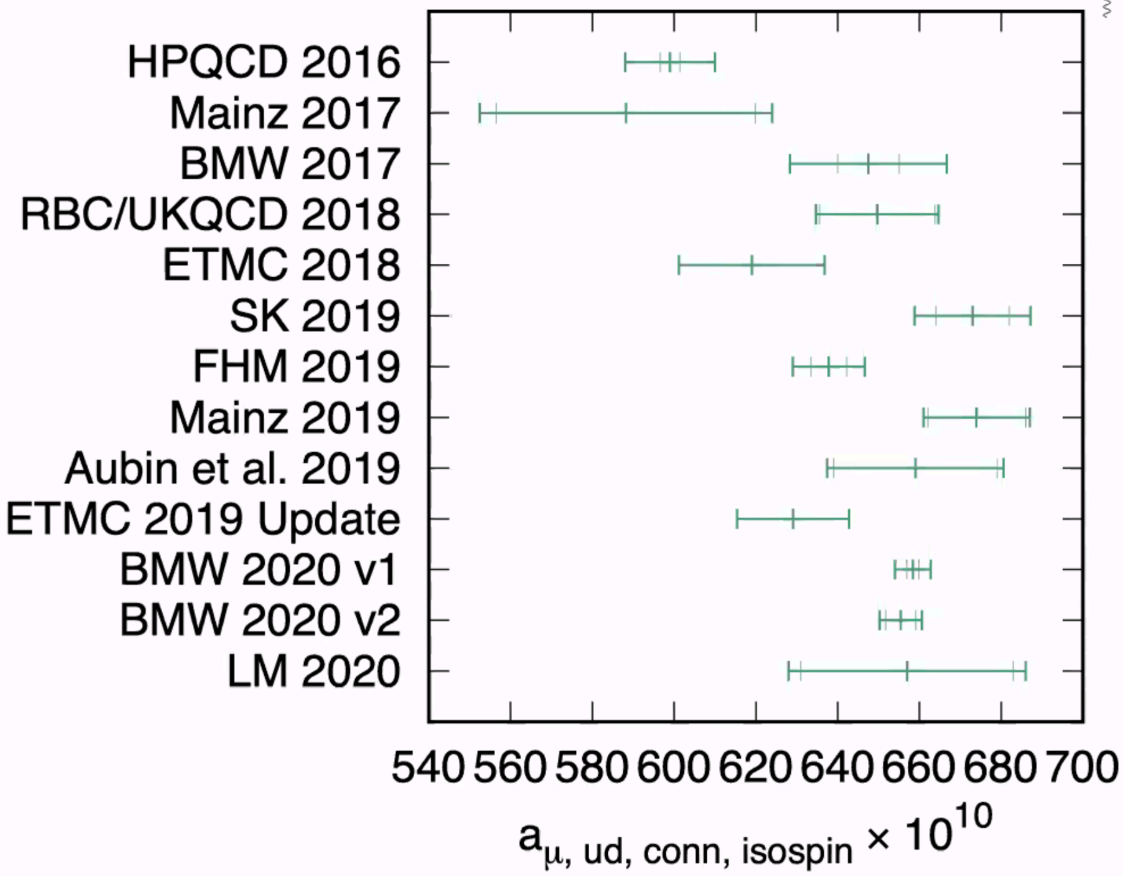
HPQCD 2014
Mainz 2017
ETMC 2017
BMW 2017
RBC/UKQCD 2018
SK 2019
Mainz 2019
BMW 2020 v1
13
13.5
14
14.5
15
15.5
16
$a_{\mu,\, c,\, conn,\, isospin} \times 10^{10}$



Consistency of BMW20 lattice result with previously know lattice results

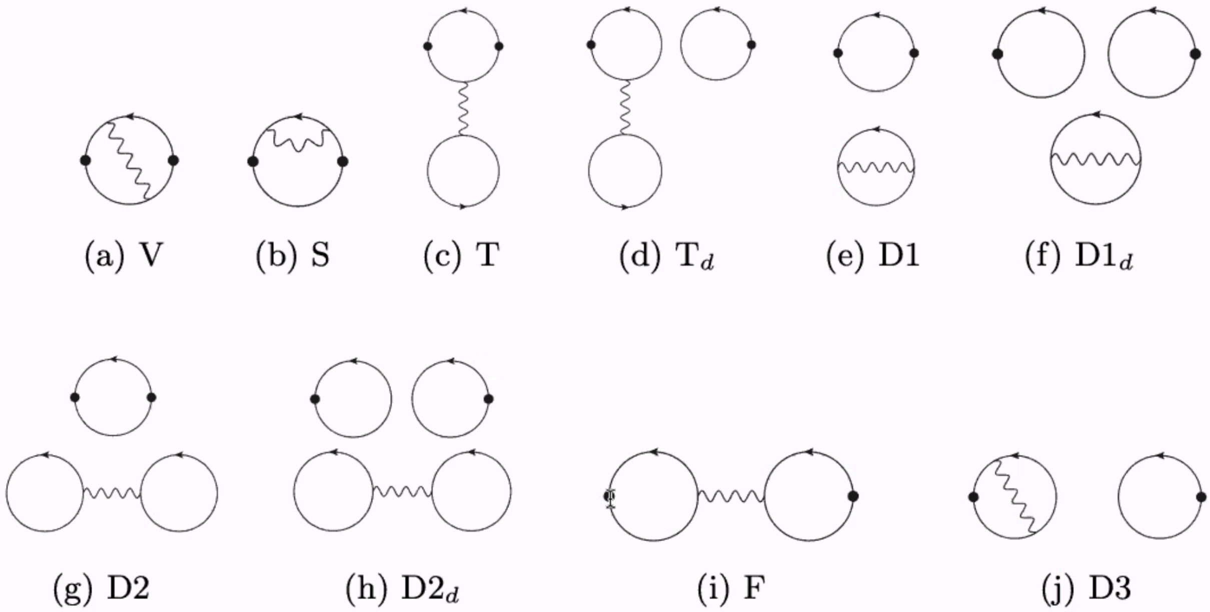
Up, down; isospin symmetric limit; $m_\pi = m_\pi^0$



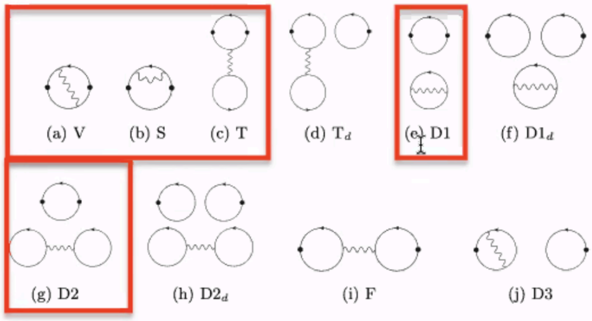


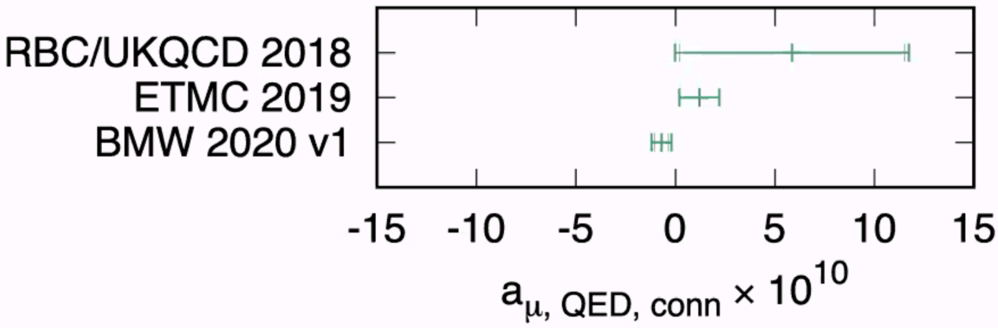
Some tensions to be understood

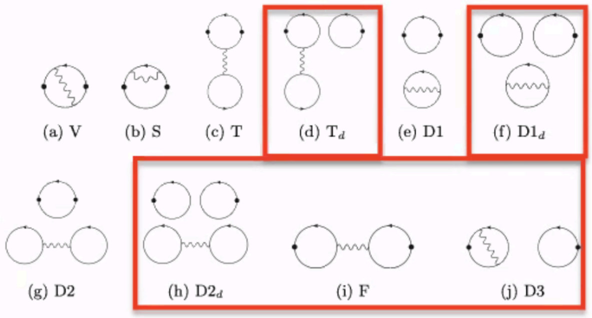
Diagrams – QED corrections

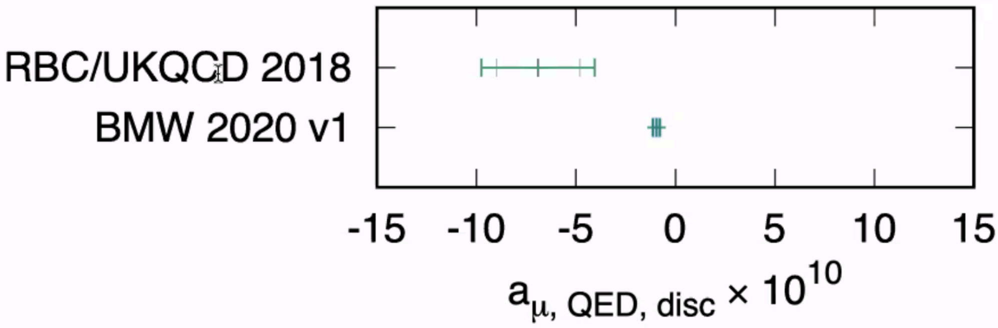


For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.


(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3

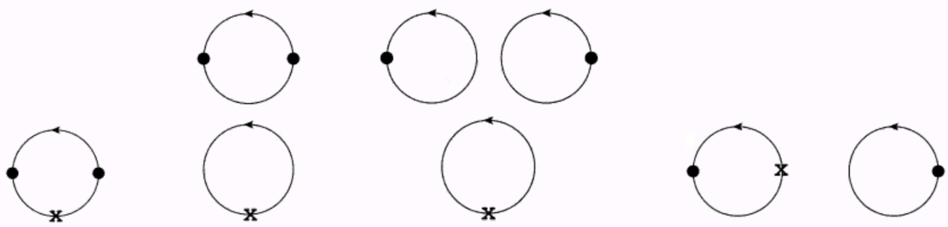

RBC/UKQCD 2018
ETMC 2019
BMW 2020 v1
-15
-10
-5
0
5
10
15
$a_{\mu,\ \mathrm{QED,\ conn}} \times 10^{10}$





Attention needed

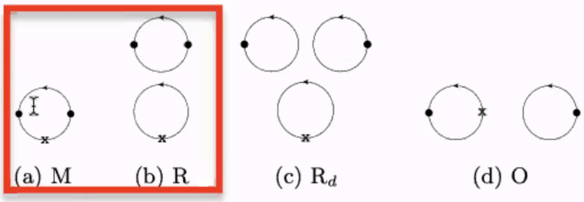
Diagrams – Strong isospin breaking

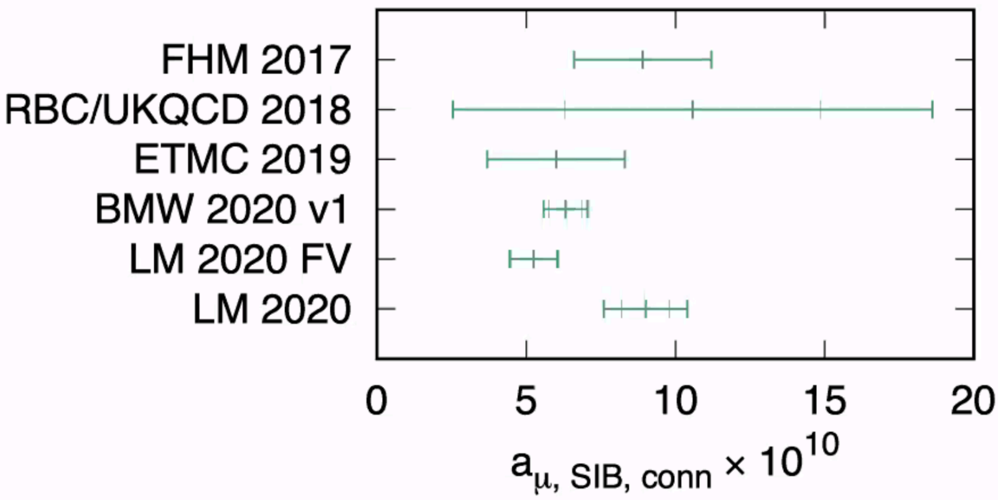


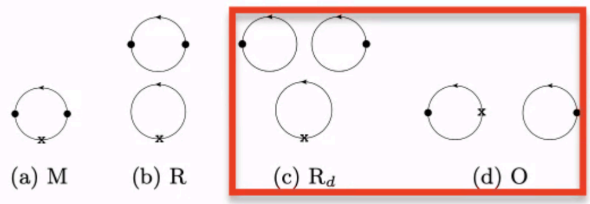
(a) M (b) R (c) R_d (d) O

For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.

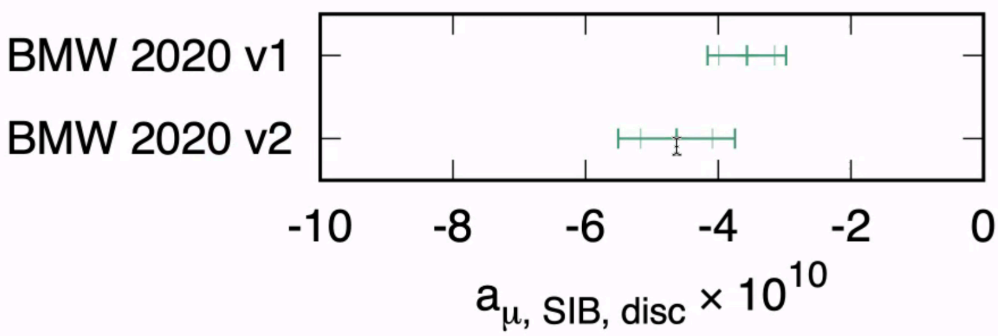
Lehner, Meyer 2020: NLO PQChPT: FV effects in connected and disconnected cancel but are each significant $O(4 \times 10^{-10})$; PQChPT expects cancellation between connected and disconnected contribution $a_\mu^{\mathrm{SIB,\ conn.}} = -a_\mu^{\mathrm{SIB,\ disc.}} = 6.9 \times 10^{-10}$


(a) M
(b) R
(c) R_d
(d) O


FHM 2017
RBC/UKQCD 2018
ETMC 2019
BMW 2020 v1
LM 2020 FV
LM 2020
0
5
10
15
20
$a_{\mu,\text{ SIB, conn}} \times 10^{10}$



(a) M
(b) R
(c) R_d
(d) O



BMW 2020 v1
BMW 2020 v2
-10
-8
-6
-4
-2
0
$a_{\mu,\,\mathrm{SIB,\,disc}} \times 10^{10}$

Attention on light-quark isospin-symmetric contribution and QED disconnected contribution

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

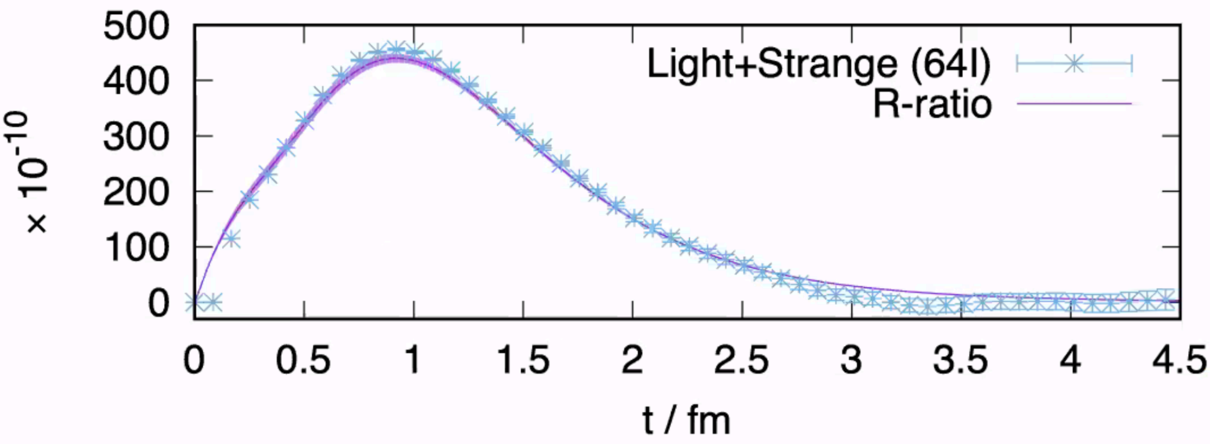
with

$$C(t) = \frac{1}{3}\sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x},t) J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Lattice QCD – Example of correlation function $C(t)$ (RBC/UKQCD18)



Large discretization errors at short distance, large finite-volume errors and statistical errors at large distance

Window method (introduced in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

with

$$a_\mu^{\mathrm{SD}} = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)] \,,$$

$$a_\mu^{\mathrm{W}} = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)] \,,$$

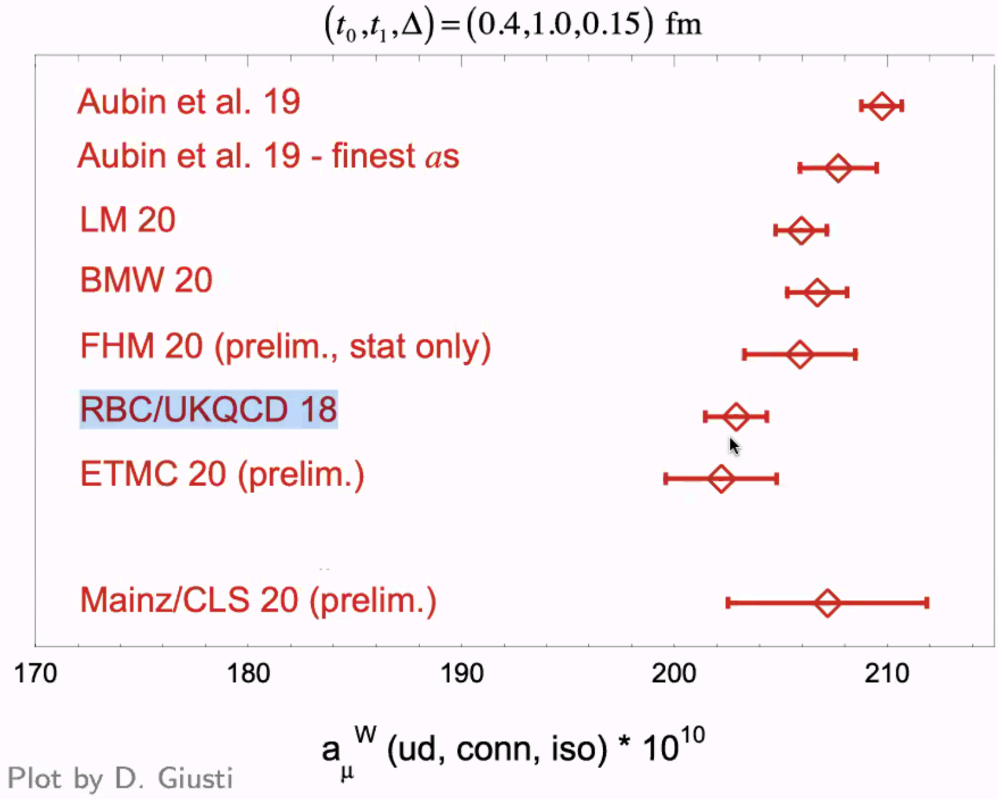
$$a_\mu^{\mathrm{LD}} = \sum_t C(t) w_t \Theta(t, t_1, \Delta) \,,$$

$$\Theta(t, t', \Delta) = [1 + \tanh[(t - t')/\Delta]] /2 \,.$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+e^- \to \mathrm{had})$.

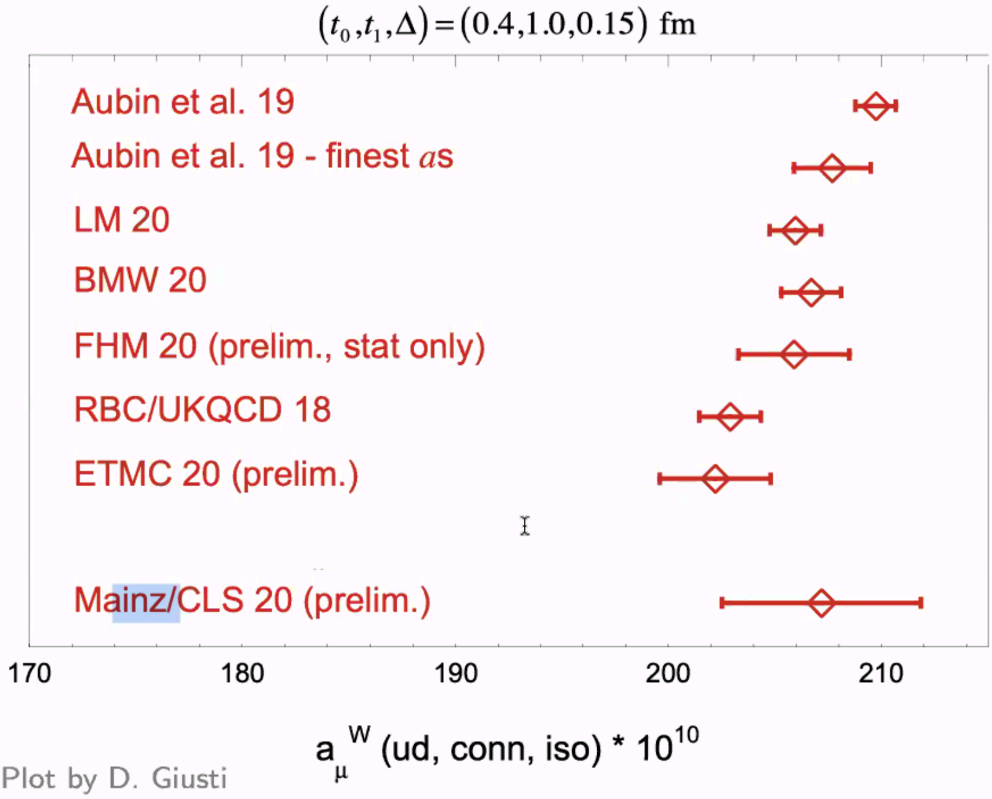
a_μ^{W} has small statistical and systematic errors on lattice!

Use these windows as a lattice internal cross-check



Plot from recent theory initiative workshop (https://indico.cern.ch/event/956699/)

Use these windows as a lattice internal cross-check



Plot by D. Giusti

Plot from recent theory initiative workshop (https://indico.cern.ch/event/956699/)

Status of consistency of lattice results

Significant difference between published high-precision LQCD results (BMW20 and RBC/UKQCD18) for window with $t_0 = 0.4\text{fm}$ and $t_1 = 1.0\text{fm}$:

$$a_{\rm W}^{\rm BMW20} = 207.3(1.4) \times 10^{-10}\,, \tag{1}$$

$$a_{\rm W}^{\rm RBC/UKQCD18} = 202.9(1.4)(0.4) \times 10^{-10} \tag{2}$$

and therefore there is a 2.2σ tension

$$a_{\rm W}^{\rm BMW20} - a_{\rm W}^{\rm RBC/UKQCD18} = 4.4(2.0) \times 10^{-10}\,. \tag{3}$$

Scaled to the total $a_\mu^{\rm HVP}$ this corresponds to 15×10^{-10} uncertainty on the lattice HVP compared to current 5.5×10^{-10} uncertainty of BMW20.

Urgently need new results for this and other windows. Update by RBC/UKQCD 2018 is in preparation. Hopefully available within two months. More groups to join. Important: different regulators!

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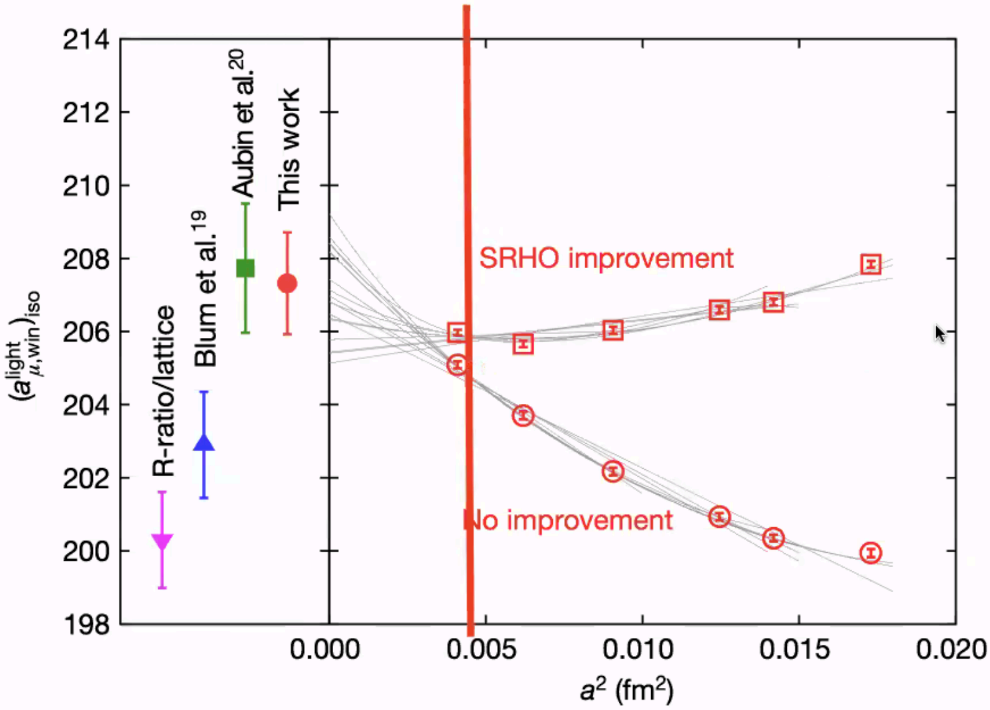
Continuum extrapolation - What lattice spacing is fine enough?

BMW 20 - light quark window

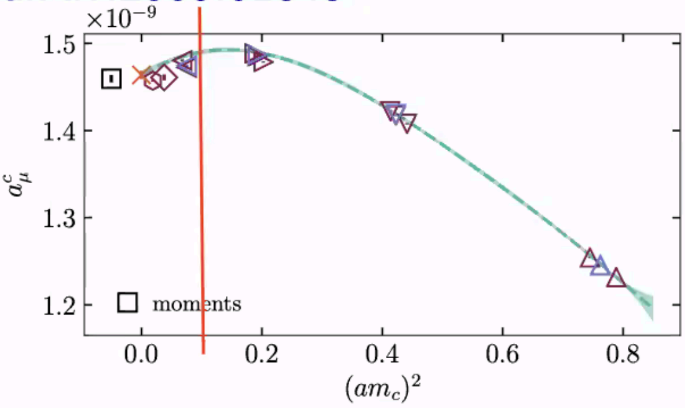


3.7σ tension between BMW20 and R-ratio for Window! Discuss in second part of talk.

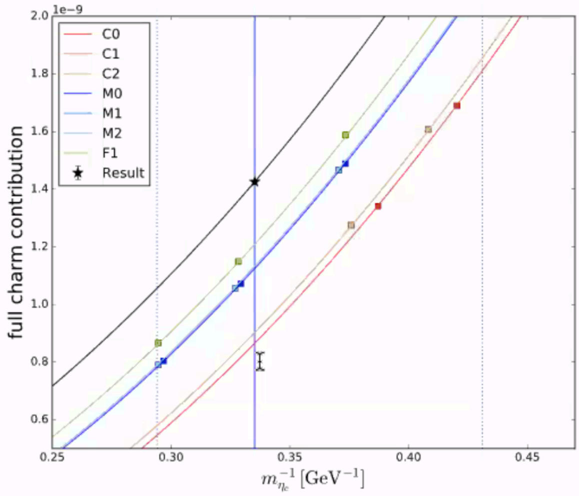
Red line for comparison with next slide

Continuum extrapolation - What lattice spacing is fine enough?

HPQCD 20 charm quark full a_μ arXiv:2005.01845



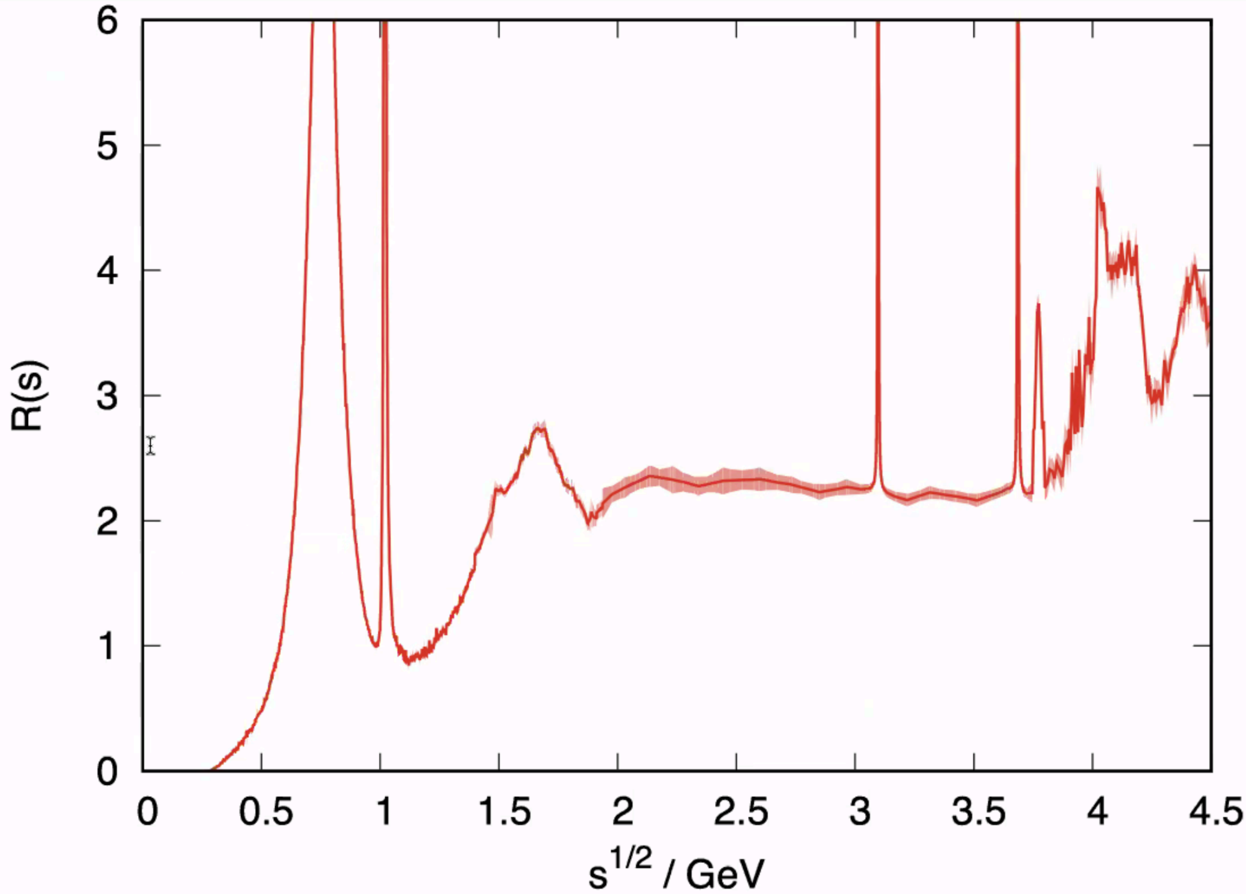
RBC 18 charm quark full a_μ



Finest lattice spacing in this extrapolation is green; approximately corresponds to red line in previous plots

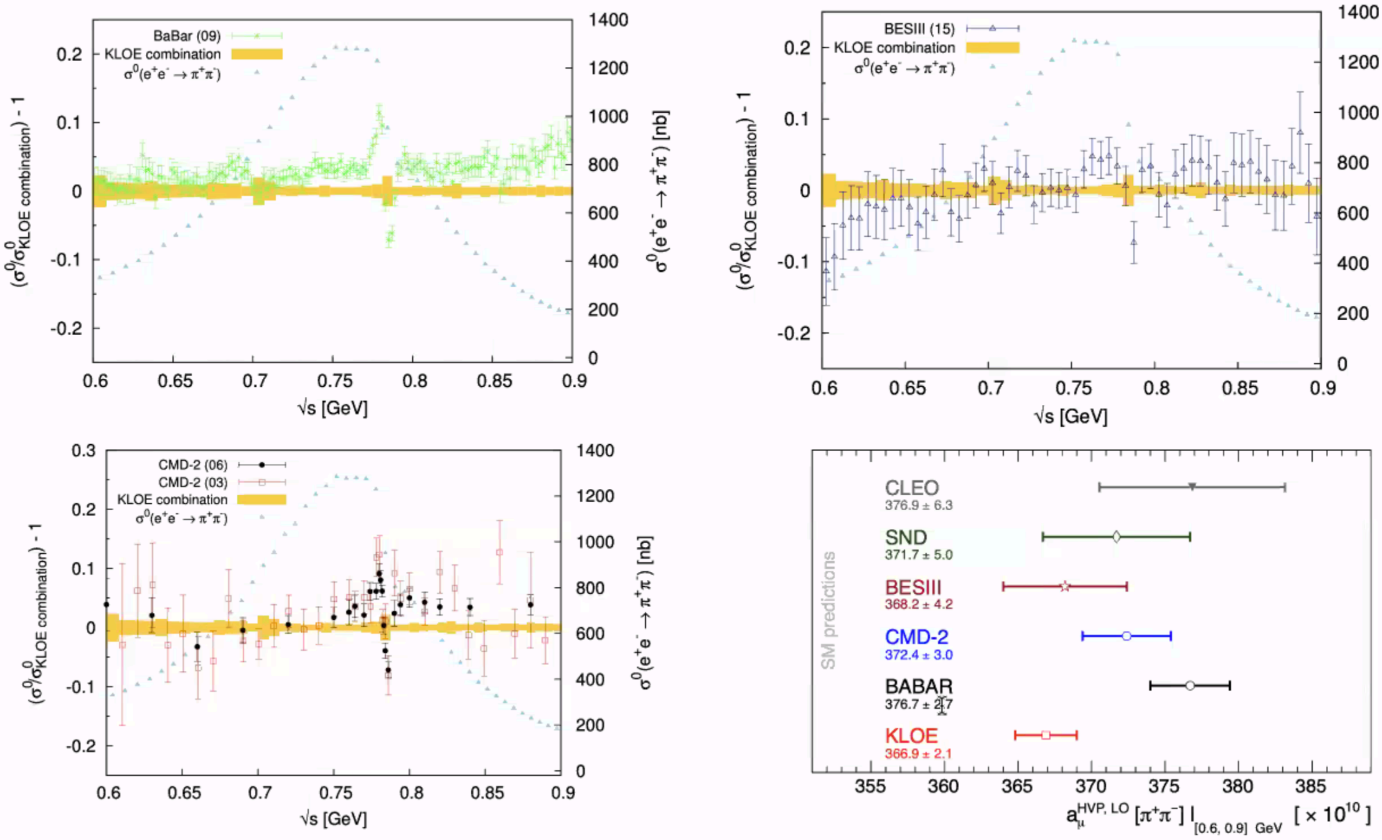
Restricting to fixed lattice spacing range can lead to different discretization errors for different UV regulators; systematically independent calculations very desirable!

Consistency of lattice result with R-ratio



$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had}), \quad C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t}$$

Tensions in input data, however, already taken into account in WP20 merger of KNT19 and DHMZ19:



What does tension in windows mean for R-ratio?

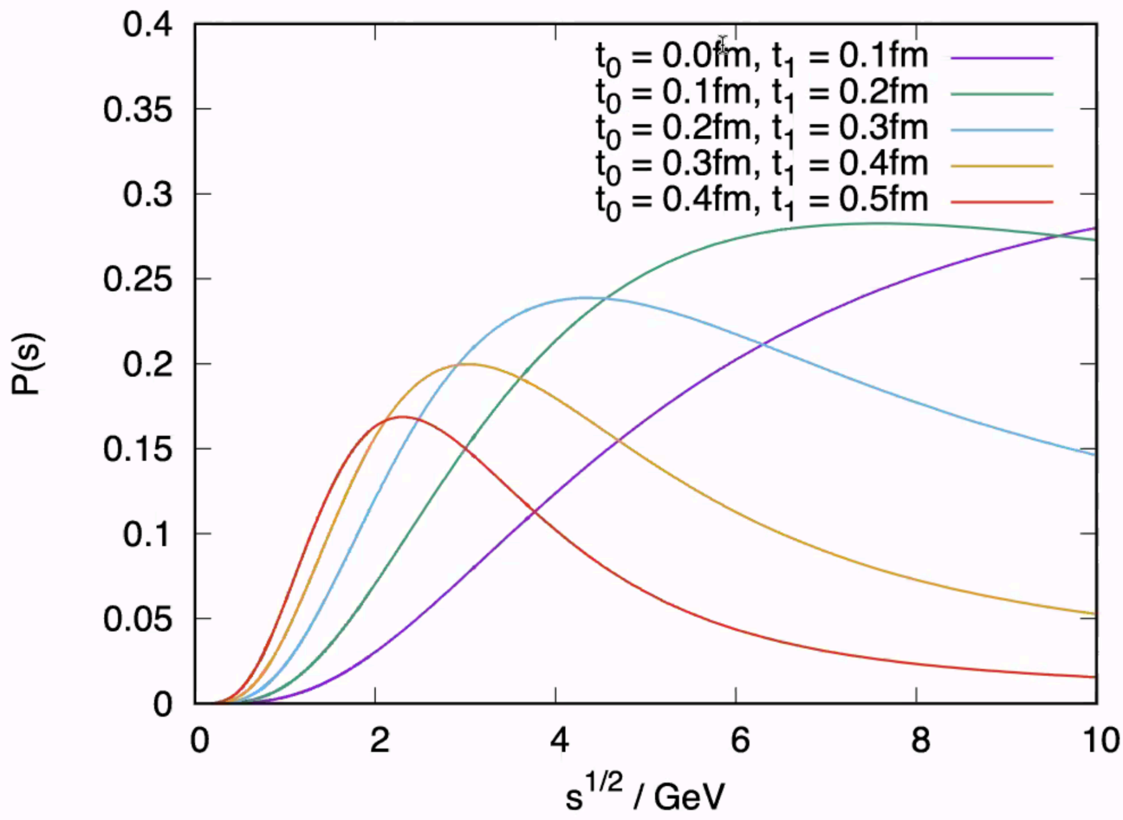
Talk by Massimo Passera last week: if there is a shift in R-ratio, it crucially depends on which energy to understand what the impact on $\Delta\alpha$ and EW precision physics is.

Express Euclidean Windows in time-like region:

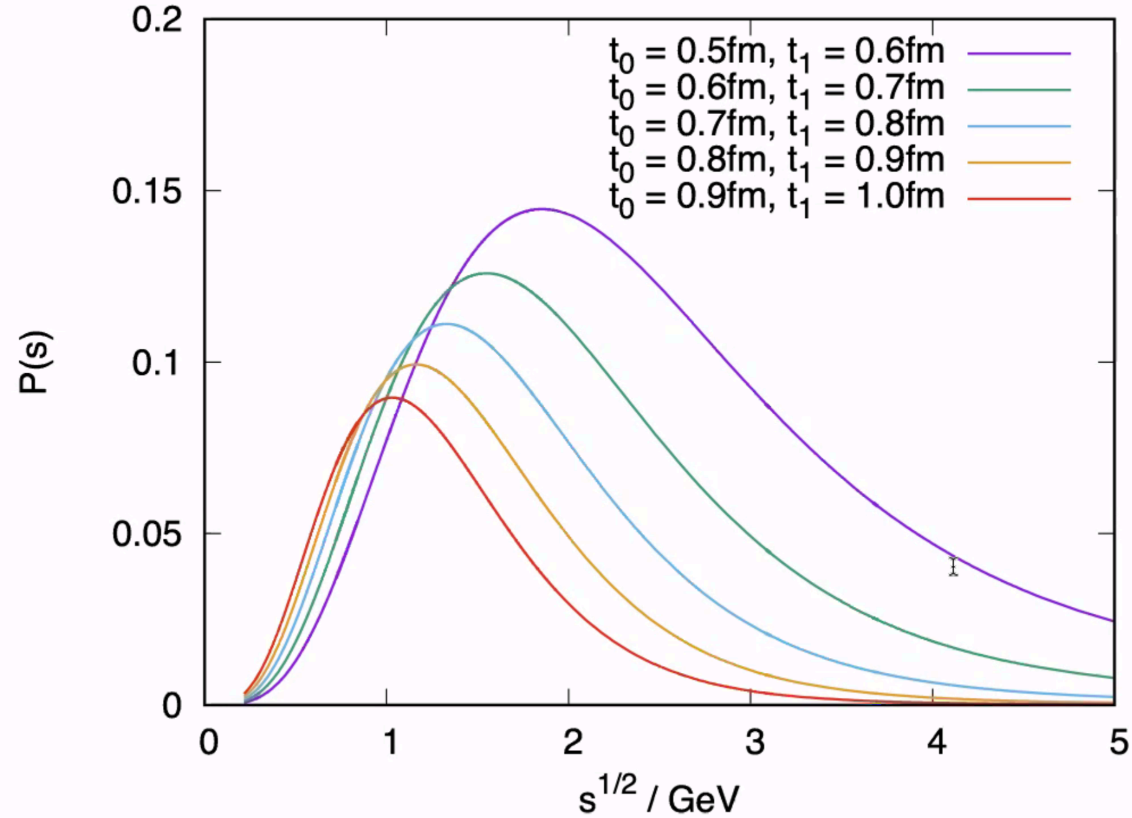
$$a_\mu = \int_0^\infty ds\, R(s)K(s) \tag{4}$$

and window

$$a_\mu^{\mathrm{W}} = \int_0^\infty ds\, R(s)K(s)P(s)\,. \tag{5}$$



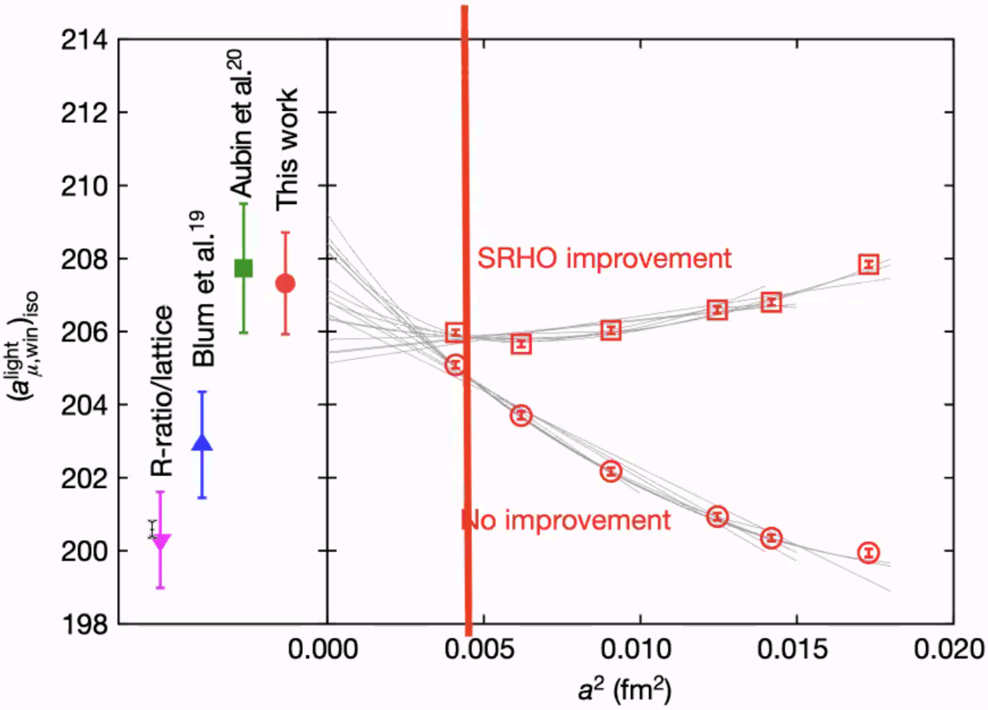
Study of windows for different t_0 and t_1 can give some energy resolution!



Study of windows for different t_0 and t_1 can give some energy resolution!

Continuum extrapolation - What lattice spacing is fine enough?

BMW 20 - light quark window



3.7σ tension between BMW20 and R-ratio for Window! Discuss in second part of talk.

Red line for comparison with next slide

What can we expect from LQCD in the coming years?

- More published results with high precision with different regulators for the standard window $t_0 = 0.4\text{fm}$, $t_1 = 1.0\text{fm}$, $\Delta = 0.15\text{fm}$. This will clarify the 2.2σ tension between BMW20 and RBC/UKQCD18 for this quantity.

- More results for different windows, which will give energy resolution to locate possible remaining tension with R-ratio in time-like energy. After this: any impact on $\Delta\alpha$ and EW precision physics?

- More results of complete high-precision HVP results from major lattice collaborations. RBC/UKQCD18 aims for end of this year.

Outlook

- Expect more lattice HVP calculations at few per-mille level precision which allows for proper scrutiny at high precision; For total a_μ as well as windows!
- Data-driven dispersive results will improve with expected experimental results from Belle II, BESIII, CMD-3, and SND
- MUonE at CERN will provide complementary measurements for the HVP
- Theory Initiative will publish updated SM predictions as experiment and theory improves; provides platform for cross-checks and establishing new methodology

Thank You!