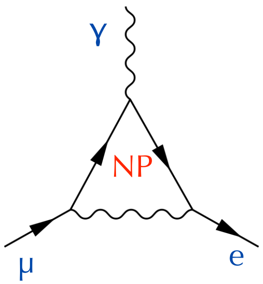
Muon g-2 and EDM

cLFV school

July 5-6, 2019

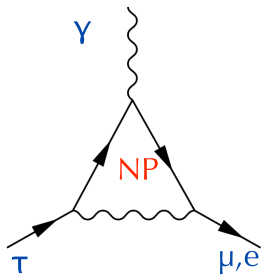
Tsutomu Mibe (IPNS, KEK)

Examples of New Physics diagrams



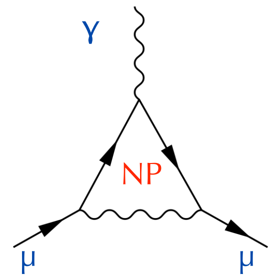
$$\mu \longrightarrow e\gamma$$

MEG II



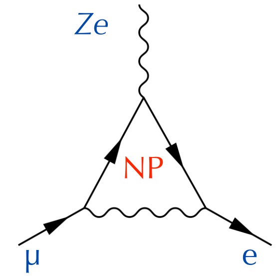
$$\tau \longrightarrow \mu\gamma$$
$$\tau \longrightarrow e\gamma$$

Belle II



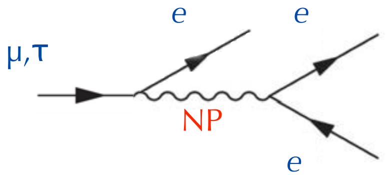
$$(g-2)_\mu$$

Fermilab
J-PARC



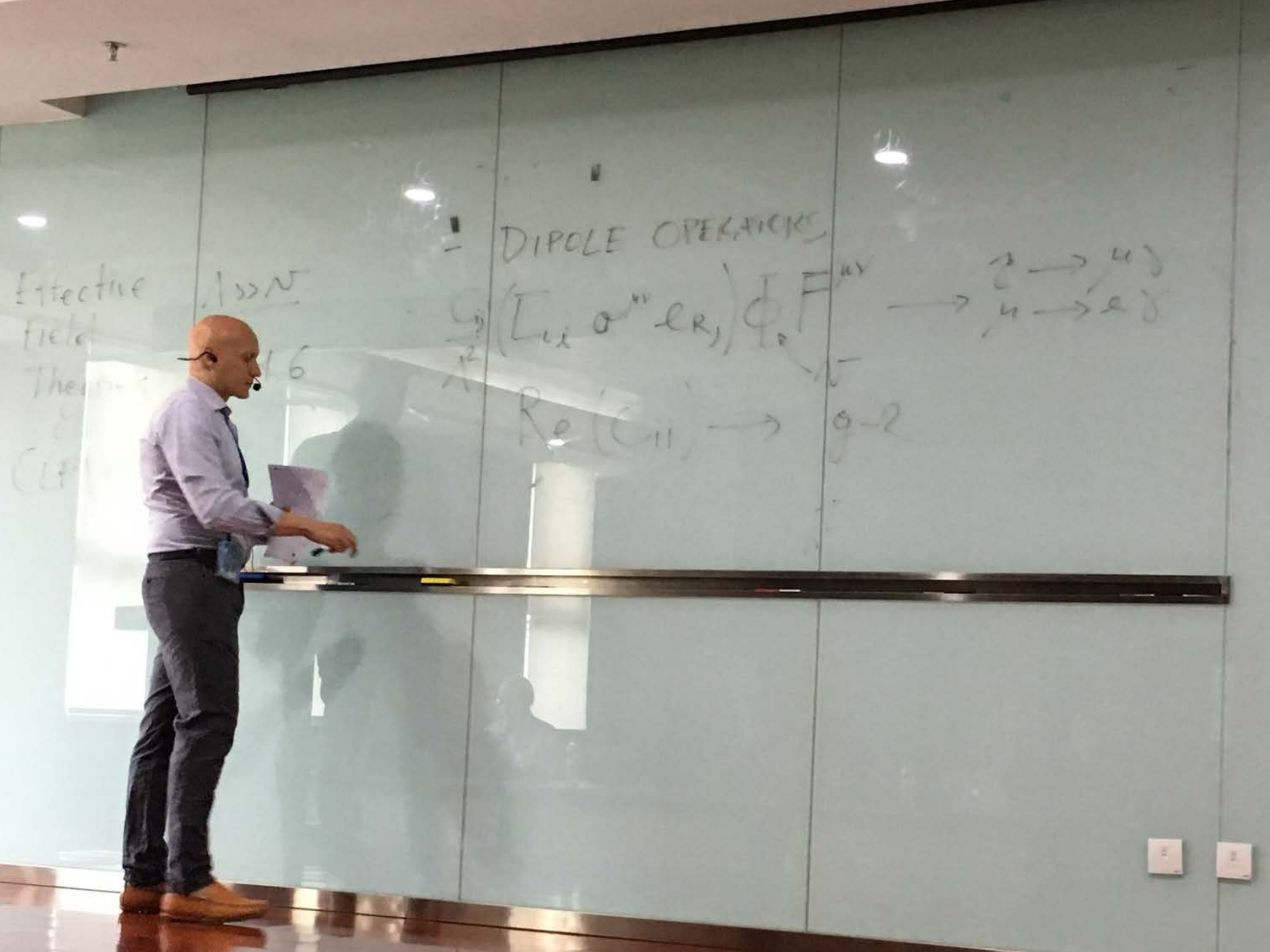
$$\mu^- \mathcal{N} \rightarrow e^- \mathcal{N}$$

COMET
mu2e



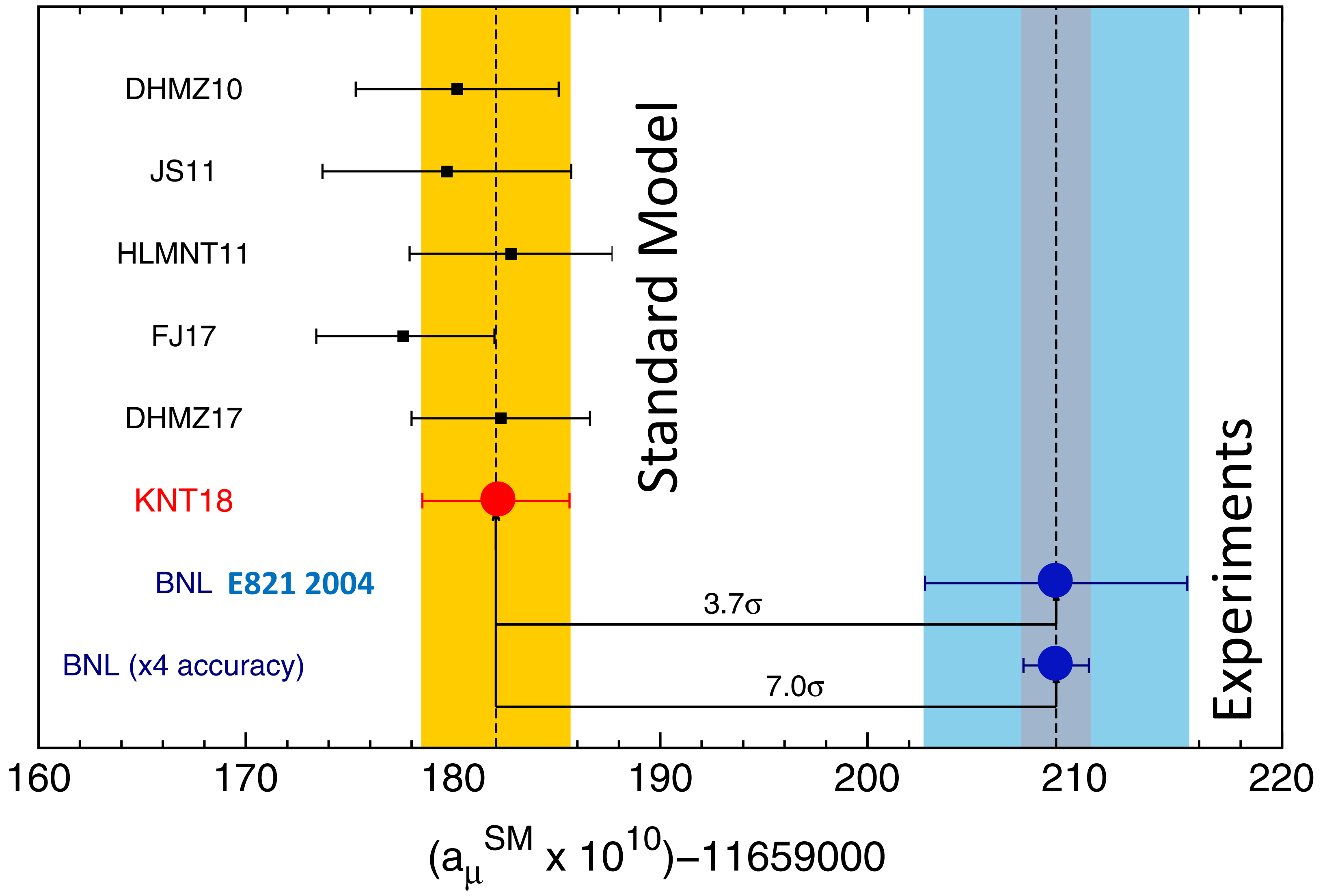
$$\mu \longrightarrow eee$$

mu3e


DIPOLE OPERATORS
Effective
Field

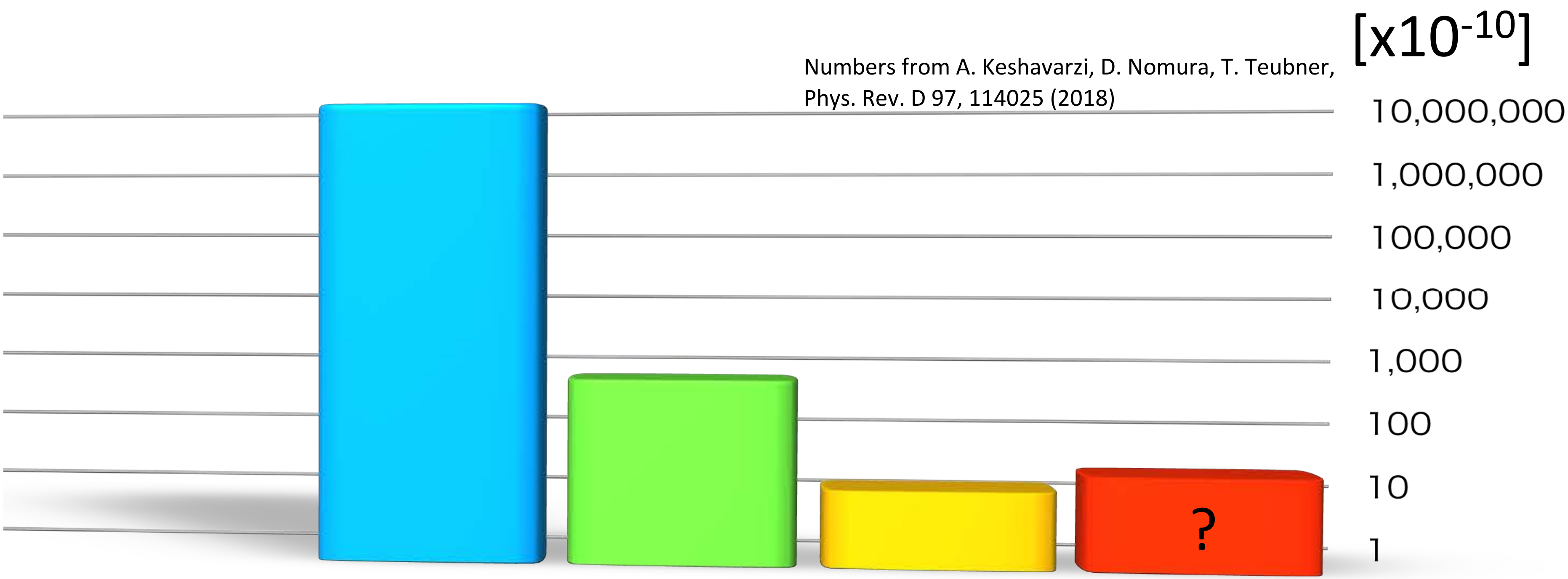
Anomaly in muon g-2

A. Keshavarzi, D. Nomura, T. Teubner, Phys. Rev. D 97, 114025 (2018)



Note that electron g-2 is consistent with the SM.

Why muon g-2 is important?



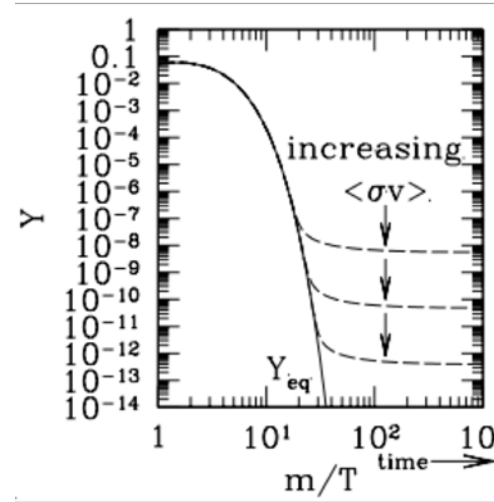
$$a_\mu = a_\mu(QED) + a_\mu(had) + a_\mu(weak) + a_\mu(BSM)$$

Anomaly effect as big as the weak contributions

Why muon g-2 is important?

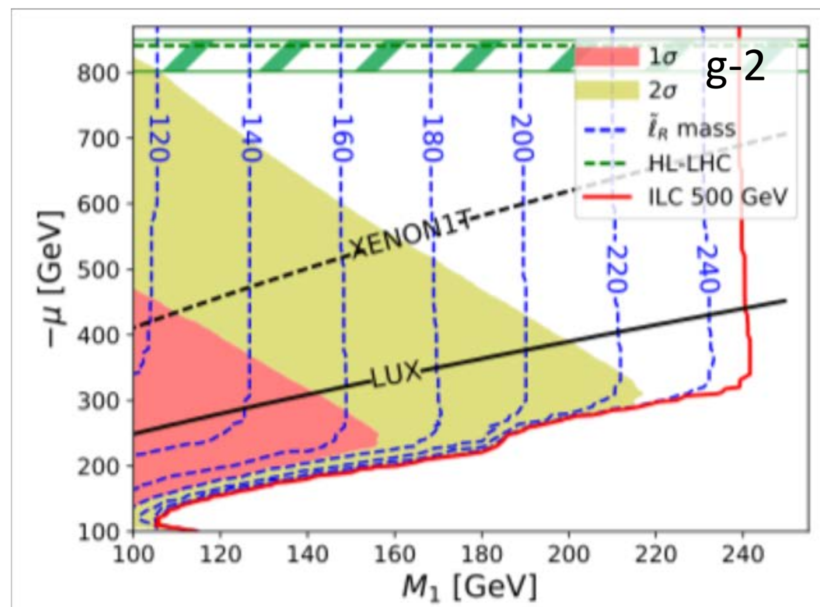
- Thermal dark matter relic density
  - $<\sigma_v> \sim 3 \times 10^{-26}\ cm^2/s$ (from CMB)

$$< \sigma_v > \sim \frac{\alpha_{new}^2}{m_{new}^2}$$

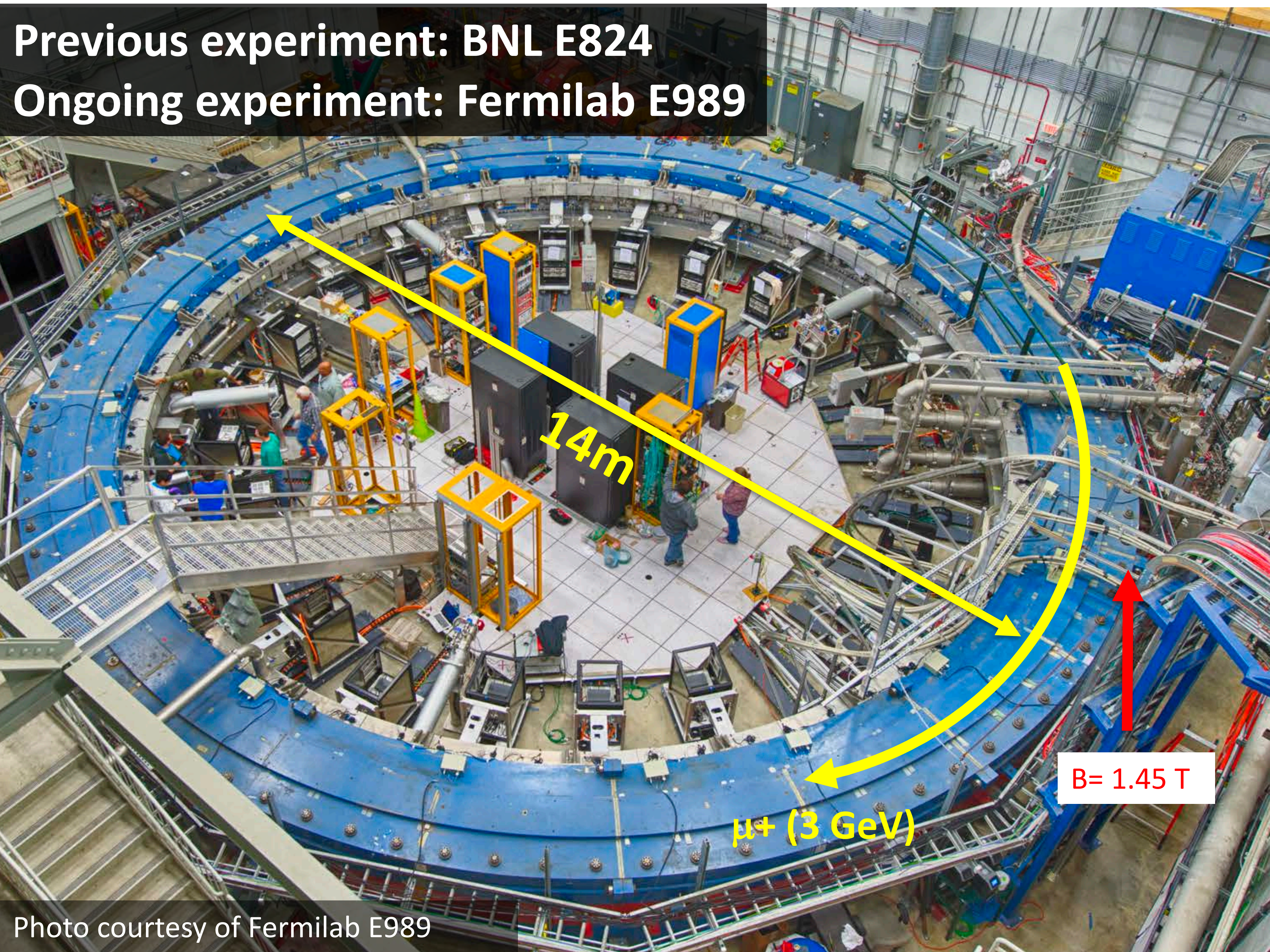


- Contribution to muon g-2

$$\Delta a_\mu \sim \frac{\alpha_{new}}{m_{new}^2}$$



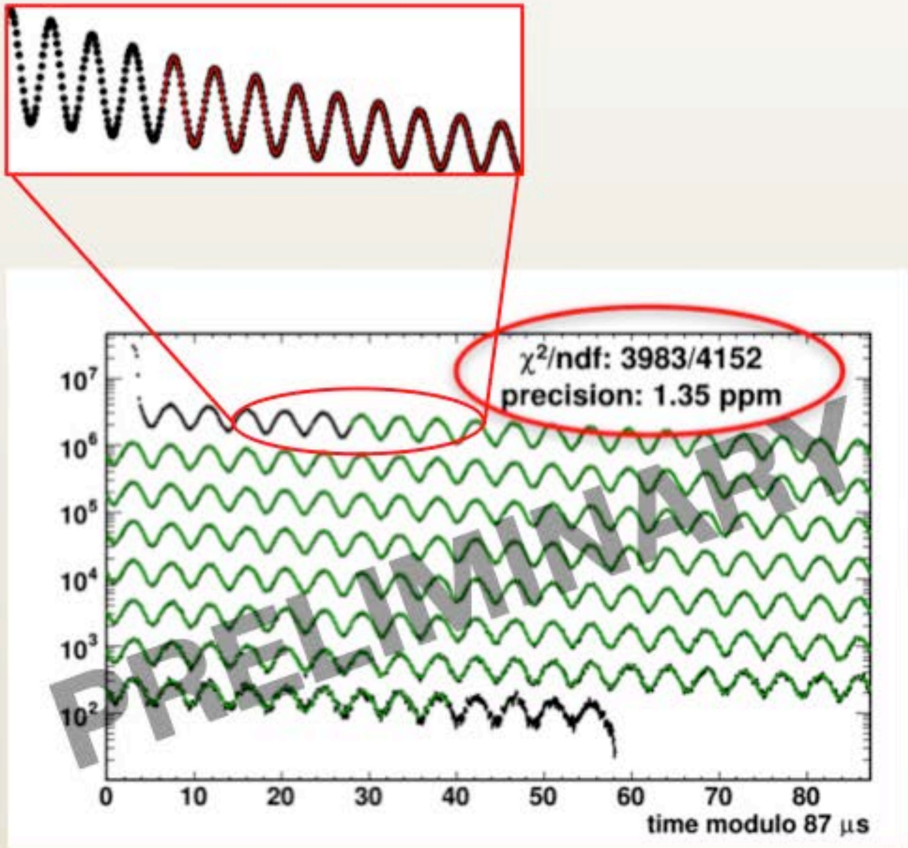
“Probing minimal SUSY scenarios in the light of muon g − 2 and dark matter” M. Endo, K. Hamaguchi, S. Iwamoto, and K. Yanagi, JHEP 1706, 031 (2017)

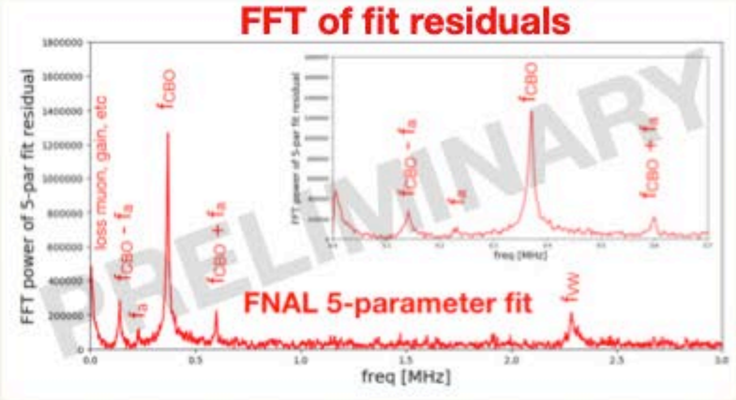

Previous experiment: BNL E824
Ongoing experiment: Fermilab E989
14m
μ+ (3 GeV)
B= 1.45 T
Photo courtesy of Fermilab E989

Slide by Nam Tran, FPCP 2019 (May 6)

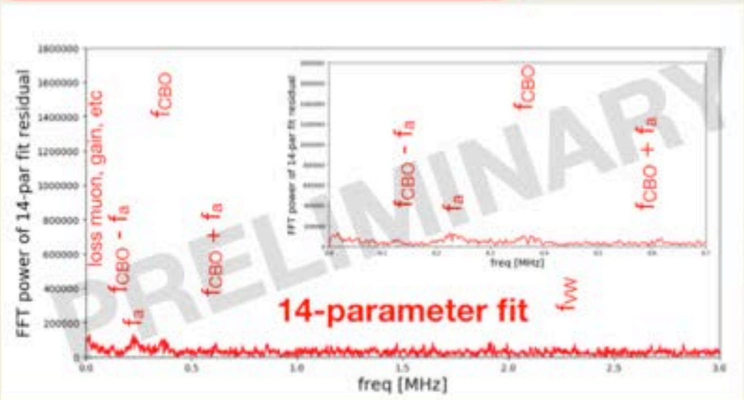
ω_a in Run 1

$$N(t) = N_0 e^{-t/\tau}\left[1 - A\cos\left(\omega_a t + \phi\right)\right]$$



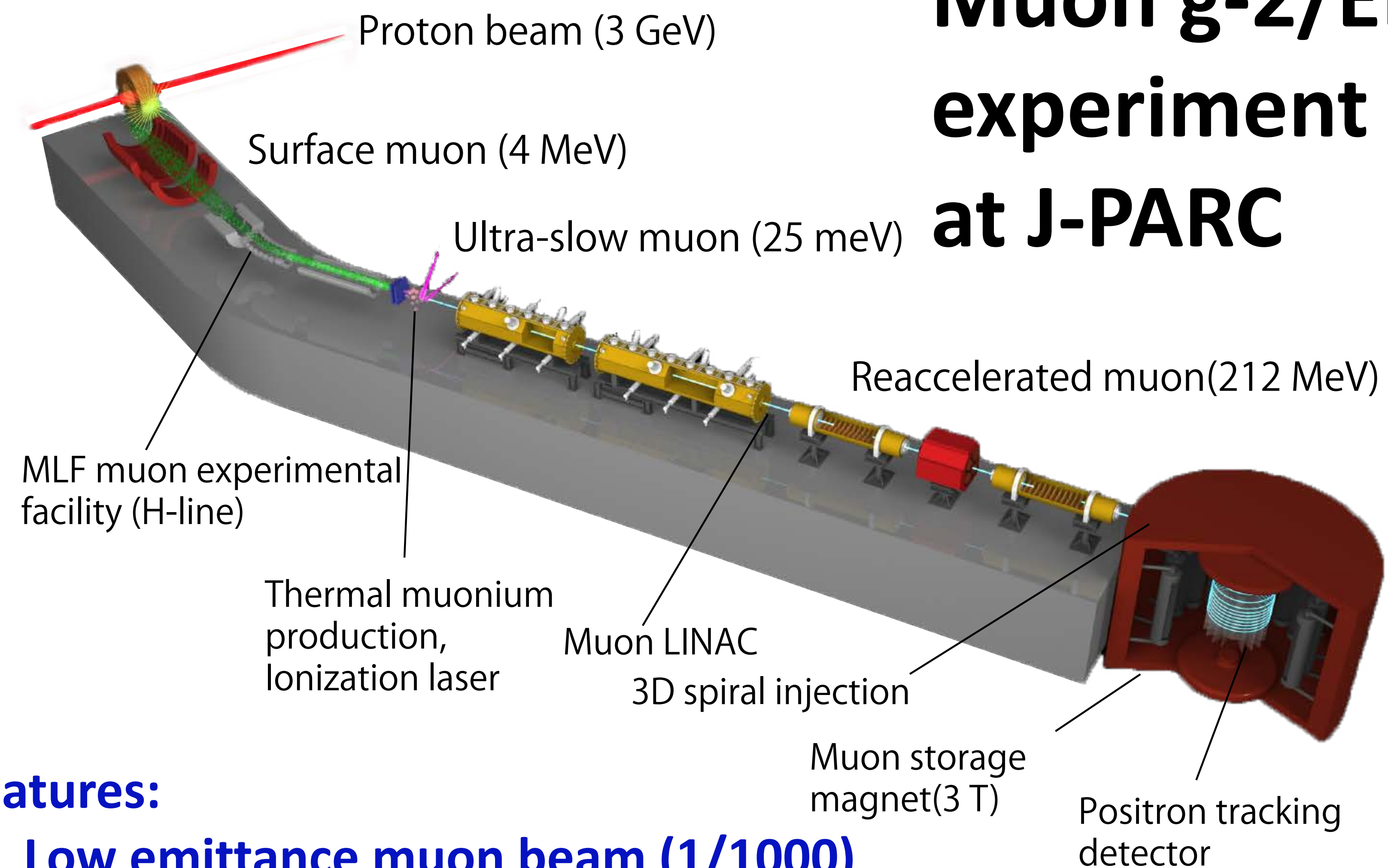


Big improvements when accounting for CBO, lost muons,...



BOSTON UNIVERSITY

Muon g-2/EDM experiment at J-PARC



Features:

- **Low emittance muon beam (1/1000)**
- **No strong focusing (1/1000) & good injection eff. (x10)**
- **Compact storage ring (1/20)**
- **Tracking detector with large acceptance**
- **Completely different from BNL/FNAL method**

Comparison of experiments

	BNL-E821	Fermilab-E989	Our experiment
Muon momentum	3.09 GeV/c		300 MeV/c
Lorentz γ	29.3		3
Polarization	100%		50%
Storage field	$B = 1.45$ T		$B = 3.0$ T
Focusing field	Electric quadrupole		Very weak magnetic
Cyclotron period	149 ns		7.4 ns
Spin precession period	4.37 μs		2.11 μs
Number of detected e^+	5.0×10^9	1.6×10^{11}	5.7×10^{11}
Number of detected e^-	3.6×10^9	–	–
a_μ precision (stat.)	460 ppb	100 ppb	450 ppb
(syst.)	280 ppb	100 ppb	<70 ppb
EDM precision (stat.)	0.2×10^{-19} $e \cdot$ cm	–	1.5×10^{-21} $e \cdot$ cm
(syst.)	0.9×10^{-19} $e \cdot$ cm	–	0.36×10^{-21} $e \cdot$ cm

Prog. Theor. Exp. Phys. **2019**, 053C02 (2019)

Contents

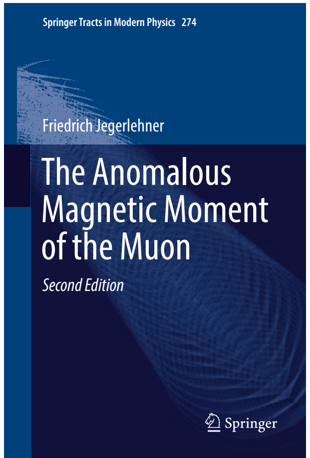
- Spin properties of muon
- Building a magnet from SM
- Measurement of g-2
- Searching for EDM
- Technical advances for higher precision
- Auxiliary measurements with muonium

References

- "Precision Muon Physics" T.P. Gorringe, D.W. Hertzog
  - Prog. Part. Nucl. Phys. 84, 73 (2015)

- "Lepton Dipole Moments" L. Roberts, W. Marciano
  - Advanced Series on Directions in High Energy Physics – Vol. 20, World Scientific (2010)

- "The Anomalous Magnetic Moment of the Muon" (2^{nd} ed.), by F. Jegerlehner
  - Springer Tracts in Modern Physics 274 (2017)

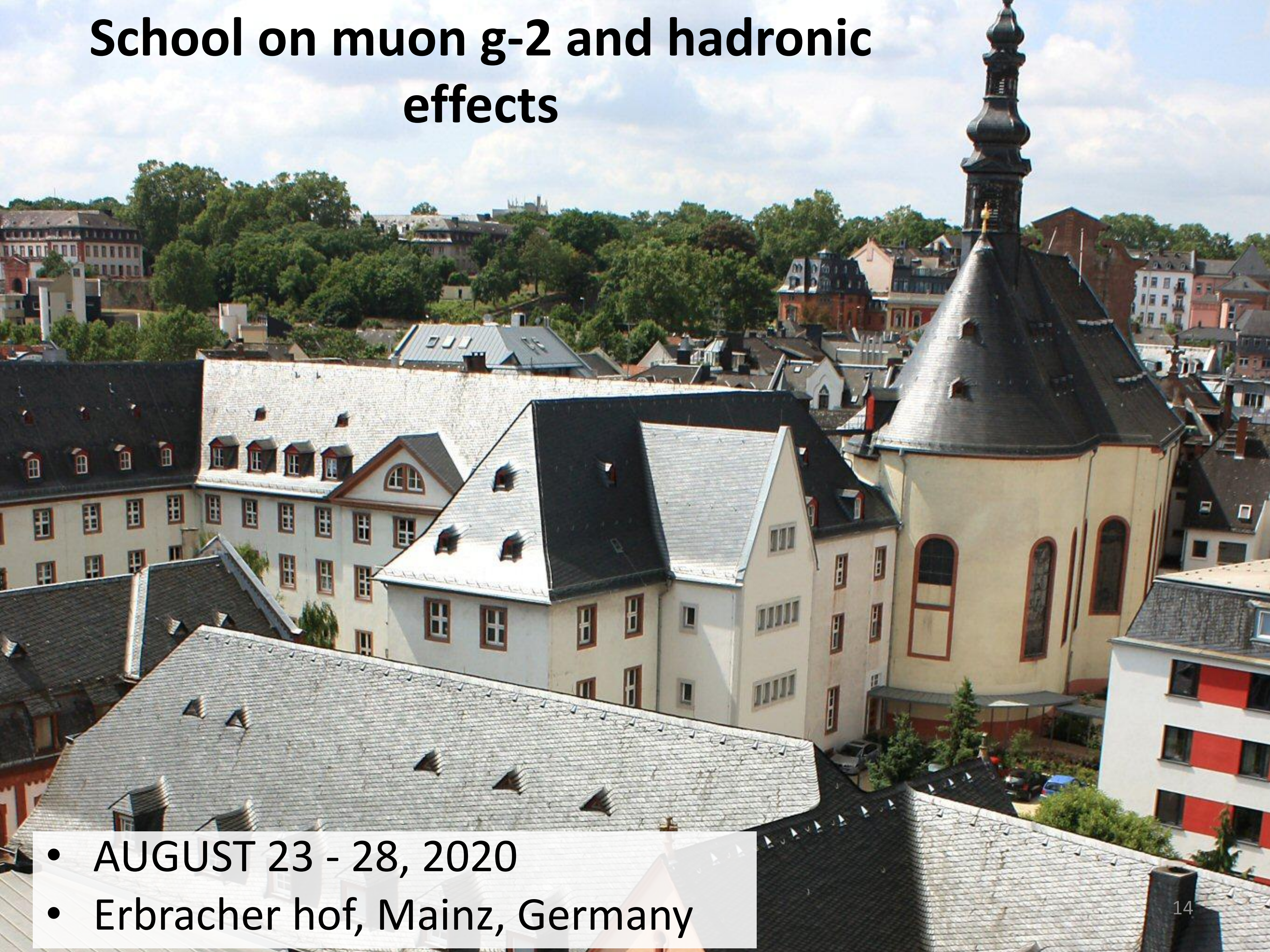


International school on muon dipole moment and hadronic effects

Sep 17-21, 2018 @BINP

https://indico.inp.nsk.su/event/14/

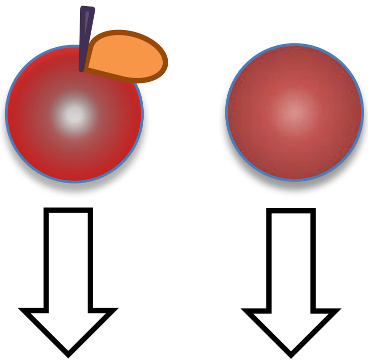
School on muon g-2 and hadronic effects



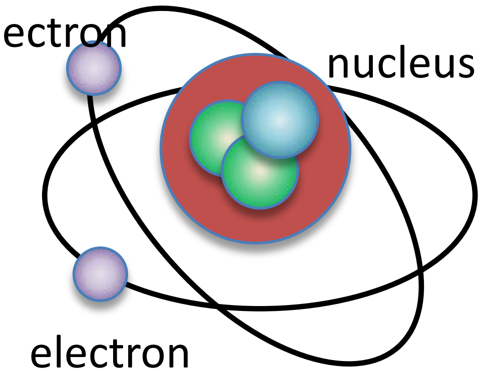
- AUGUST 23 - 28, 2020
- Erbracher hof, Mainz, Germany

Particle Interactions

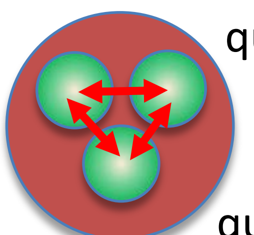
Gravity



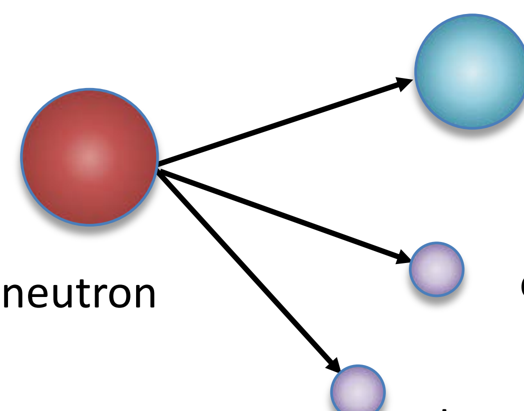
Electromagnetic force



Strong force

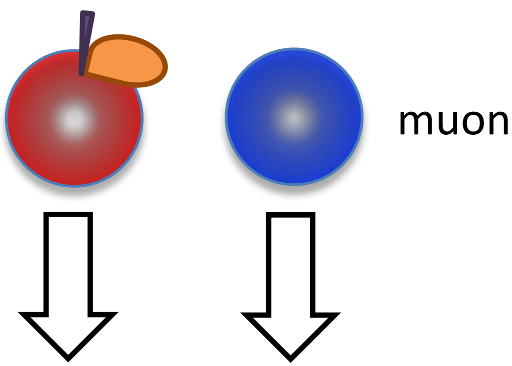


Weak force

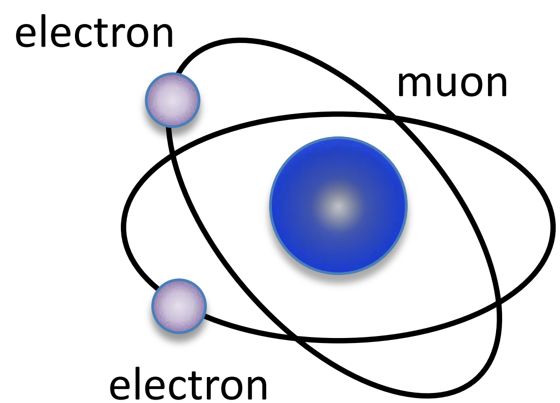


Particle Interactions

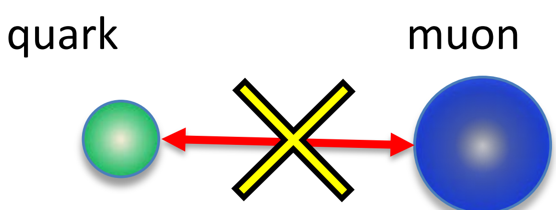
Gravity



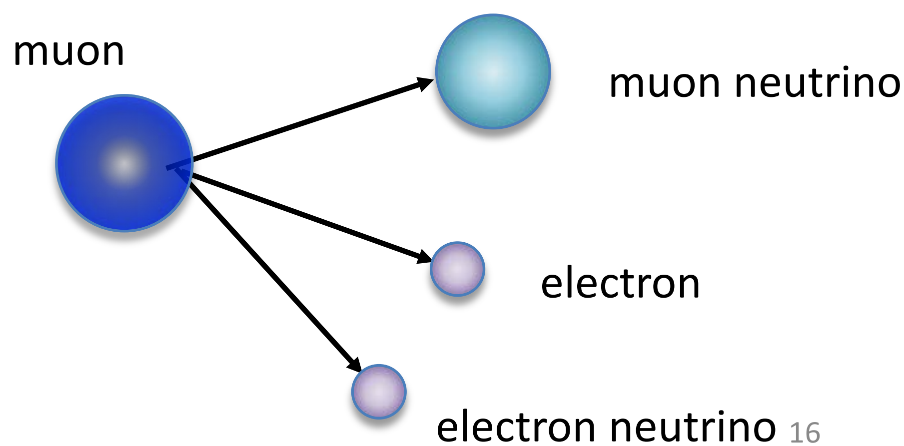
Electromagnetic force



Strong force

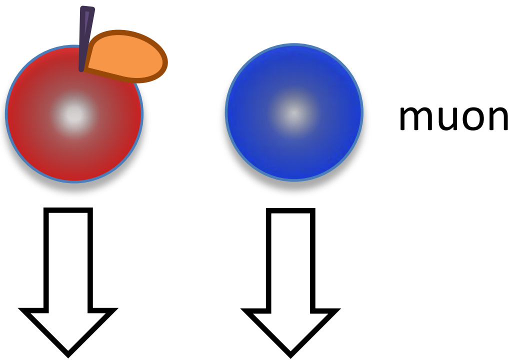


Weak force

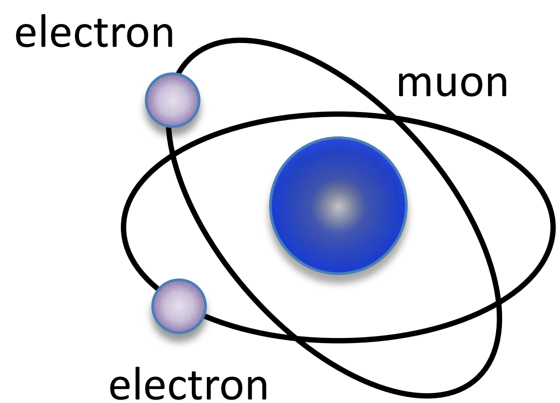


Particle Interactions

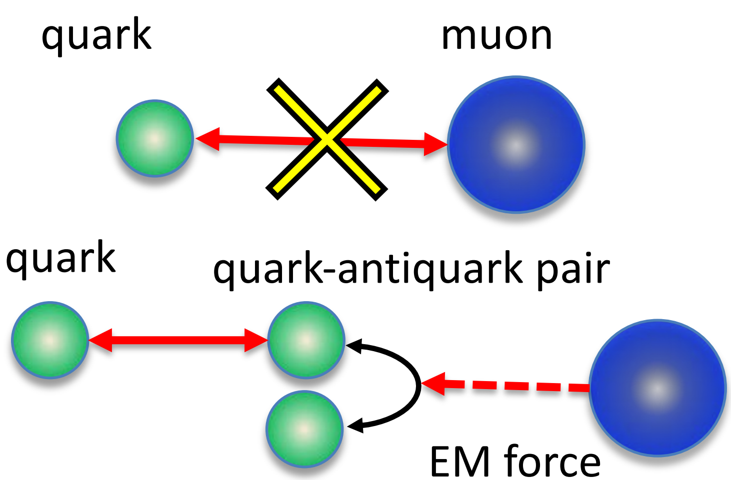
Gravity



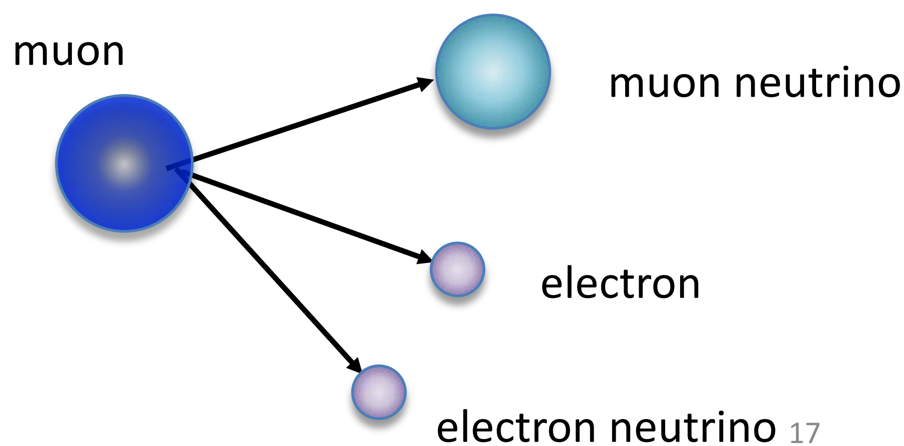
Electromagnetic force



Strong force



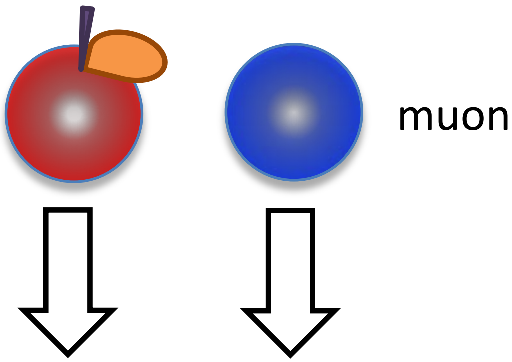
Weak force



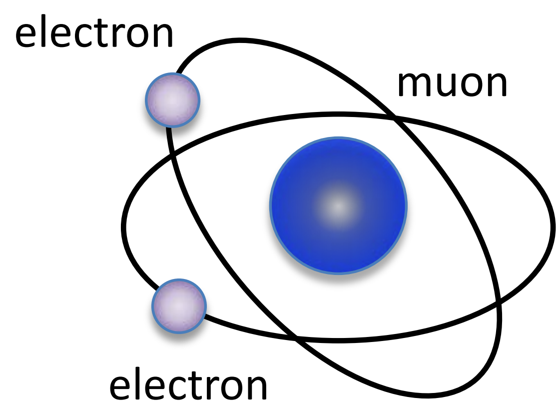
Particle Interactions

Muon "feels" all four interactions.

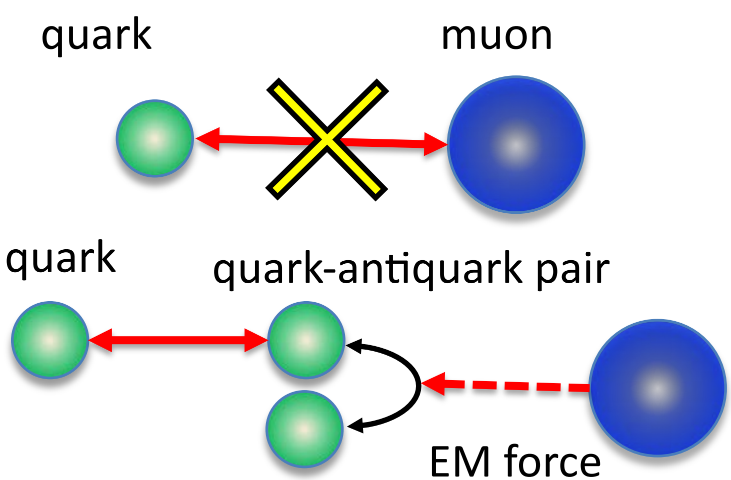
Gravity



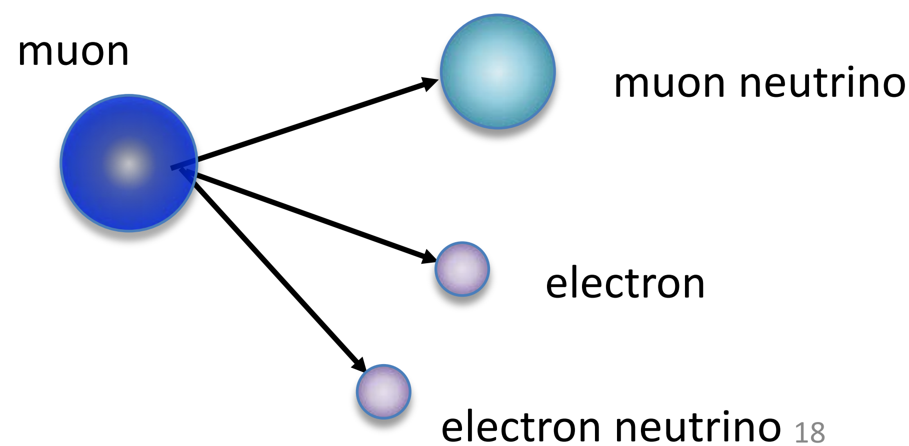
Electromagnetic force



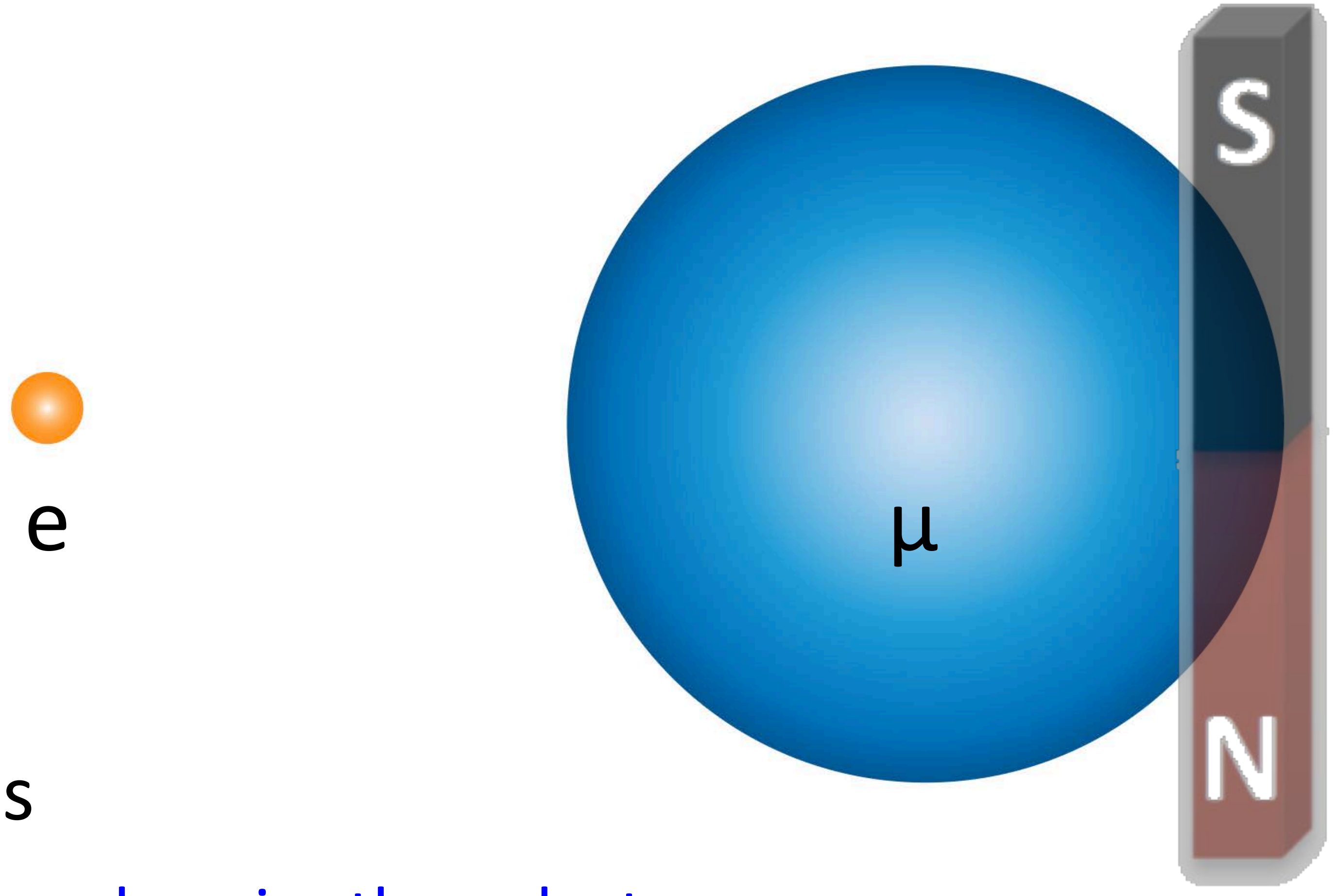
Strong force



Weak force



Muon



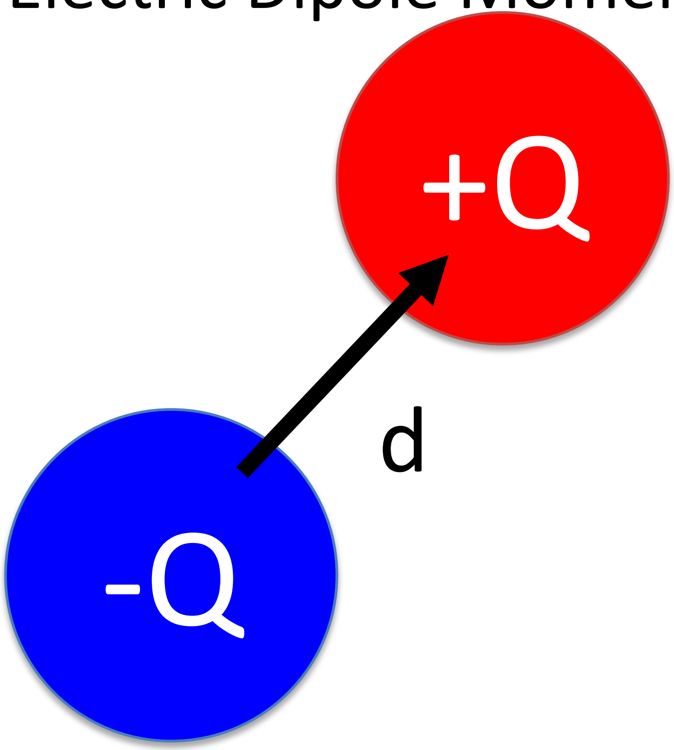
Muon is

- 200 times heavier than electron
- Decays in 2.2 μsec (conserving "lepton flavor")
- Has a spin ½
- Feels all interactions (including unknown ones if any)

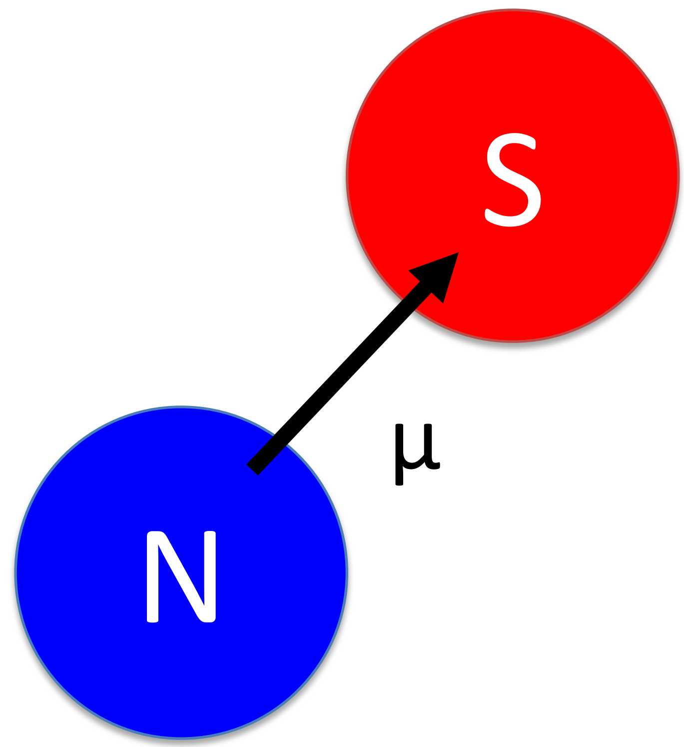
Dipole moments

- A pair of spatially separated (electric,magnetic) charges

Electric Dipole Moment



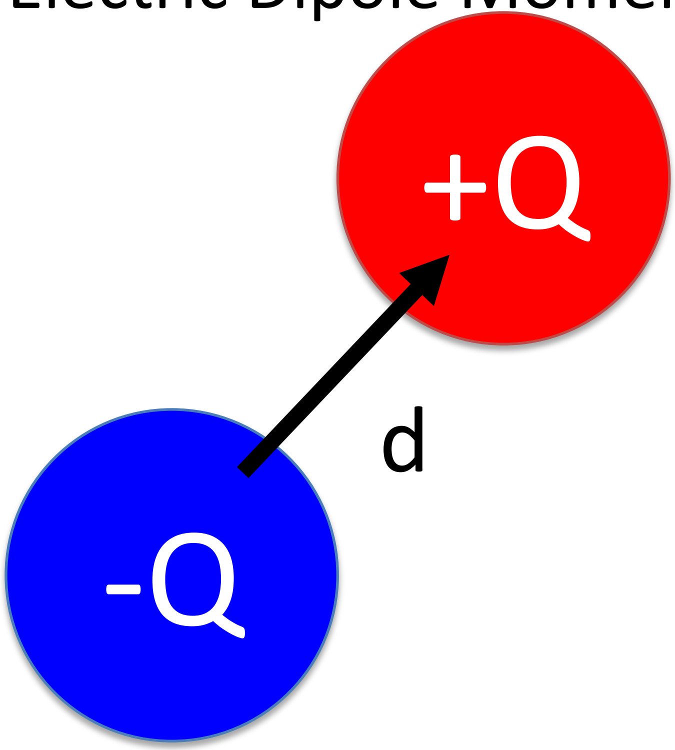
Magnetic Dipole Moment



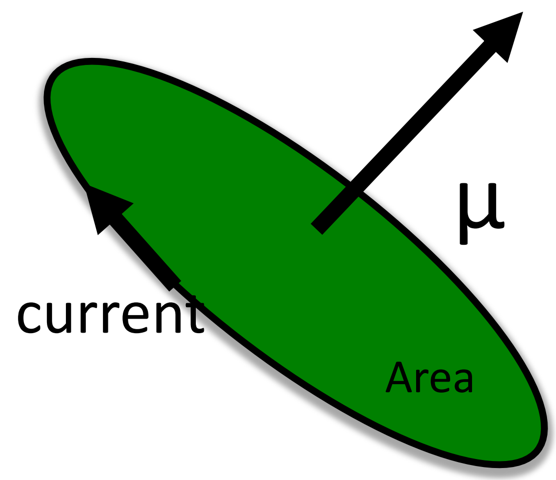
Dipole moments

- A pair of spatially separated (electric,magnetic) charges

Electric Dipole Moment



Magnetic Dipole Moment



Classical Electro-Magnetizm

- Magnetic moment

$$\vec{M} \equiv \frac{1}{2}\int \vec{x} \times \vec{J}(\vec{x}) d^3\vec{x} = \frac{q}{2}\sum \vec{x}_i \times \vec{v}_i$$

- Angular momentum

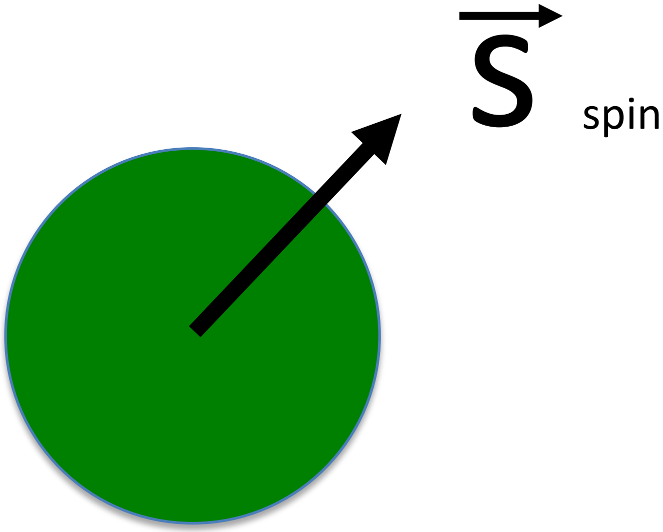
$$\vec{L} = \sum \vec{x}_i \times m\vec{v}_i$$

- Using above relation,

$$\vec{M} = \frac{q}{2m}\vec{L}$$

Dipole moments of elementary particle

- Dipole moment = "charge" x "spacial distance"
- Spin : only quantity with directional property
- Dipole moment $\propto$ Spin



Electron spin and magnetic dipole moment

- Hamiltonian

$$H = -\vec{M} \cdot \vec{B}$$

- Magnetic dipole moment

$$\vec{M} = g_L \frac{e}{2m} \vec{L} + g_S \frac{e}{2m} \vec{S}$$

g_L , g_S : Lande's g factor. "1" in classic mechanics

$$\vec{M} = \frac{q}{2m} \vec{L}$$

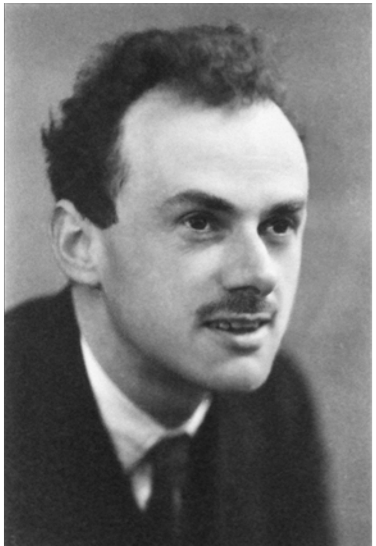
Quantum theory of the electron (1928)

Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 117 (778): 610.

- In a non-relativistic limit of the Dirac equation:

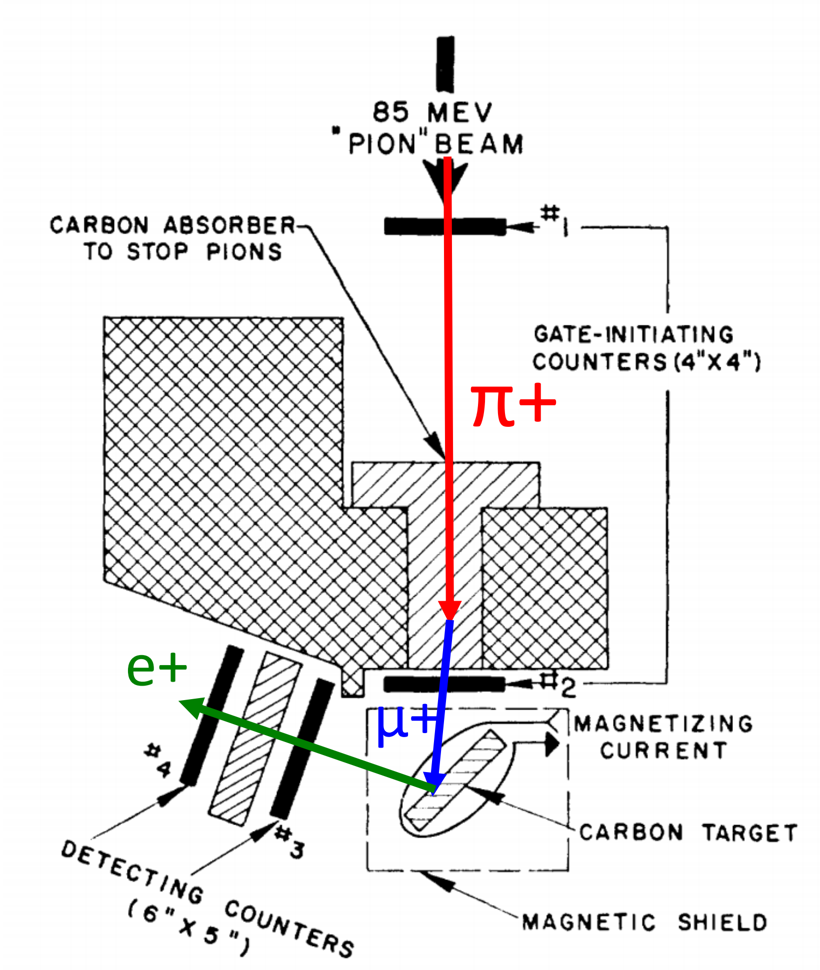
$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{p^2}{2m} - \frac{e}{2m}(\vec{L} + 2\vec{S})\right]\psi$$

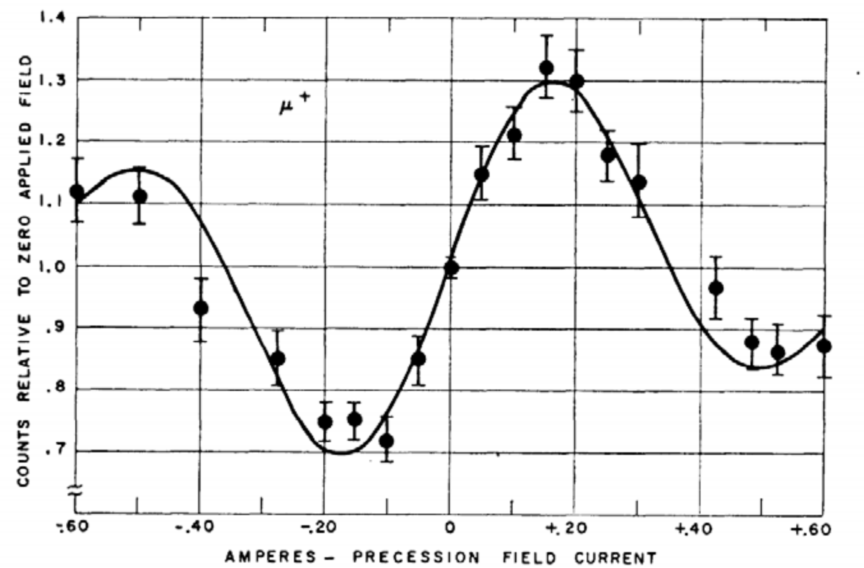
- From this, $g_L = 1$ and $g_S = 2$.



Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: The Magnetic Moment of the Free Muon

R. Garwin, L. Lederman, M. Weinrich, Phys.Rev. 105 (1957) 1415–1417.

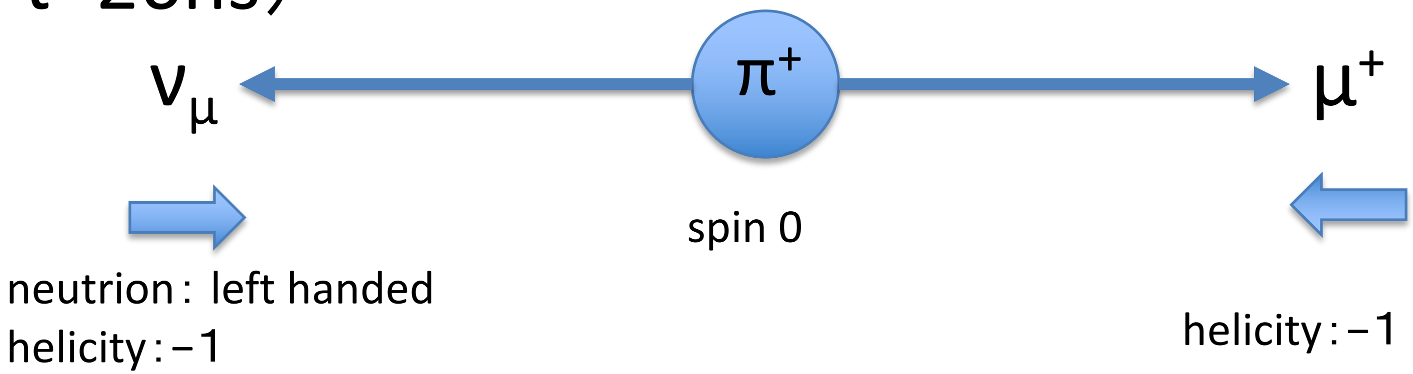




Pion decay and P-violation

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

- Two body decay of charged pion（BR=99.9877%, τ=26ns）



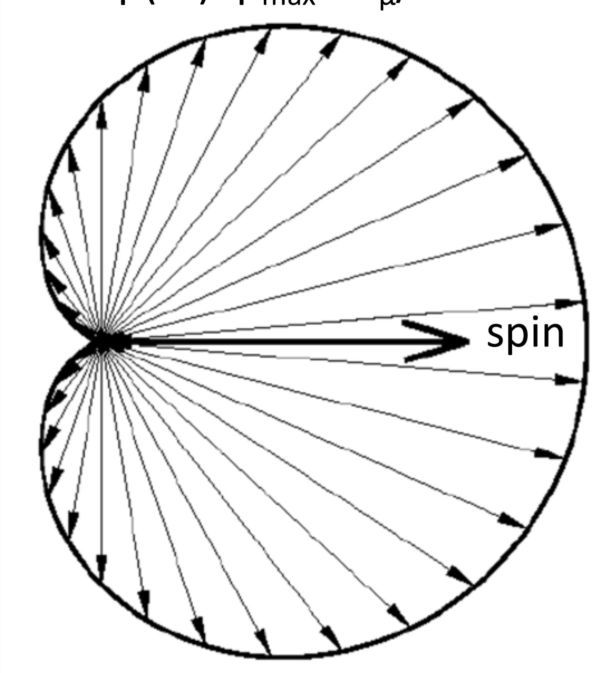
$$\frac{\vec{s}\cdot\vec{p}}{|\vec{s}||\vec{p}|}$$

P-violating quantity

Muon decay and Parity violation

$$\mu \rightarrow \nu_\mu + e + \bar{\nu}_e$$

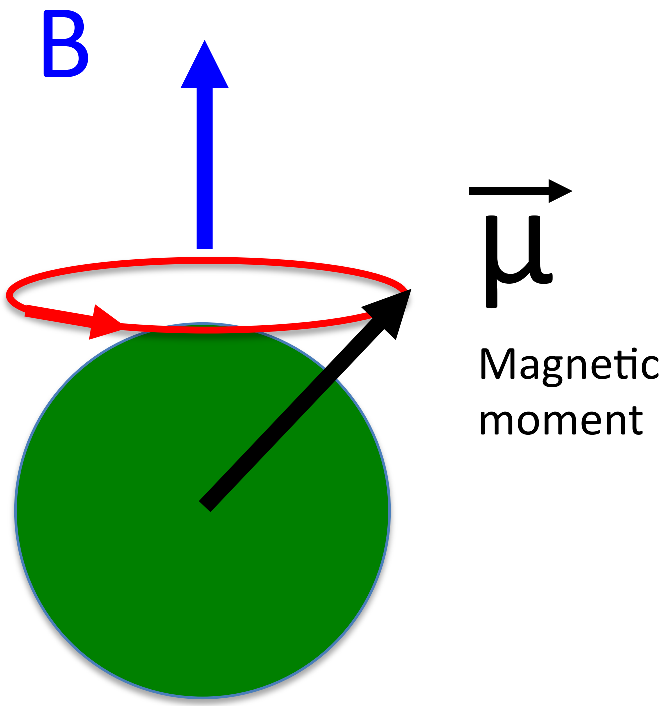
Positron emission angle
at $p(e^+)=p_{max}=m_\mu/2$



- Positron emission angle follows the structure of weak interaction (V-A type).
- Higher energy positron tends to emit along muon's spin direction.
  - Parity violation

Muon magnetic moment

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B}$$



Larmor precession

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \vec{B}$$

Magnetic moment (spin) precesses under magnetic field.

Equation of vector rotation

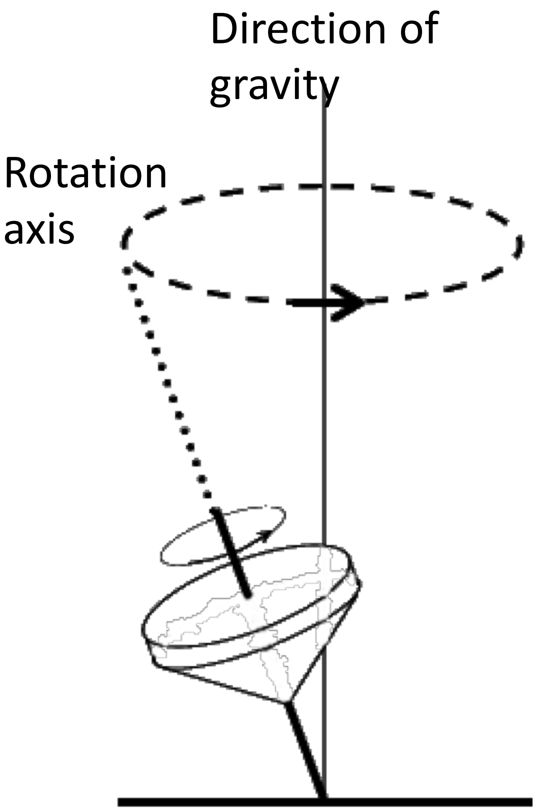
$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

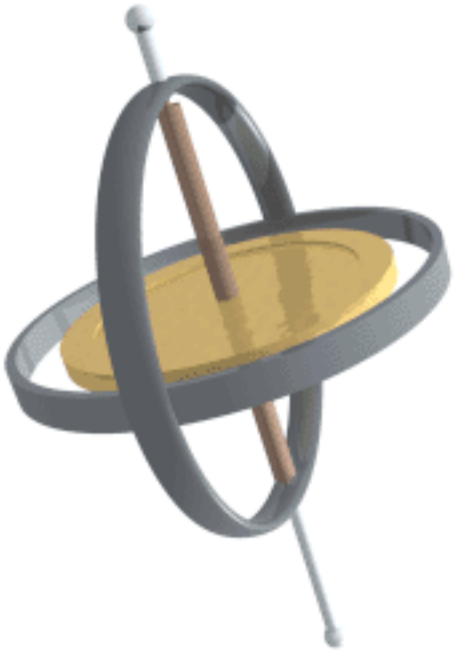
Rotation axis

Rotating vector

Goldstein, Classical Mechanics (2nd ed.), p174

Precession





Courtesy: LucasVB

Equation of spin precession

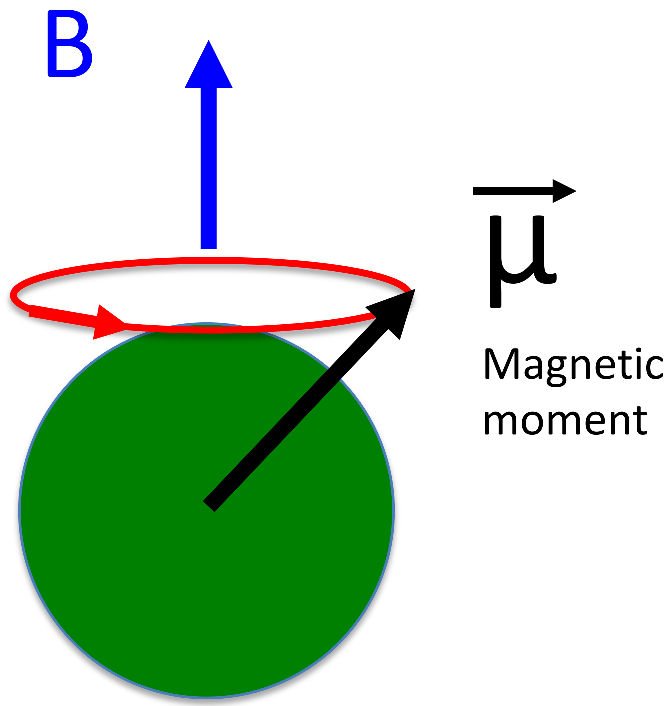
$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s}$$

Rotation axis

spin

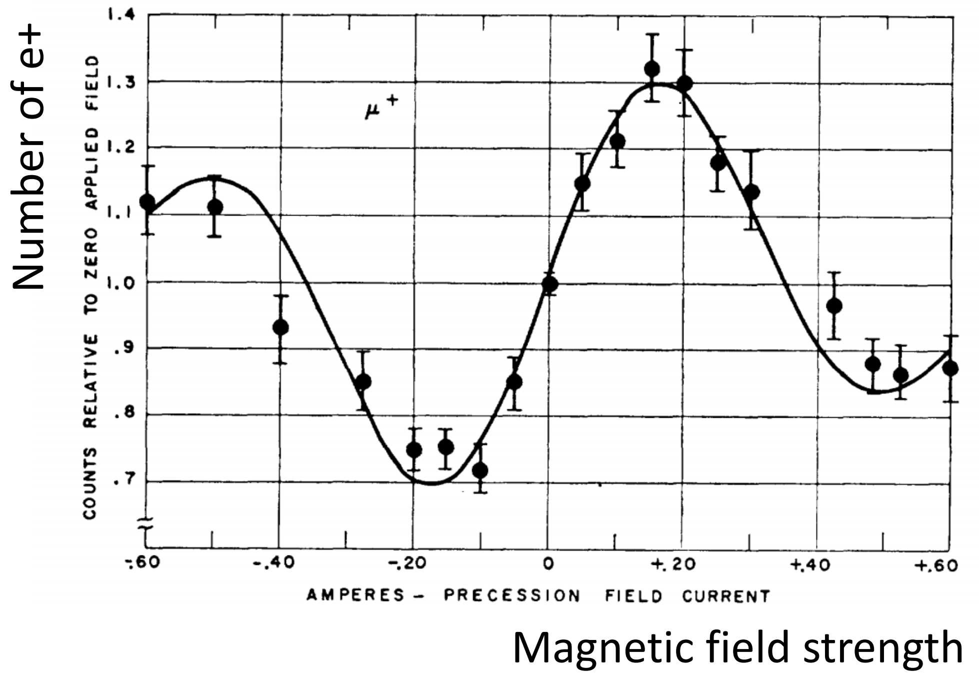
Muon magnetic moment

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B}$$



Magnetic moment (spin) precesses under magnetic field.

R. Garwin, L. Lederman, M. Weinrich, Phys.Rev. 105 (1957) 1415–1417.



$$g = 2.00 \pm 0.10$$

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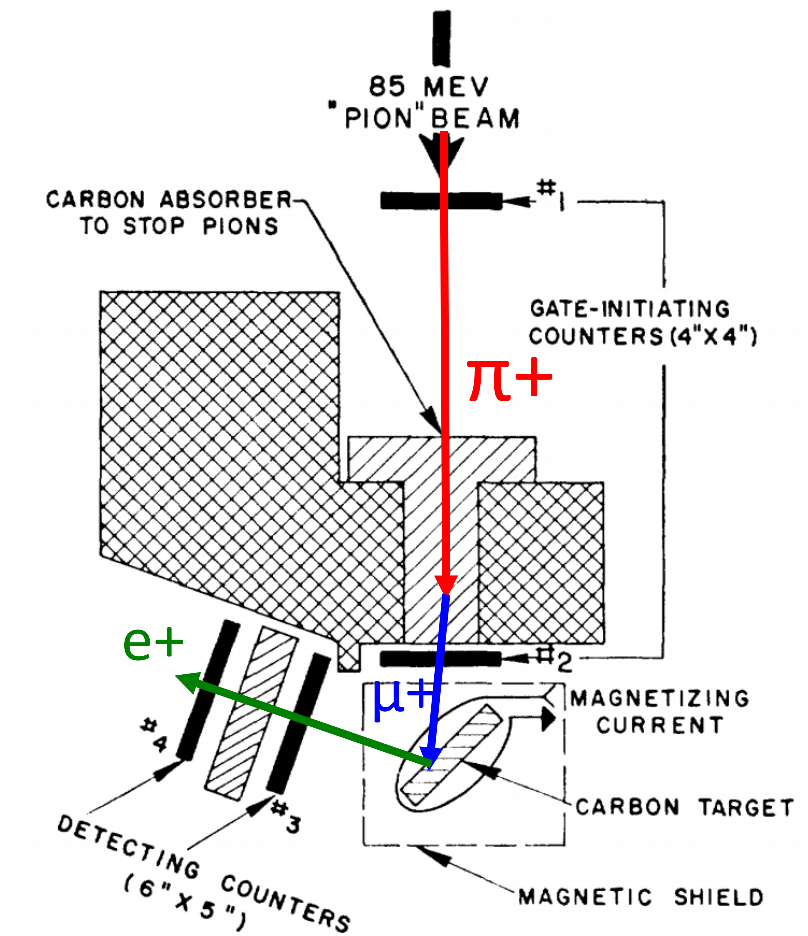
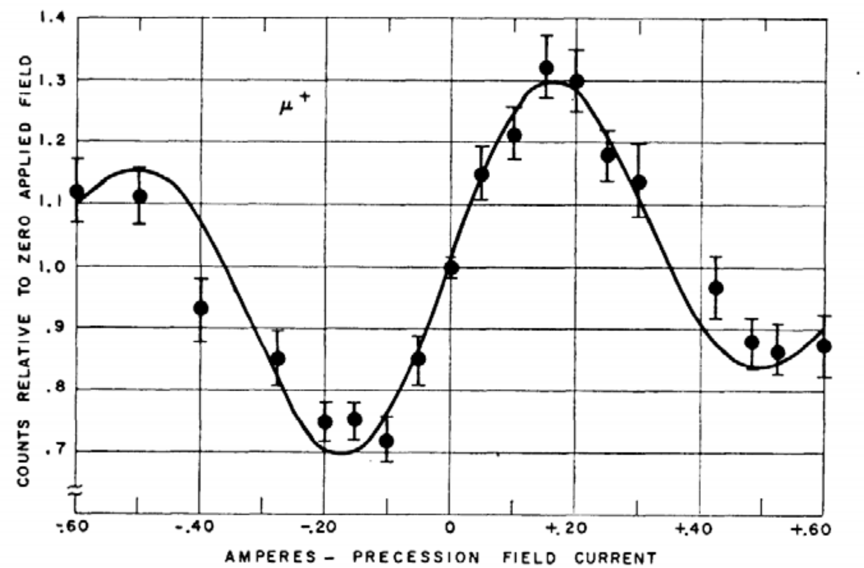




Three important discoveries

1）P-violation in pion decay

2）P-violation in muon decay

3） Muon's magnetic moment

Measurement of magnetic moment of electron

Phys. Rev. 74, 250 (1948)

The Magnetic Moment of the Electron†

P. KUSCH AND H. M. FOLEY
Department of Physics, Columbia University, New York, New York

(Received April 19, 1948)

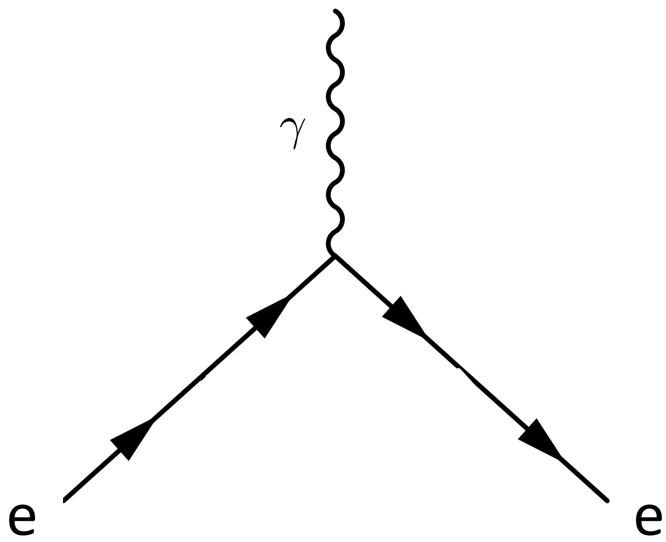
A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{1/2}$ states, In in the $^2P_{1/2}$ state, and Na in the $^2S_{1/2}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L=1$ and $g_S=2(1.00119\pm0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

$$\mathcal{H}=g_L\mu_0 L_z H_z+g_S\mu_0 S_z H_z,$$

$g_S > 2$!?

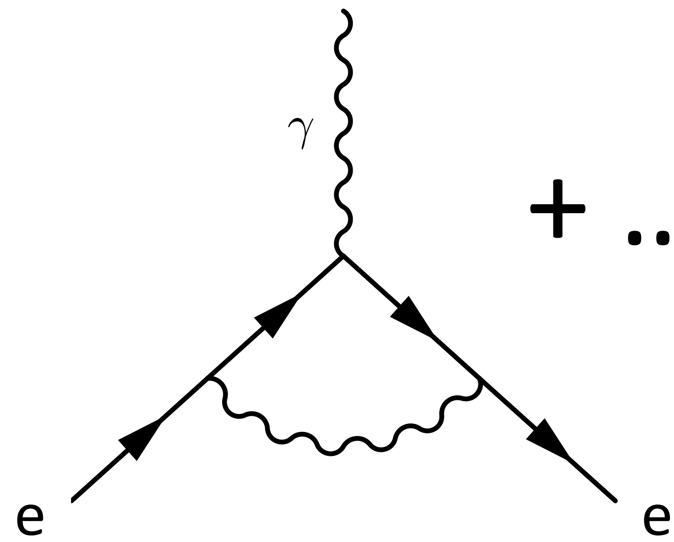
Quantum corrections

Dirac's theory



$g = 2$

QED



+ ...

$g = 2\,(1 + \alpha/2\pi + \ldots)$

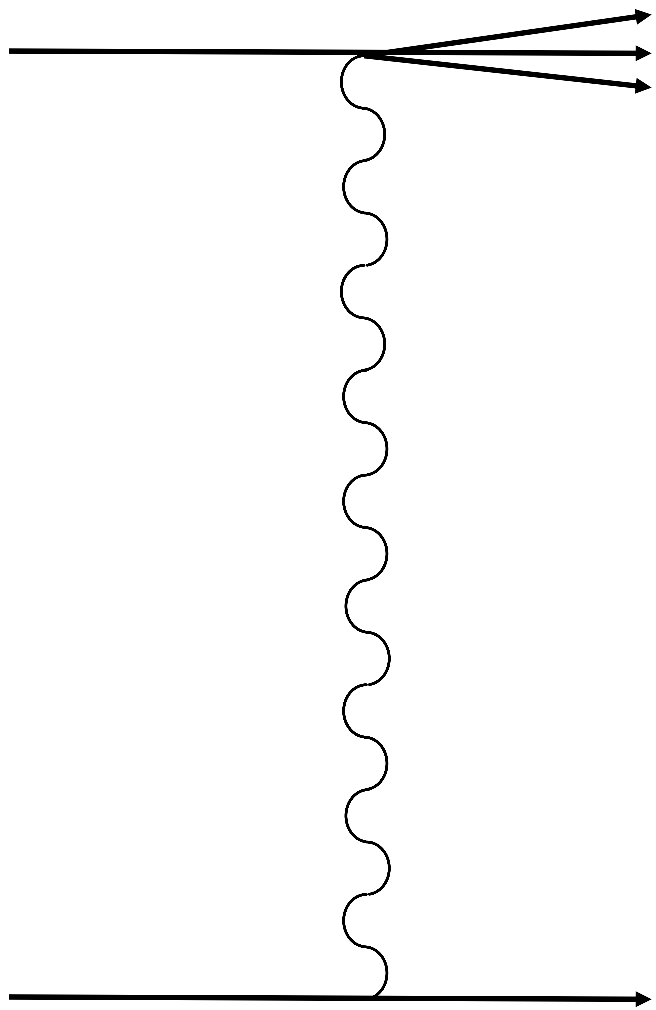
Schwinger

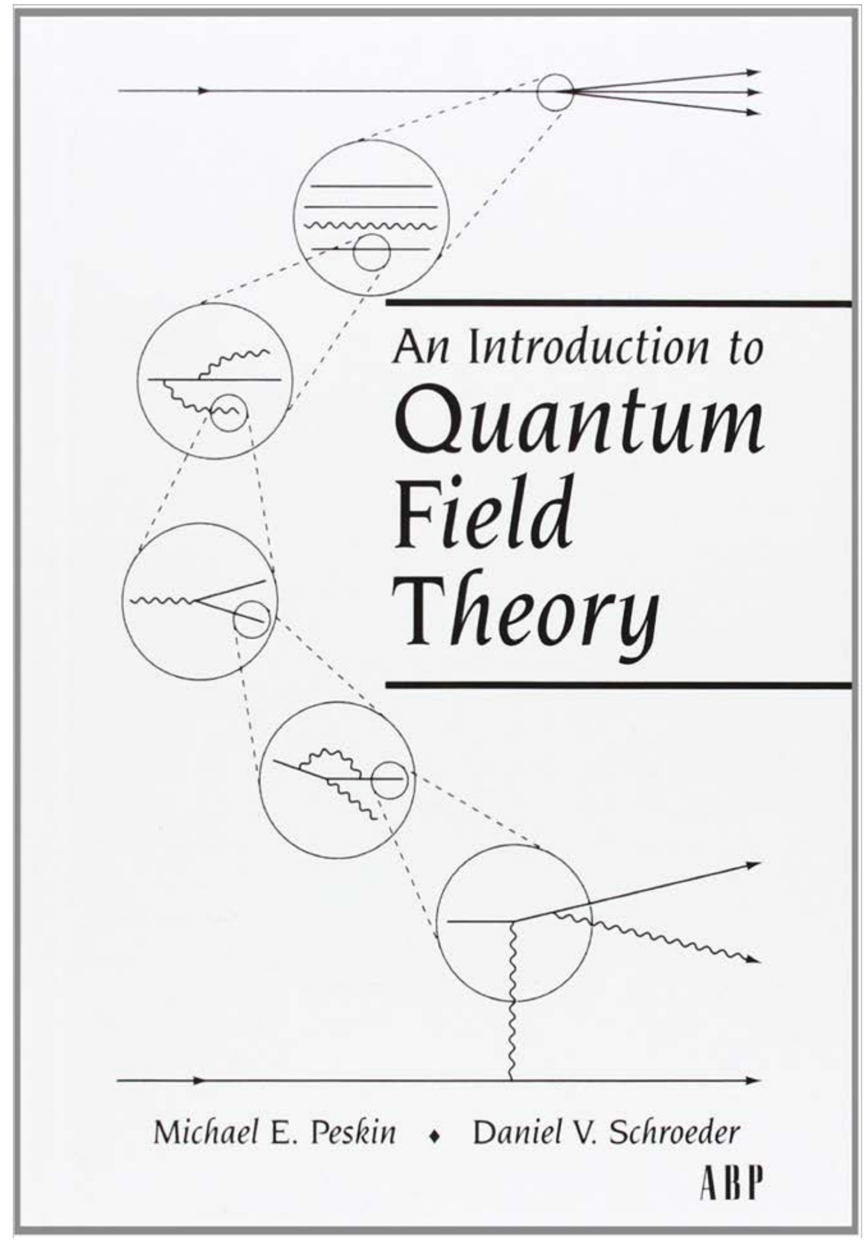
g-2 and Julian Schwinger

Schwinger's Memorial Stone

The centennial : 2018 December
A dedicate workshop in UCLA

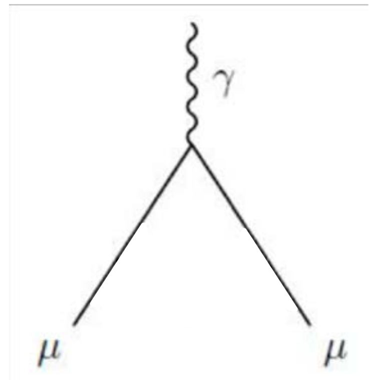
Vacuum fluctuate!



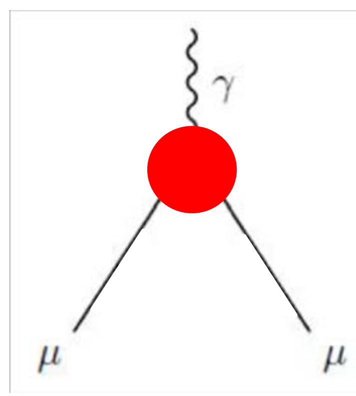


Anomalous magnetic momet (g–2)

- The Lande's *g* factor is 2 in tree level (Dirac equation)



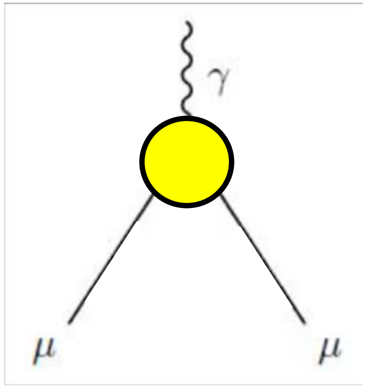
- In quantum field theory, *g* factor gets corrections:



Anomalous magnetic moment (g–2)

$$g = 2\,(1 + a)$$

Anomalous magnetic moment



$$a_\mu = a_\mu(QED) + a_\mu(had) + a_\mu(weak) + a_\mu(BSM)$$

All interactions, *including ones we don't know*, appear in quantum loops, and add up to contribute a_μ

Electron's anomalous magnetic moment (a_e) and Quantum Electro Dynamics (QED)

Electron's anomalous magnetic moment a_e

- Anomalous magnetic moment is defined as

$$a_e = \frac{g_e - 2}{2}$$

- Kusch and Foley : Zeeman splitting of Ga, In, Na atoms (1947)

P. Kusch, H.M. Foley, PR 72, 1256 (1947)

$$a_e = 1.19(5) \times 10^{-3}$$

- Schwinger's radiative correction;

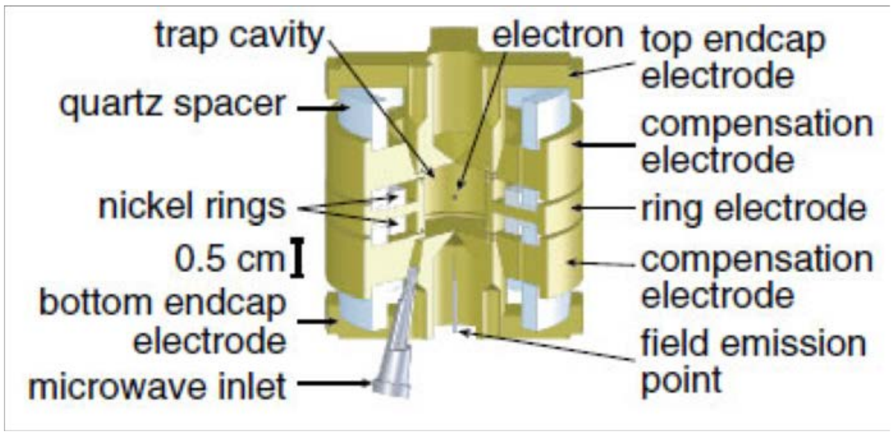
J. Schwinger, PR 73, 416L (1948); PR 75, 898 (1949)

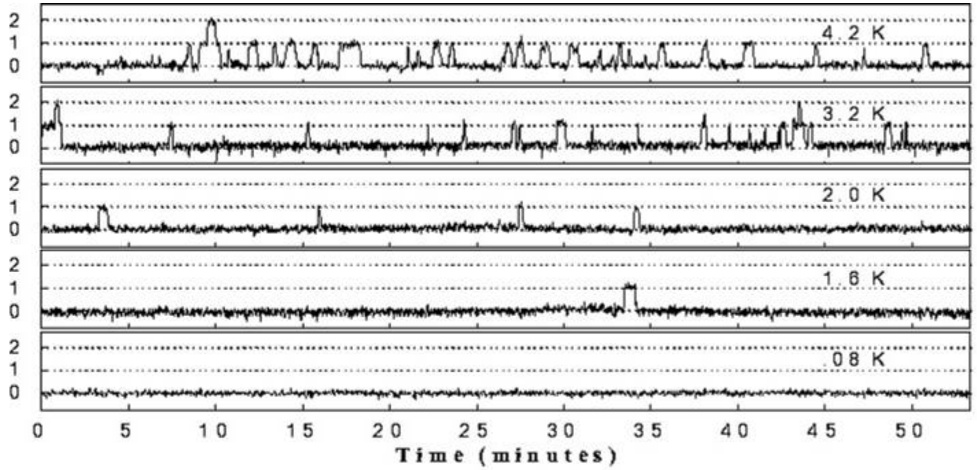
$$a_e = \frac{\alpha}{2\pi} = 1.161... \times 10^{-3}$$

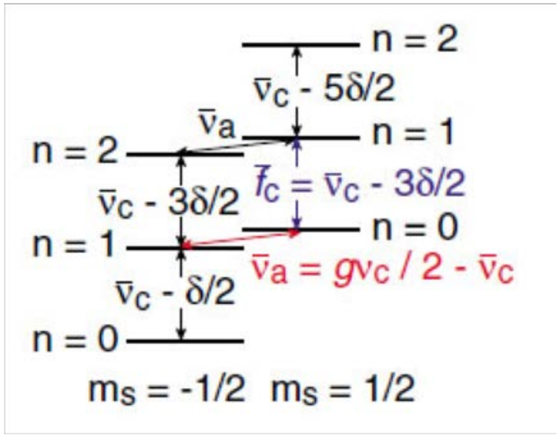
Anomalous magnetic moment of electron a_e

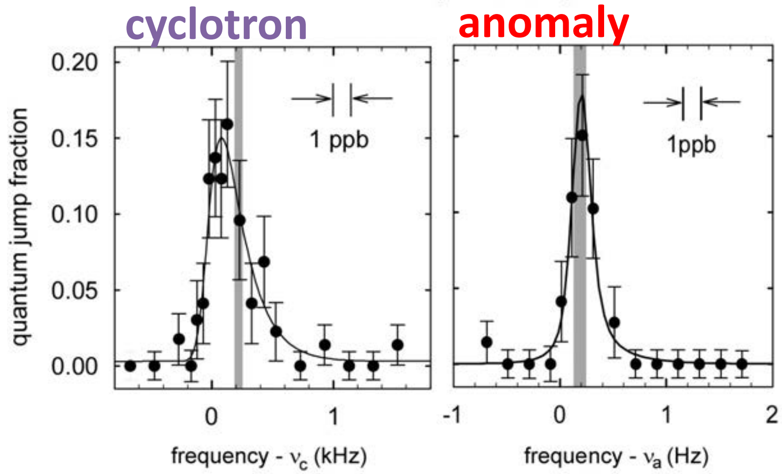
D. Hanneke, S. Fogwell, G. Gabrielse, PRL 100, 120801 (2008)
D. Hanneke, S Fogwell Hoogerheide, G. Gabrielse, PRA 83, 052122 (2011)

- Measurement of spin motion in a quantum cyclotron (Penning trap) by Gabrielse's group at Harvard









David Hanneke   Gerald Gabrielse

Electron's anomalous magnetic moment a_e

- Harvard group measured with cylindrical Penning trap

D. Hanneke, S. Fogwell, G. Gabrielse, PRL 100, 120801 (2008)
D. Hanneke, S Fogwell Hoogerheide, G. Gabrielse, PRA 83, 052122 (2011)

$$a_e = 1\ 159\ 652\ 180.73(28) \times 10^{-12}$$

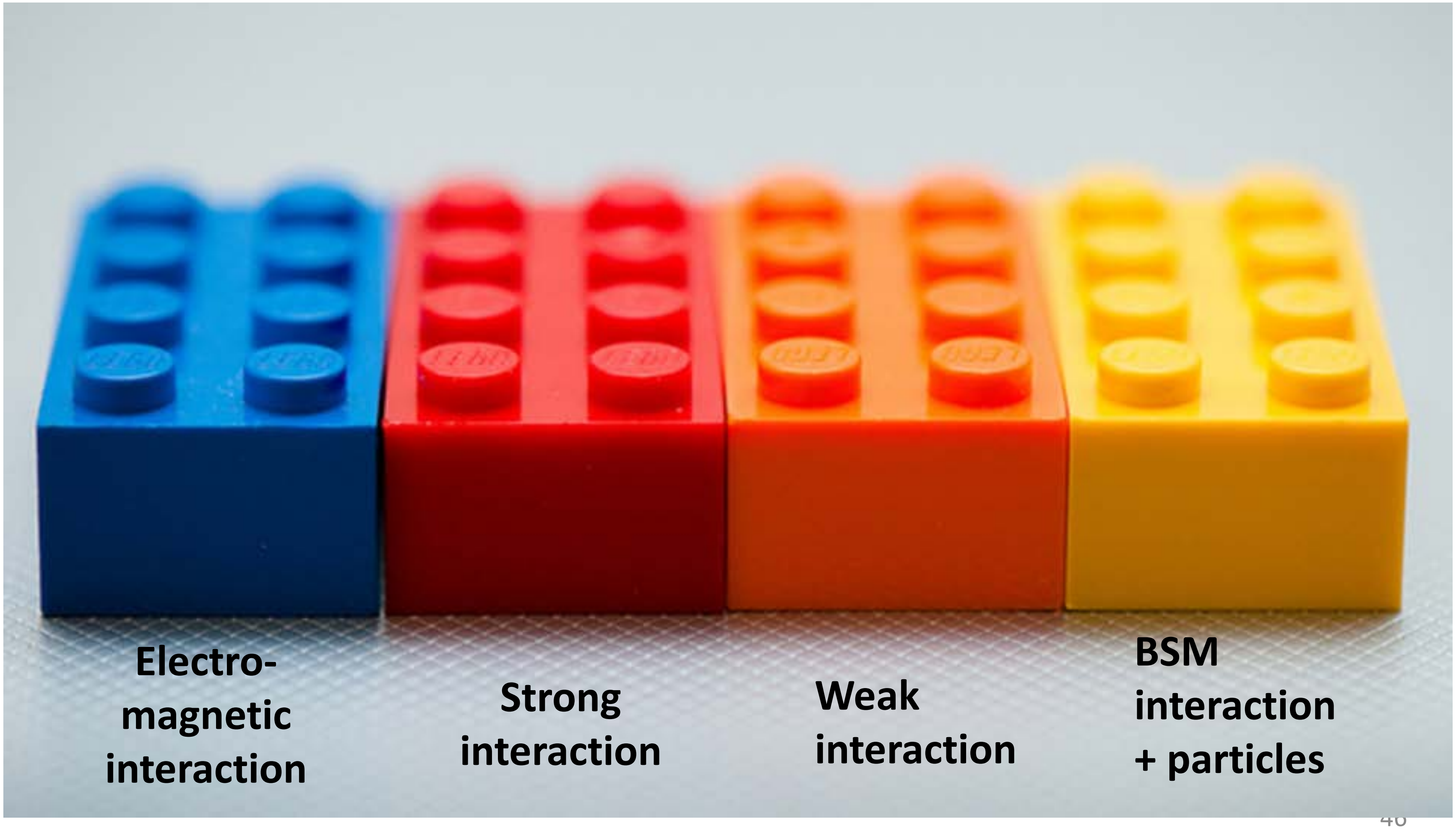
- Aoyama, Kinoshita, Nio completed 10th order QED calculations. Together with non-QED contributions one obtains

T. Aoyama, T. Kinoshita, M. Nio, Atoms, 7(1), 28 (2019)

$$a_e = 1\ 159\ 652\ 181.606\ \underset{10^{th}\text{ order}}{(11)}\underset{\text{had\&EW}}{(11)}\underset{\alpha(Cs)}{(229)} \times 10^{-12}$$

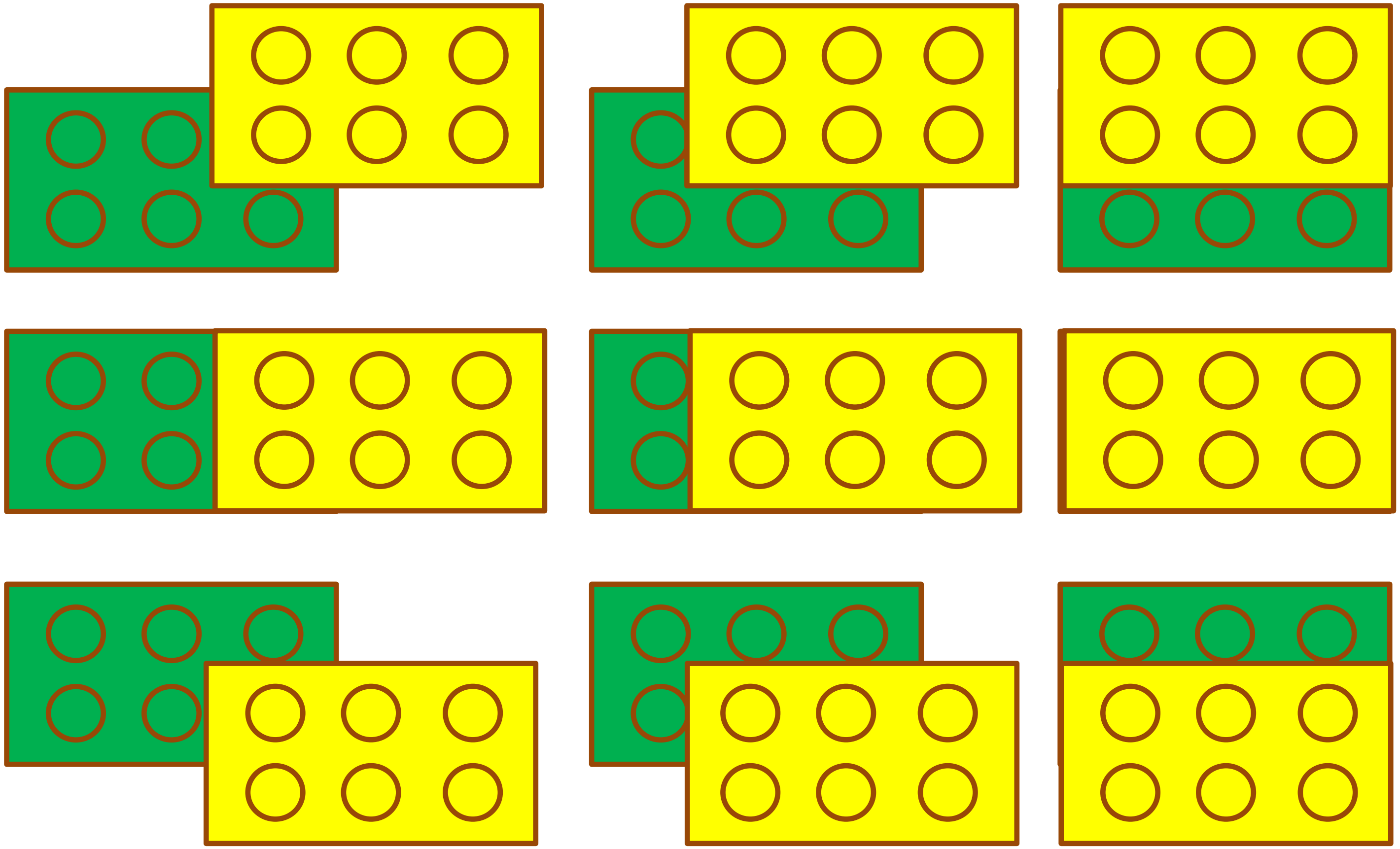
$$\Delta a_e = a_e(exp) - a_e(SM) = -0.88\ (36) \times 10^{-12}$$

Building a magnet from SM (+ BSM)



How many combinations to connect two lego blocks?

How many combinations to connect two lego blocks?



9 combinations

Building a magnet from SM (+ BSM)

Recipe:
Consider all combinations of parts and sum them up

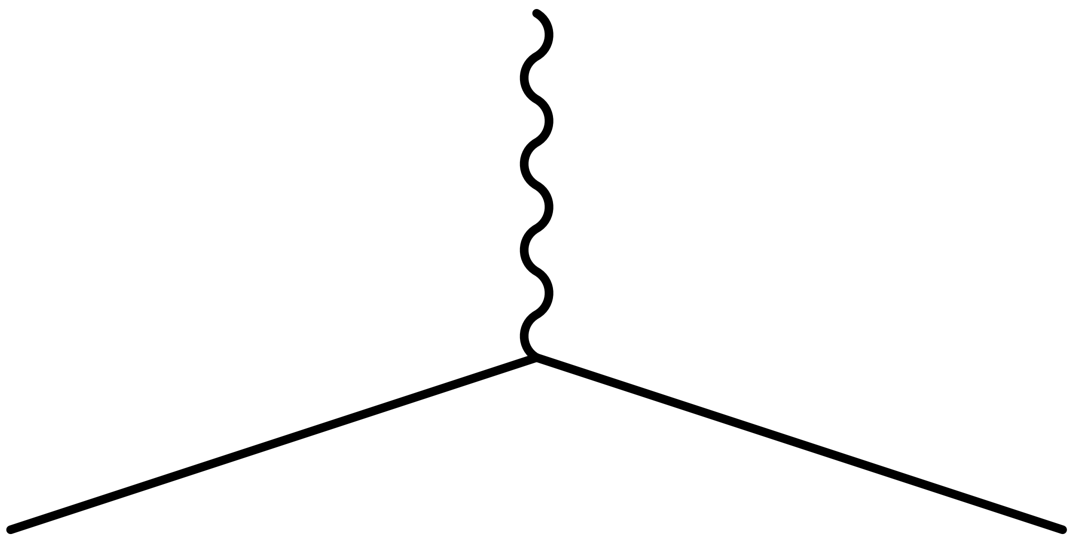
Parts

Photon

Particles
(electron, muon, etc.)

Building a magnet from SM (+ BSM)

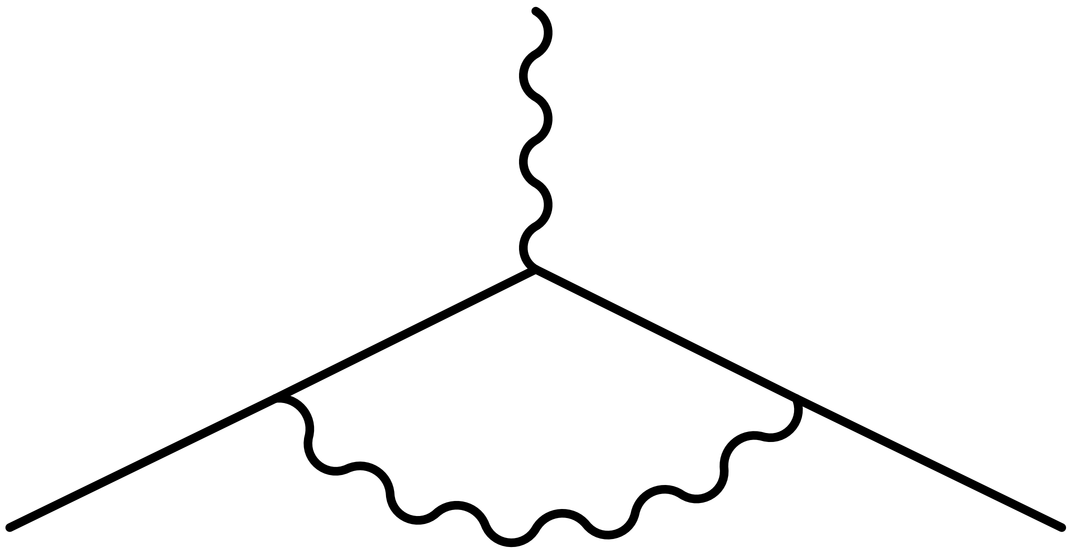
Simplest case (1 photon + 1 particle)



This term has a magnitude of 1

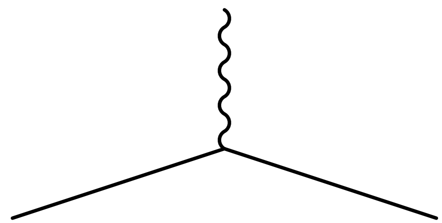
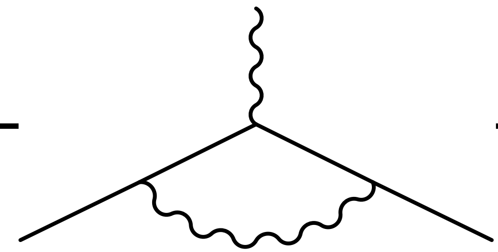
Building a magnet from SM (+ BSM)

Next simplest case (2 photons + 1 particle)



This term has a magnitude of $\alpha/2\pi$(=0.00116...).

Building a magnet from SM (+ BSM)

g/2 =  +  + …

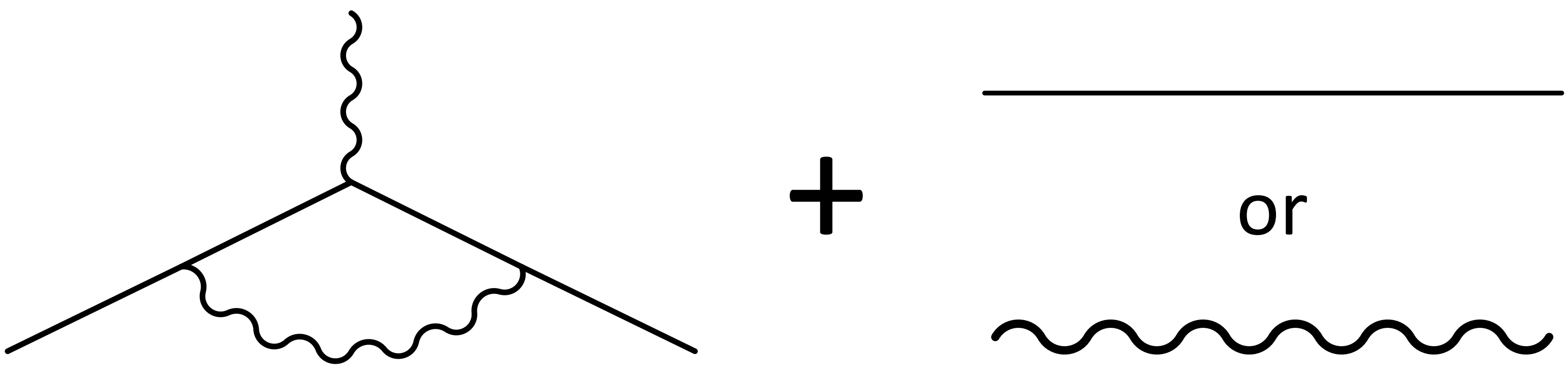
1   0.00116…

= 1.00116… + …

Sum up all combinations.

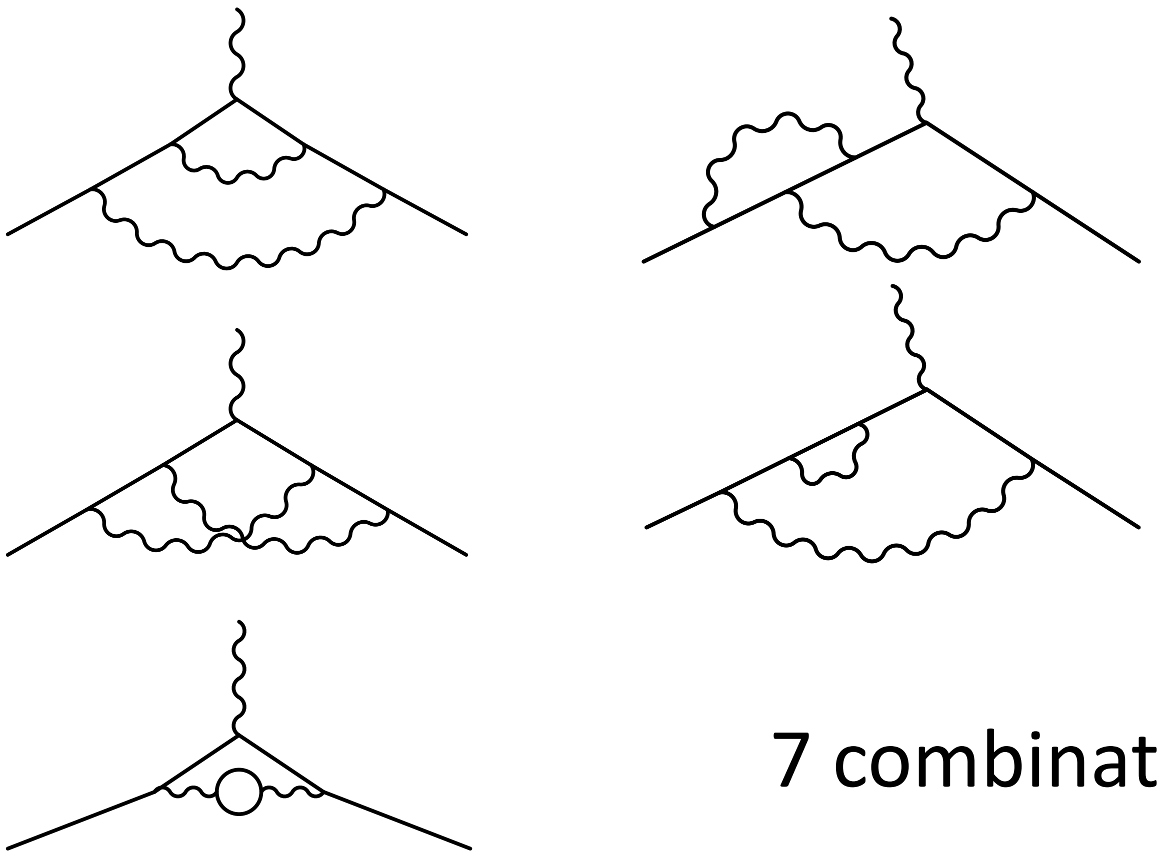
Building a magnet from SM (+ BSM)

How many combinations?



Building a magnet from SM (+ BSM)

Ans.:



×2

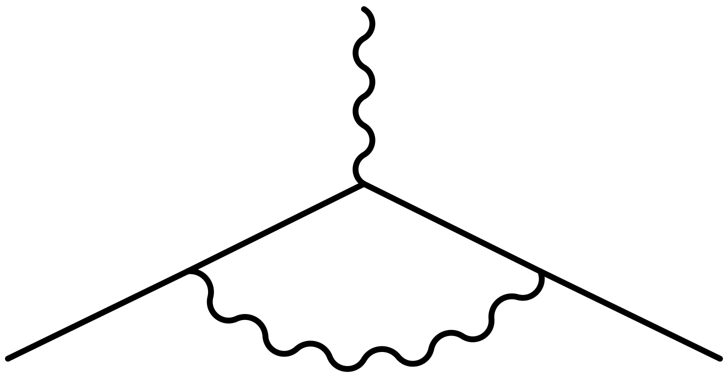
×2

7 combinations

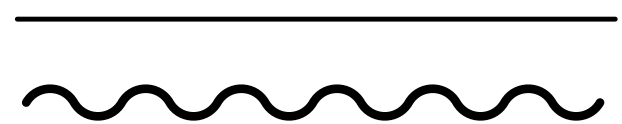
Magnitude of these terms is $-0.32\text{x}(\alpha/\pi)^2$(=0.0000017).

Building a magnet from SM (+ BSM)

How many combinations?



+

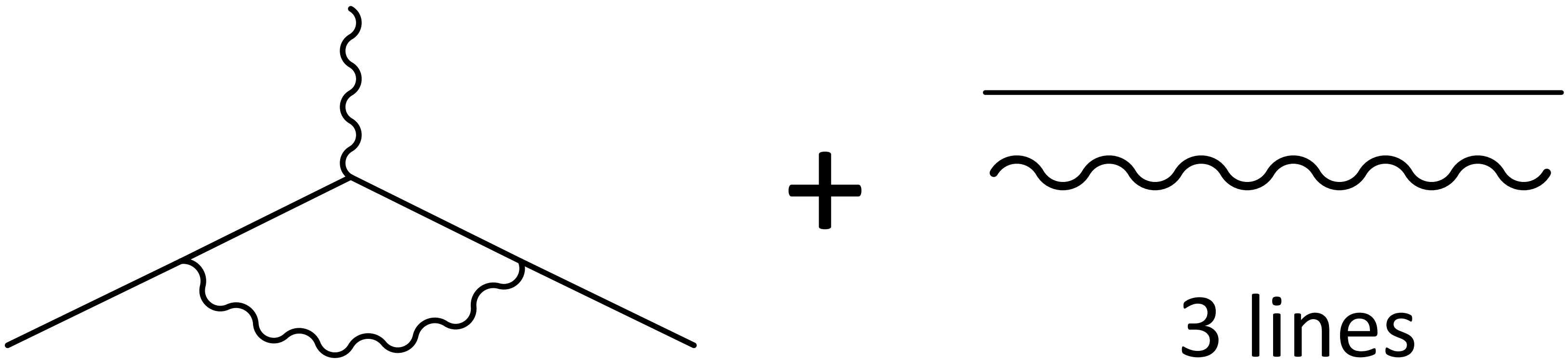


2 lines

Ans. : 72 combinations

Building a magnet from SM (+ BSM)

How many combinations?

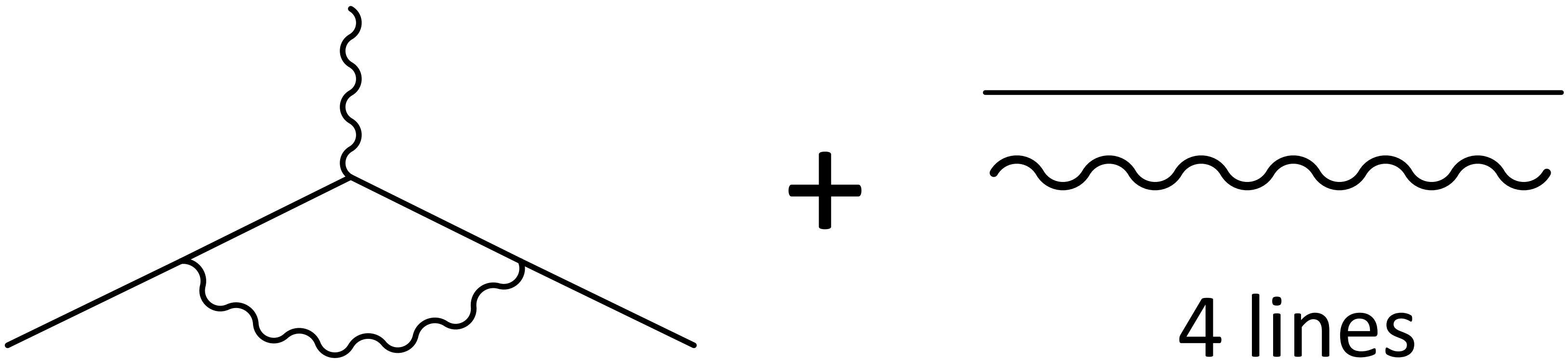


3 lines

Ans. : 891 combinations

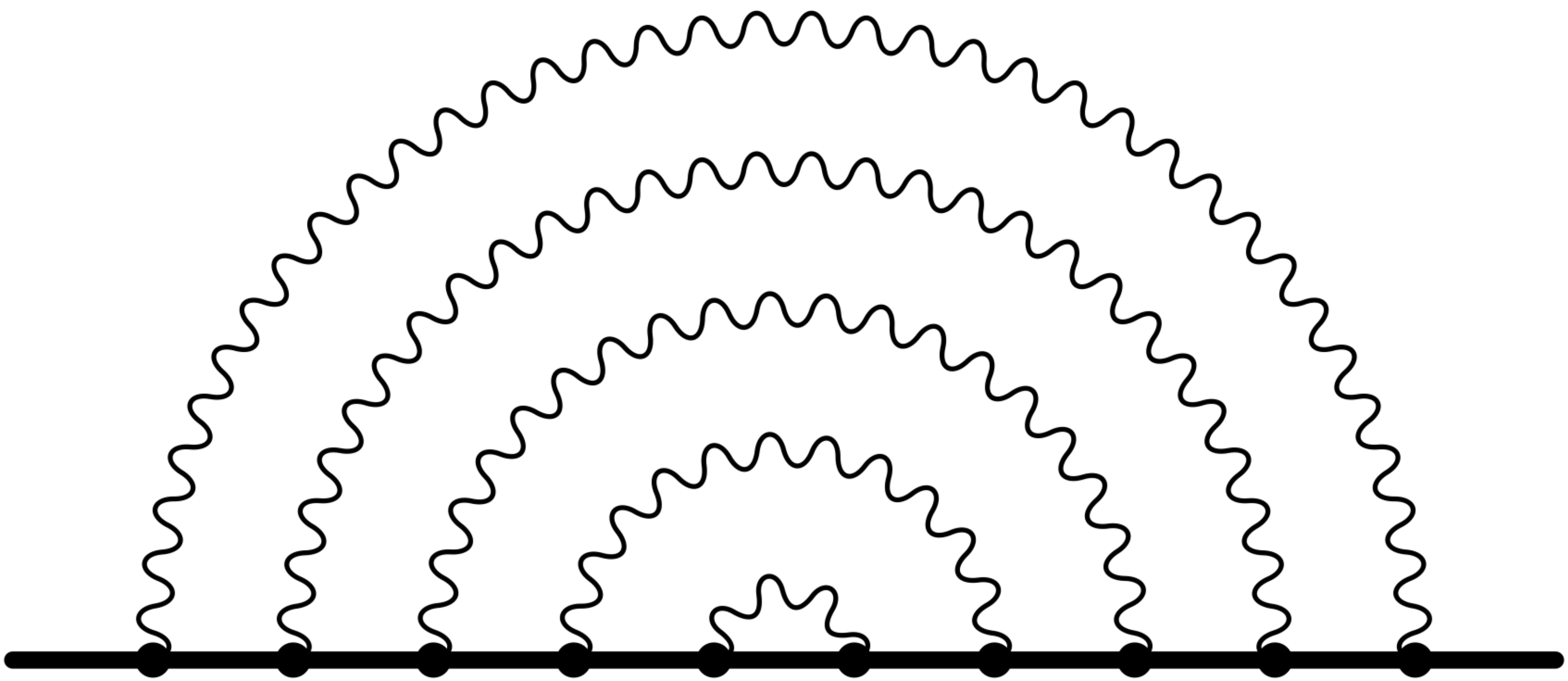
Building a magnet from SM (+ BSM)

How many combinations?

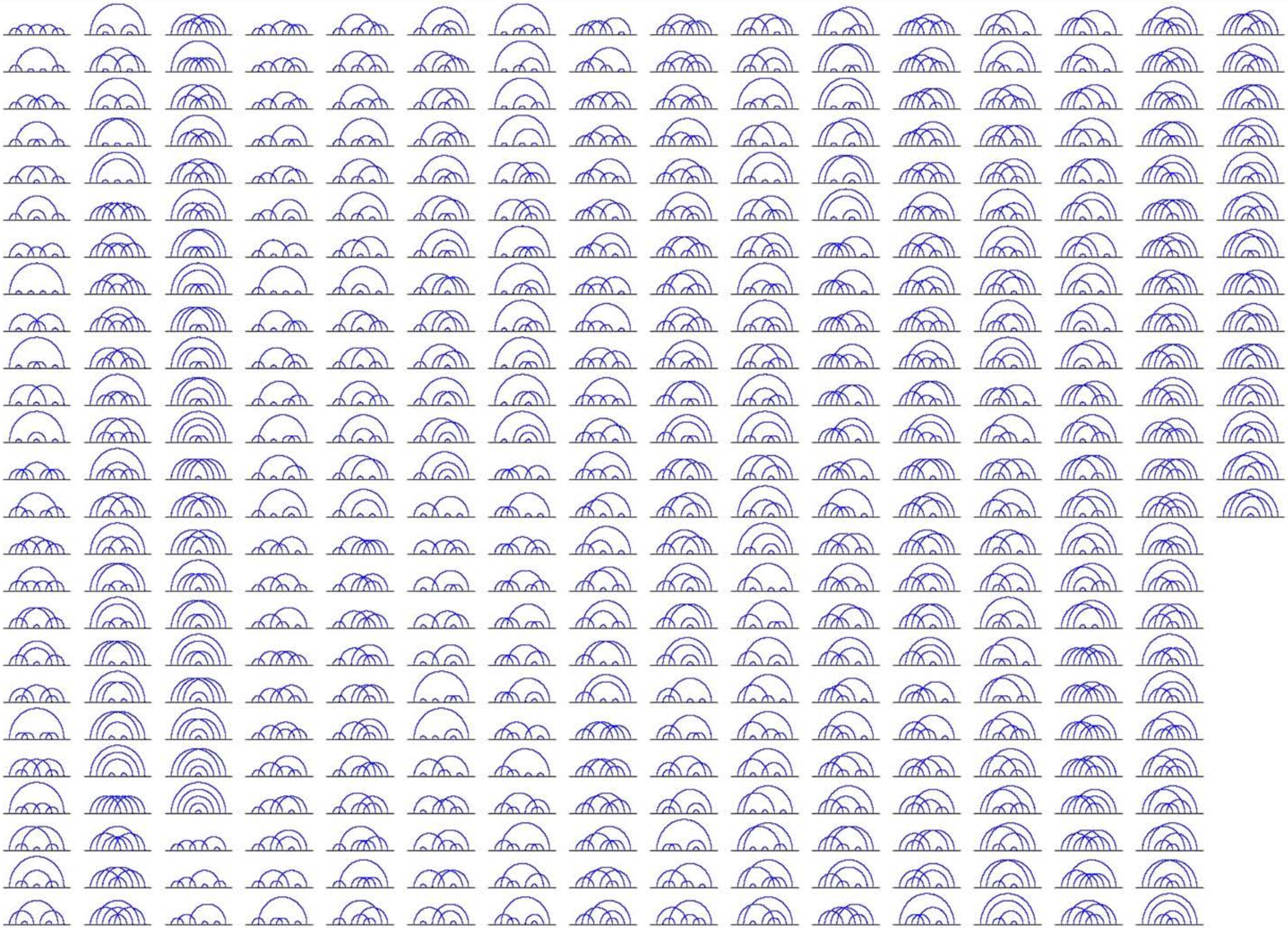


Ans. : 12672 combinations!

Examples of 12672 combinations



T. Aoyama, T. Kinoshita, M. Nio, Atoms 7, 28 (2019)



Electron's anomalous magnetic moment a_e

- In the standard model,

$$a_e = a_e(\mathrm{QED}) + a_e(\mathrm{EW}) + a_e(\mathrm{had}).$$

- QED contributions can be written as

$$a_e(\mathrm{QED}) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau).$$

- where

$$A_i = A_i^{(2)} \left(\frac{\alpha}{\pi}\right) + A_i^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + \ldots, \; i = 1, 2, 3,$$

Electron's anomalous magnetic moment a_e

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \ldots, \quad i = 1, 2, 3,$$

- $A_1(n)$ is know up to n = 10

$A_1^{(2)} = 0.5$	1 diagram (analytic)	(1948)	Schwinger
$A_1^{(4)} = -0.328\,478\,965\ldots$	7 diagrams (analytic)	(1957)	Sommerfield, Petermann
$A_1^{(6)} = 1.181\,241\,456\ldots$	72 diagrams (analytic)	(1995)	Laporta and Remiddi
$A_1^{(8)} = -1.912\,89\,(90)$	891 diagrams (numerical, July 2014)		Kinoshita, Nio
$A_1^{(10)} = 7.651\,(353)$	12672 diagrams (numerical, July 2014)		Hayakawa, Aoyama

Slide by G. Gabrielse

Basking in the Reflected Glow of Theorists

$$\frac{g}{2} = 1 + C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \ldots\ \delta a$$

2004

Remiddi Kinoshita Gabrielse

Electron's anomalous magnetic moment a_e

- Harvard group measured with cylindrical Penning trap

D. Hanneke, S. Fogwell, G. Gabrielse, PRL 100, 120801 (2008)
D. Hanneke, S Fogwell Hoogerheide, G. Gabrielse, PRA 83, 052122 (2011)

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T. Aoyama, T. Kinoshita, M. Nio, Atoms, 7(1), 28 (2019)

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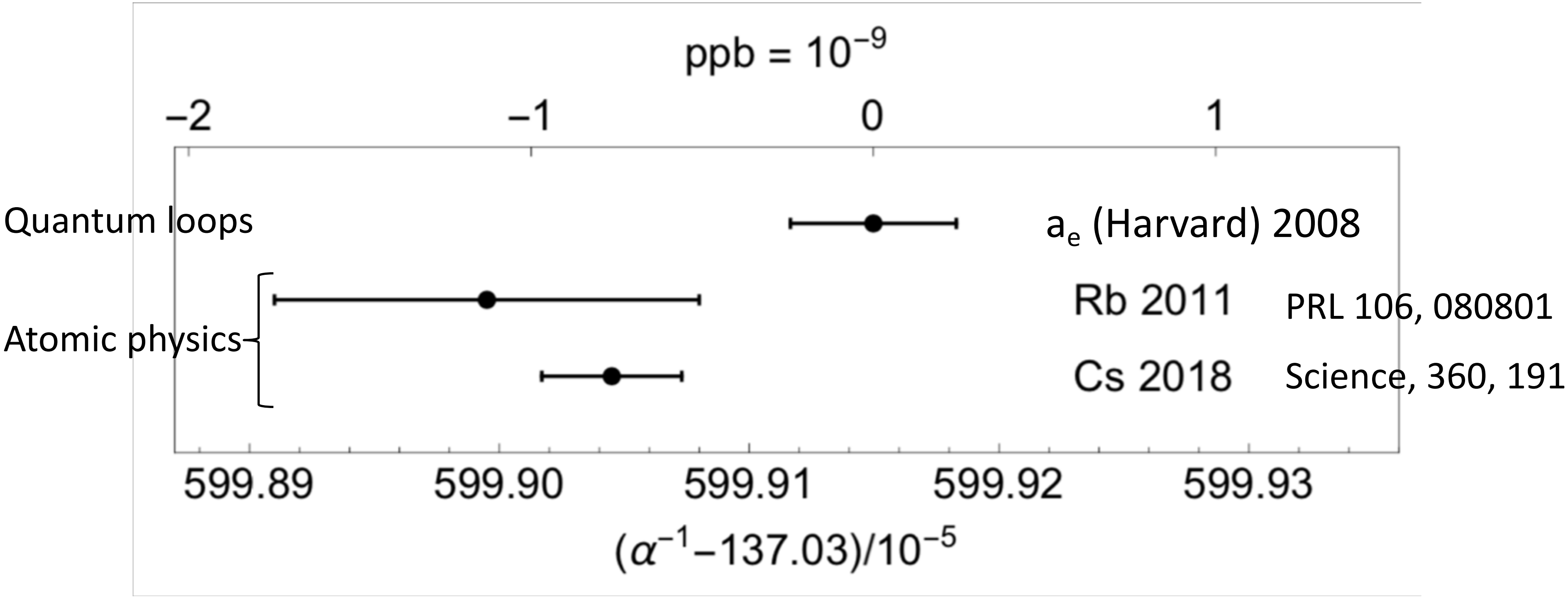
$$\Delta a_e = a_e(exp) - a_e(SM) = -0.88\;(36) \times 10^{-12}$$

Fine structure constant: α

- One can extract α from a_e assuming the SM is valid.

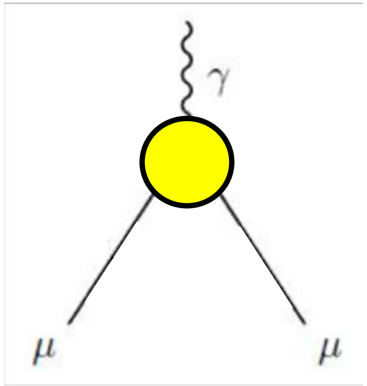


G. Gabrielse, S. E. Fayer, T.G. Myers and X. Fan, Atoms 7, 45 (2019)

Precision test of QED and internal consistency of Quantum Mechanics

Prof. T. Kinoshita gave a seminar on 10^{th} order QED corrections at KEK, 2012

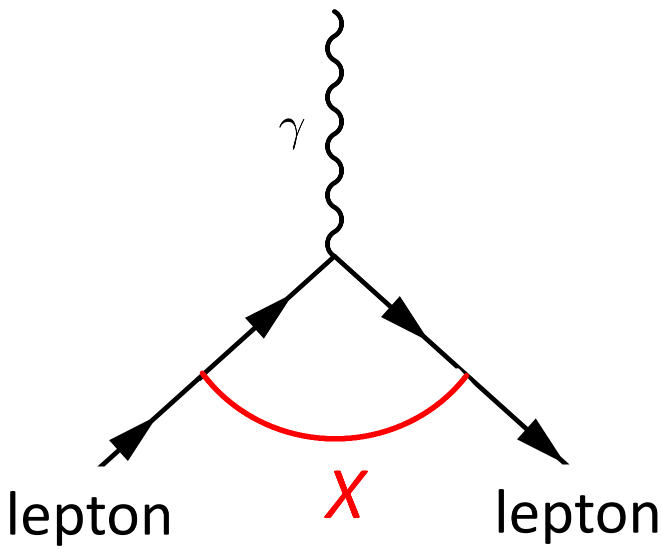
Anomalous magnetic moment of muon



$$a_\mu = a_\mu(QED) + a_\mu(had) + a_\mu(weak) + a_\mu(BSM)$$

Lepton g-2

- Contributions to lepton g-2 can be written as



$$a_l(X) \sim C_X \left(\frac{m_l}{\Lambda_X} \right)^2$$

C_x : Coupling strength

Λ : Mass Scale

$$\left(\frac{m_\mu}{m_e} \right)^2 \sim 43,000$$

Much larger contributions to muon than electron.

$$\left(\frac{m_\tau}{m_\mu} \right)^2 \sim 170$$

Even larger for tau, but difficult to measure.

Comparison of SM contributions

	electron (in unit of 10^{-13})	muon (in unit of 10^{-10})
QED contribution	115 965 218 00.7 $(7.6)_\alpha$	11 658 471.808 (0.015)
EW contribution	0.385 (0.004)	15.4 (0.2)
Hadronic contributions		
LO hadronic	**18.66 (0.11)**	**694.9 (4.3)**
NLO hadronic	−**2.23 (0.01)**	−9.8 (0.1)
light-by-light	0.39 (0.13)	10.5 (2.6)
Theory total	**115 965 218 17.9 $(7.6)_\alpha$**	**11 659 182.8 (4.9)**
Experiment	**115 965 218 07.3 (2.8)**	**11 659 208.9 (6.3)**
Theory − Exp	**10.6 (8.1)**	**26.1 (8.0)**

Numbers are from slides by D. Nomura

Comparison of SM contributions

	electron (in unit of 10^{-13})		muon (in unit of 10^{-10})
QED contribution	115 965 218 00.7 $(7.6)_\alpha$	(x 1.005)	11 658 471.808 (0.015)
EW contribution	0.385 (0.004)	(x 4.0E+4)	15.4 (0.2)
Hadronic contributions			
LO hadronic	**18.66 (0.11)**	(x 3.7E+4)	**694.9 (4.3)**
NLO hadronic	−**2.23 (0.01)**	(x 4.3E+3)	−9.8 (0.1)
light-by-light	0.39 (0.13)	(x 2.7E+4)	10.5 (2.6)
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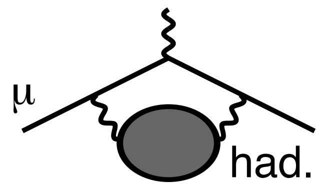
Numbers are from slides by D. Nomura

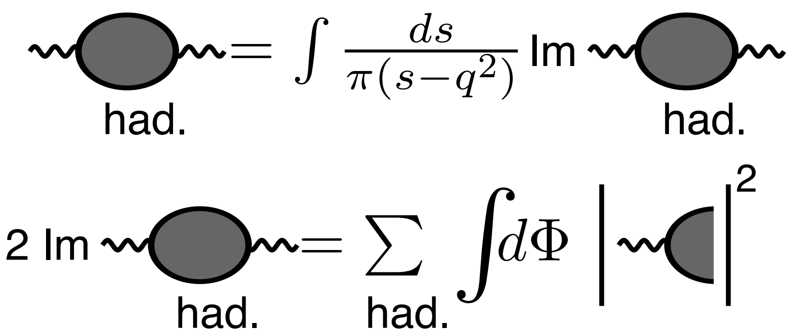
Hadronic contributions to a_μ

Slide by D. Nomura

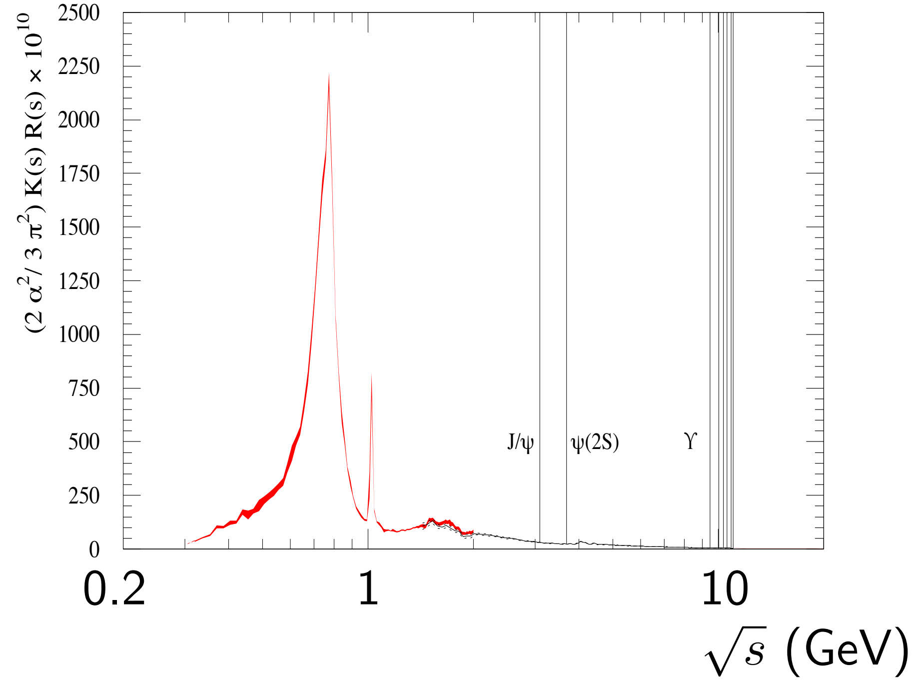
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



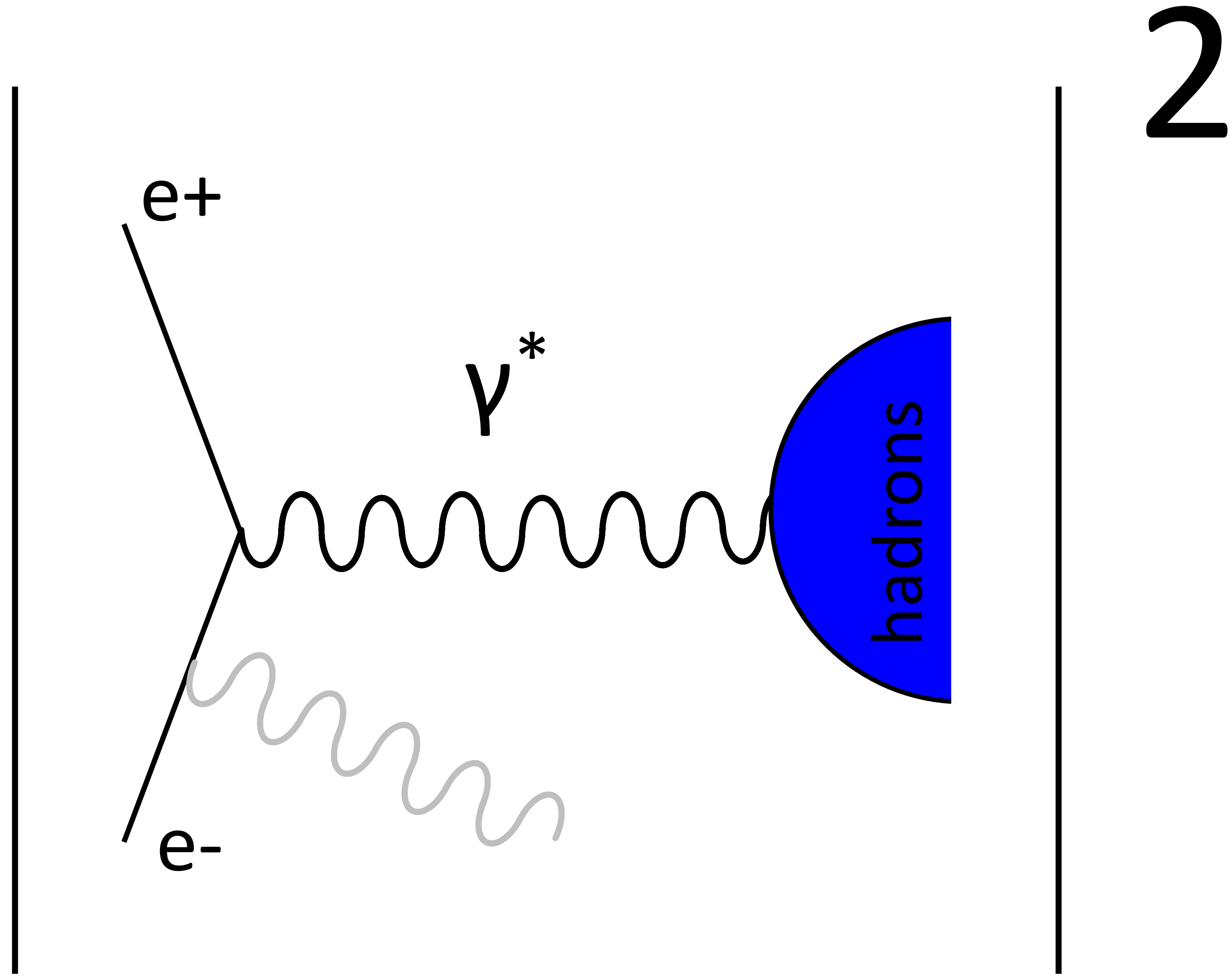
$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s} \hat{K}(s) \sigma_{\mathrm{had}}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

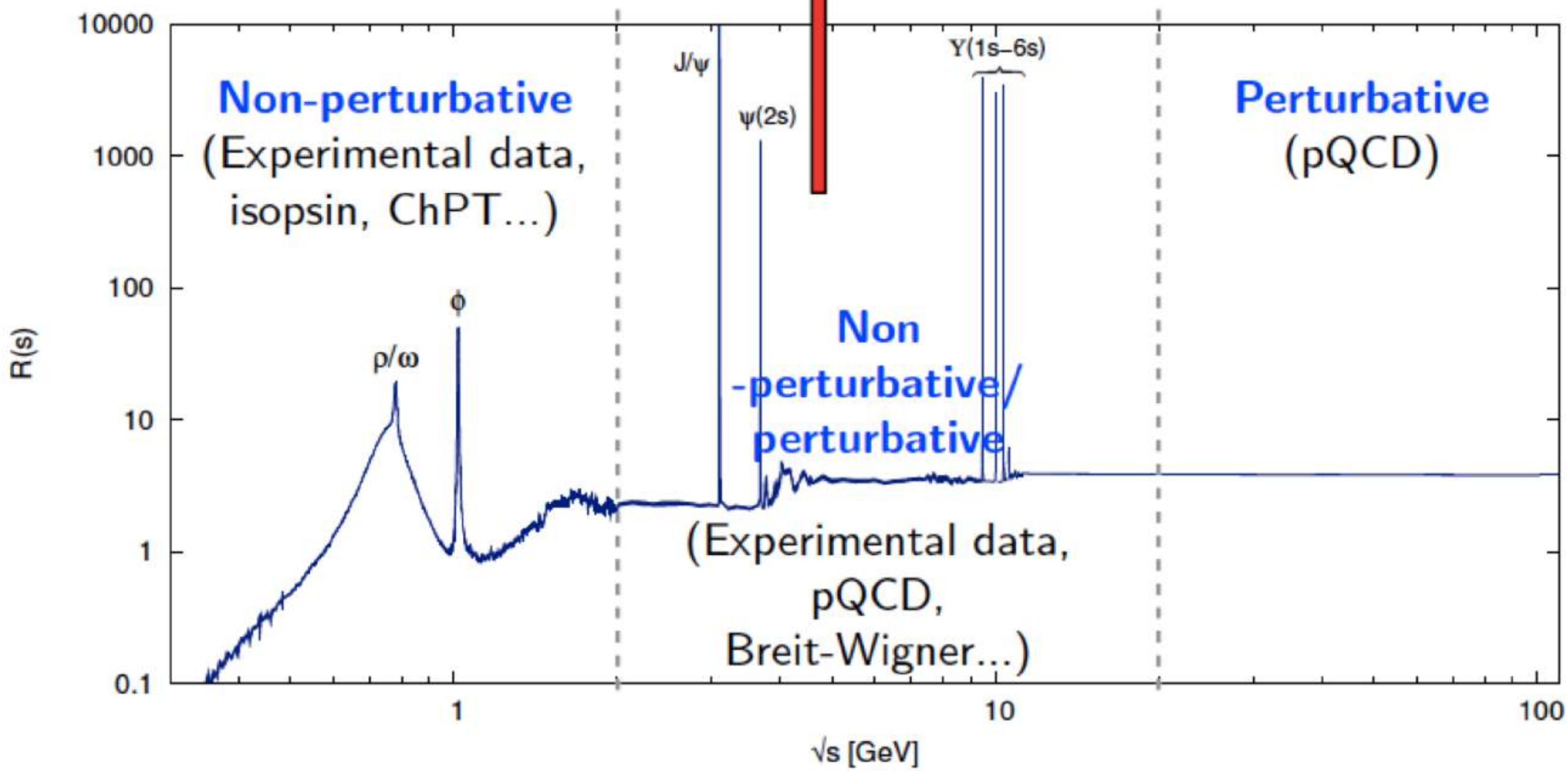
$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

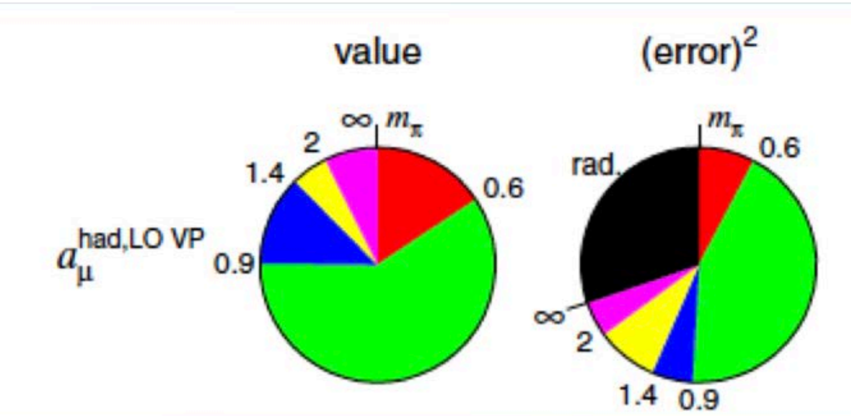

2
e+
γ*
hadrons
e-

Building the hadronic *R*-ratio

$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\text{had},\gamma}(s)}{4\pi\alpha^2/3s}$$

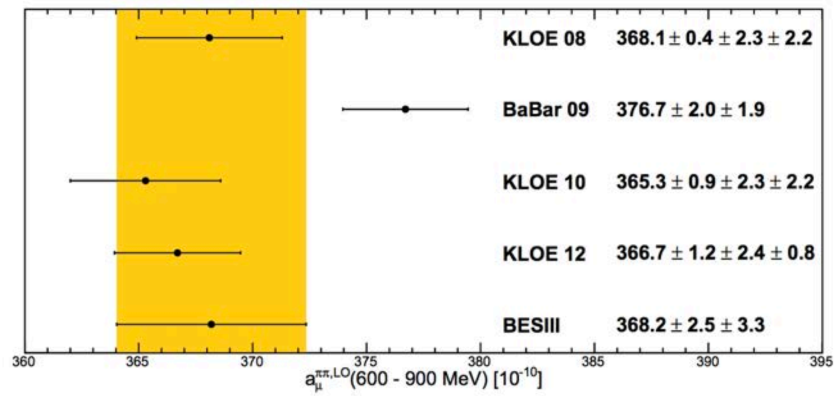


Critical inputs : $e^+e^- \rightarrow \pi^+\pi^-$ cross section



Hagiwara et al., J. Phys. G: Nucl. Part. Phys. 38 (2011) 085003

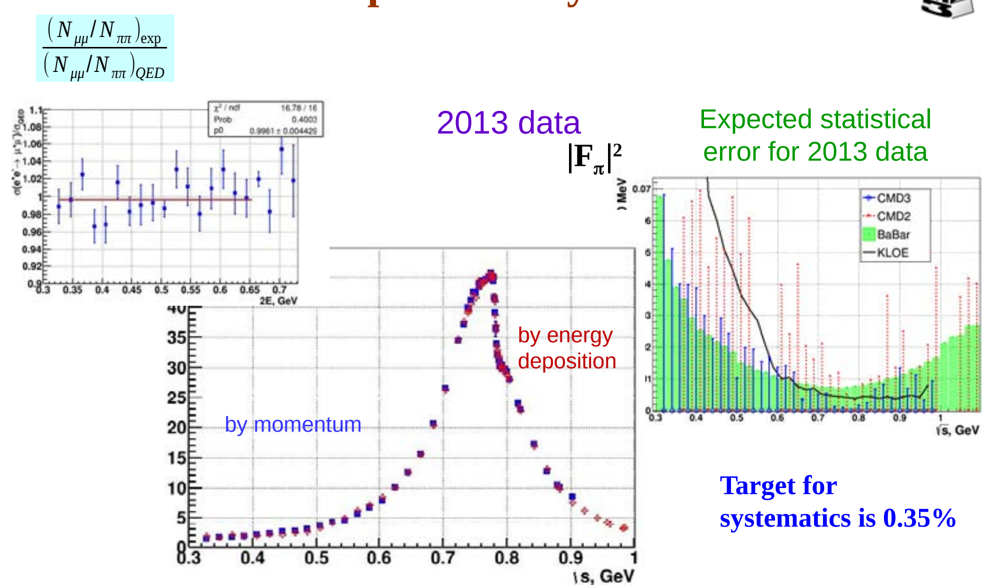
Phys. Lett. B 753, 629 (2016)



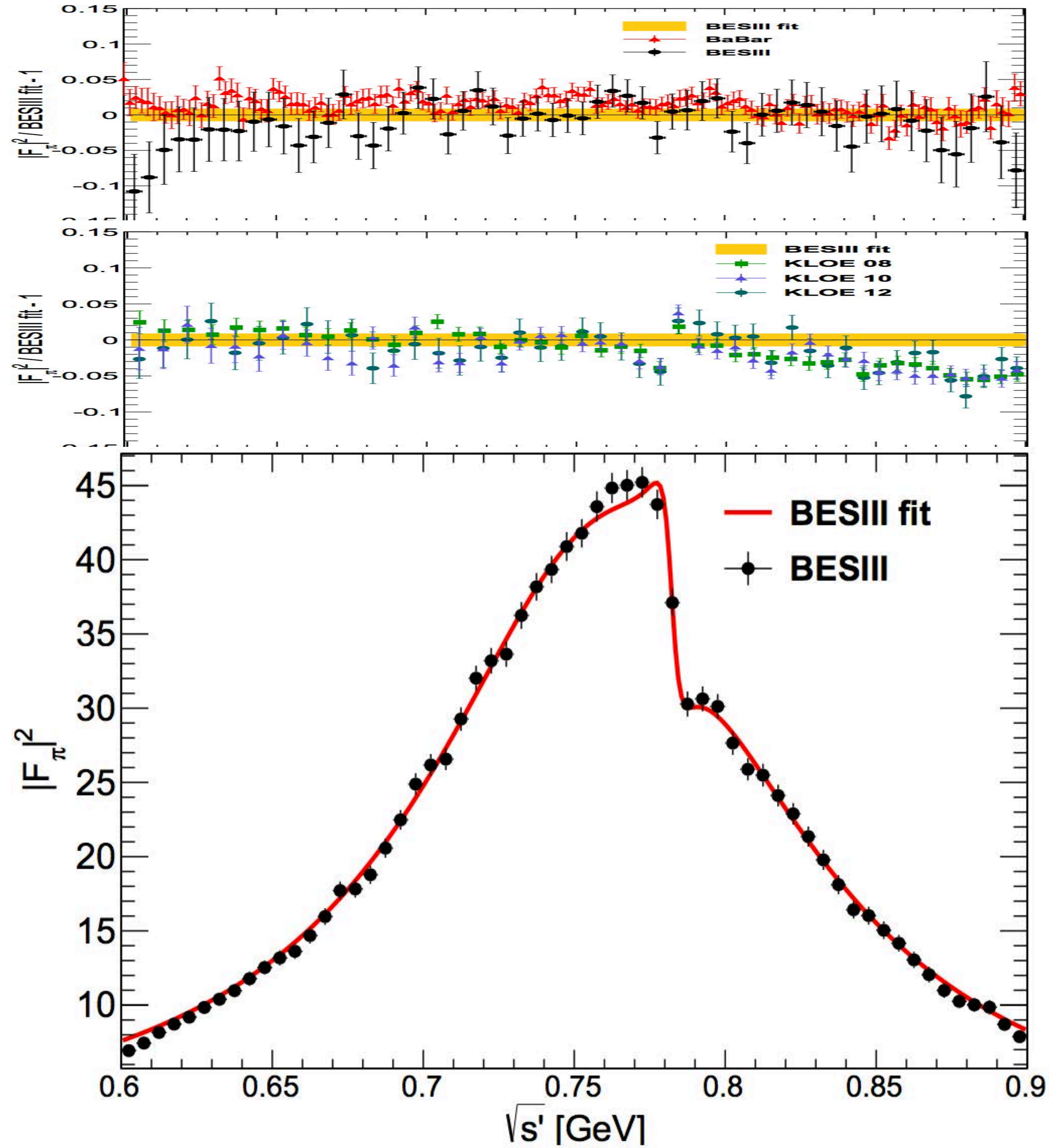
- Dominant uncertainty on a_μ(had,LO) comes from uncertainty (inconsistency) on e+e- data.
- Data from BESIII, VEPP-2000, (and Belle-II in the future) is critical to improve the situation.

Slide by Boris Shwartz (BINP)

$e^+e^- \rightarrow \pi^+\pi^-$: preliminary results



BES-III and Babar,KLOE data



Phys. Lett. B 753, 629 (2016)

$a_\mu^{\text{had, VP}}$ from KNT18 [KNT18: Phys. Rev. D 97 (2018) 114025]

KNT18 only: excluding preliminary updates (slides 15-17)

$$\text{HLMNT(11)}: \ 694.91 \pm 4.27$$

$$\downarrow$$

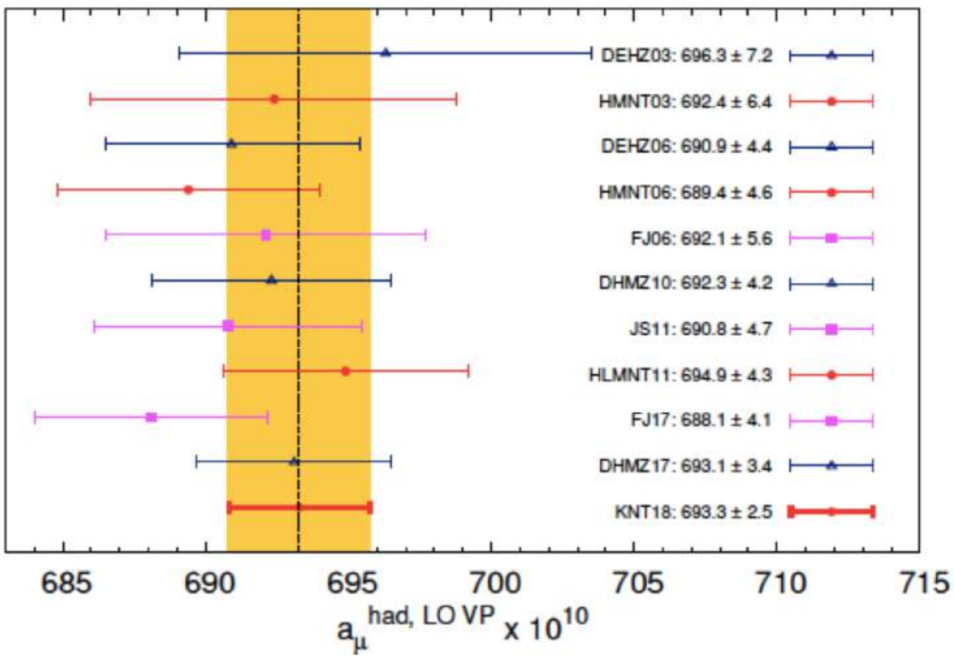
This work: $a_\mu^{\text{had, LO VP}} = 693.26 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$

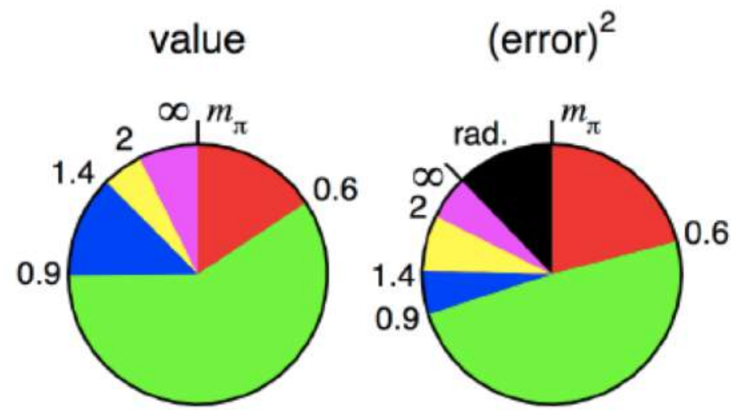
$$= 693.26 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$$

$$= 693.26 \pm 2.46_{\text{tot}}$$

$$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$$

⇒ Accuracy better then 0.4% (uncertainties include all available correlations and local χ^2 inflation)

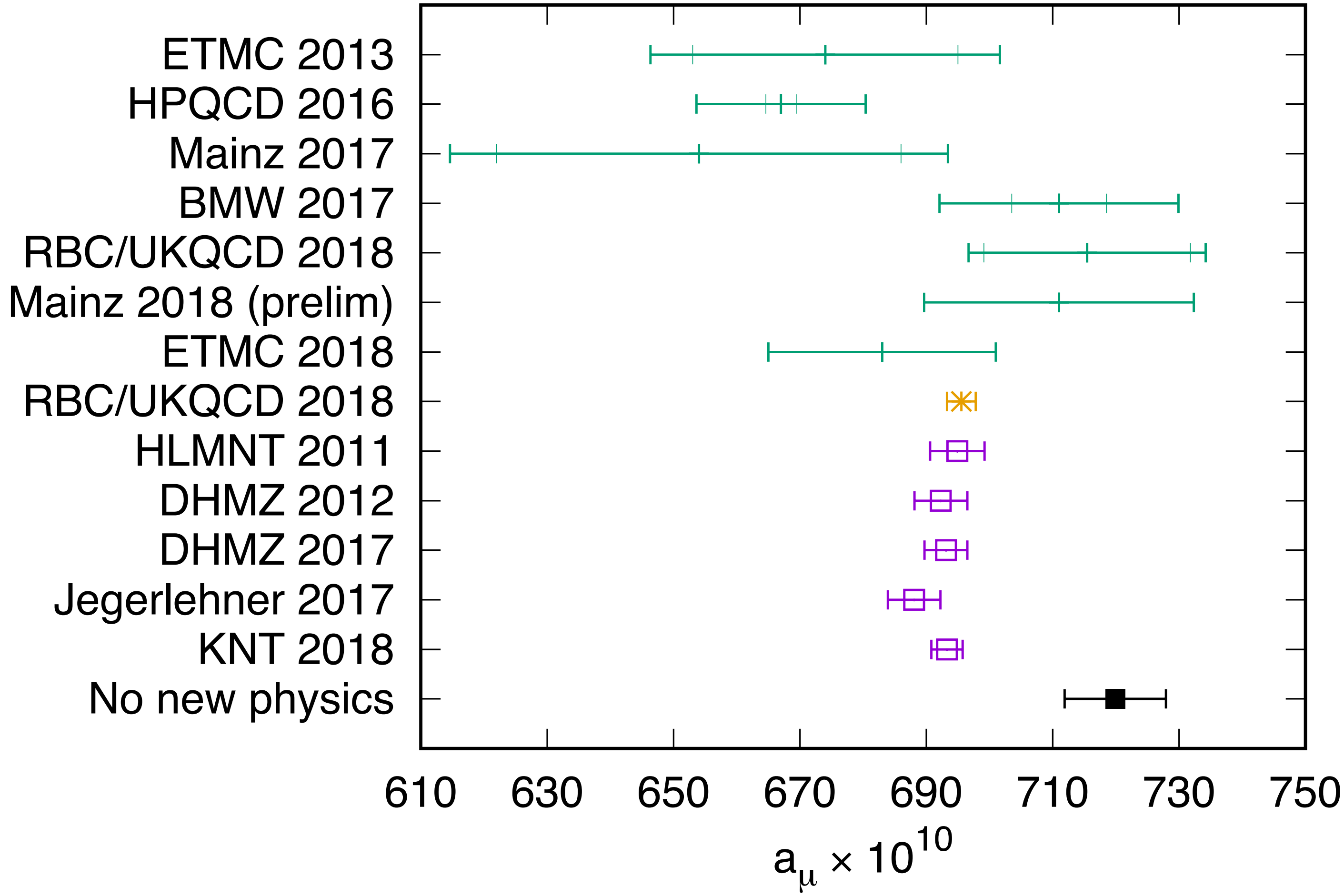




⇒ 2π dominance

New avenue: Lattice QCD calculations

Status of HVP determinations



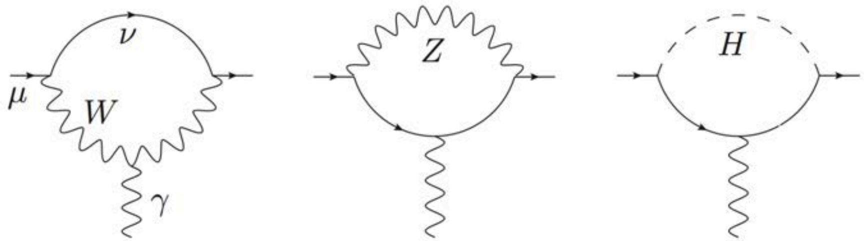
Green: LQCD, Orange: LQCD+Dispersive, Purple: Dispersive

Slide by Christoph Lehner (BNL) in Schwingerfest, UCLA, Dec 2018

Electro-Weak contributions

Czarnecki, Krause, Marciano, Vainshtein;   Knecht, Peris, Perrottet, de Rafael;

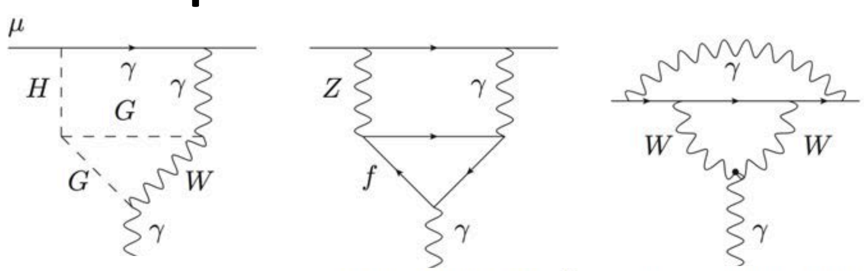
One Loop



+ 38.9 x 10^{-10} - 19.4 x 10^{-10} ≤ 3.3 x 10^{-14}

$$a_\mu^{\rm EW}(1\,{\rm loop}) = \frac{5}{3}\frac{{\rm G}_\mu {\rm m}_\mu^2}{8\sqrt{2}\pi^2} \times \left[1 + \frac{1}{5}(1-4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M^2}\right)\right] \approx 195\times 10^{-11}$$

Two Loop



$(-19.97\pm 0.03)\times 10^{-11}$ $(-4.64\pm 0.10)\times 10^{-11}$ $-(8.21\pm 0.10)\times 10^{-11}$

with $114\ {\rm GeV} \lesssim M_H \lesssim 250\ {\rm GeV}$

$$a_\mu^{\rm EW}(2\ {\rm loop}) = -41(1)(2)\times 10^{-11}$$

$$a_\mu^{\rm EW(1+2)} = (154 \pm 2)\times 10^{-11}$$

Gnendiger+Stoeckinger+S-Kim, Phys.Rev.D.88 (2013)

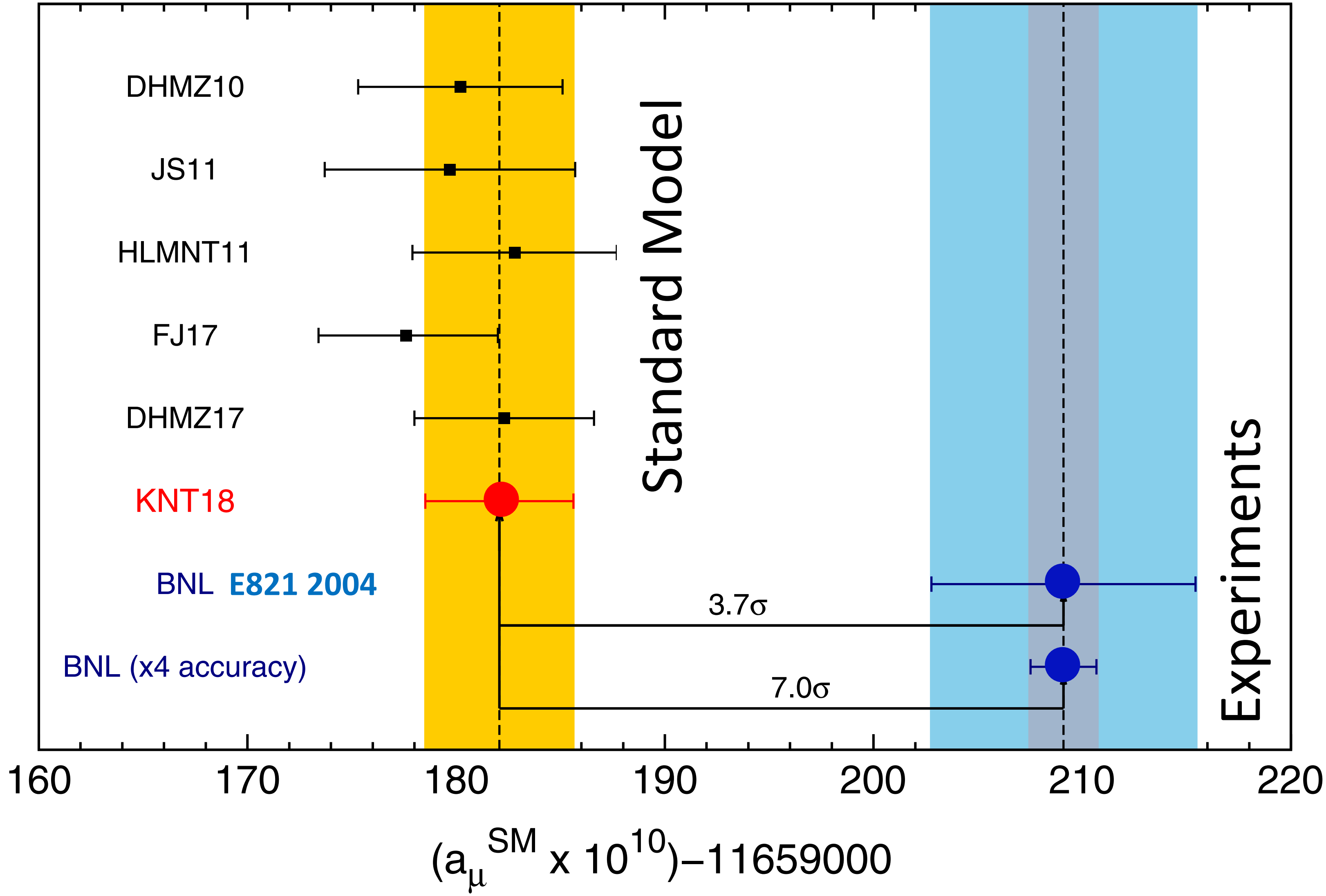
with $M_H = 125.6 \pm 1.5\ {\rm GeV}$ $a_\mu^{\rm EW(1+2)} = (153.6 \pm 1.0)\times 10^{-11}$

Theory collaboration
"Muon g-2 theory initiative"
meets at Mainz, June 18-22, 2018

Next meeting : INT, University of Washington, Seattle) 9-13 September 2019.

Comparison between SM and a_μ

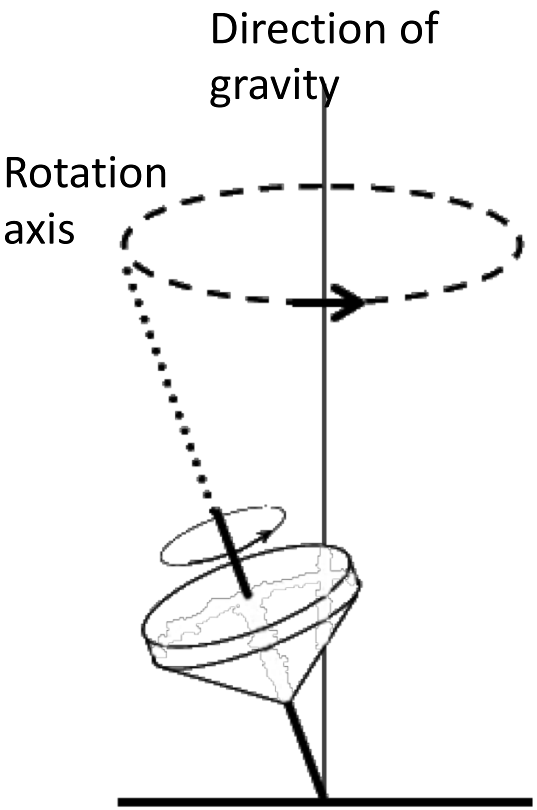
A. Keshavarzi, D. Nomura, T. Teubner, Phys. Rev. D 97, 114025 (2018)

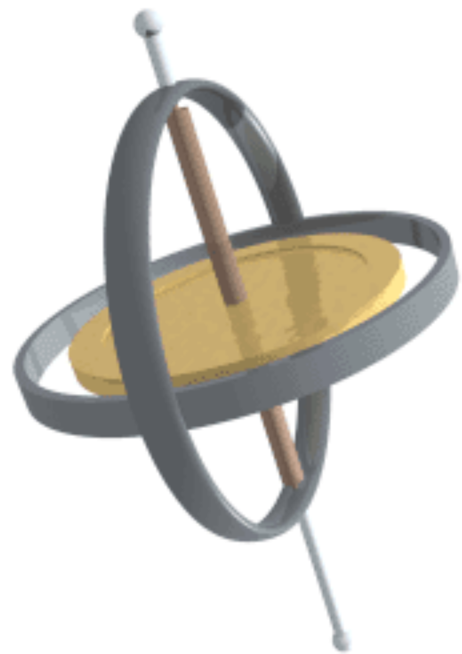


Note that electron g-2 is consistent with the SM.

Principle of measurements

Precession





Courtesy: LucasVB

Equation of spin precession

$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s}$$

Rotation axis

spin

Dipole moments and E- and B-field

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

Magnetic Dipole Moment

$$\vec{\mu} = g \left(\frac{q}{2m} \right) \vec{s}$$

CP even

Electric Dipole Moment

$$\vec{d} = \eta \left(\frac{q}{2mc} \right) \vec{s}$$

CP odd

g, η: dimension less quantities

EM fields introduces a torque on dipole moments

Spin precession in EM field

- In particle rest frame,

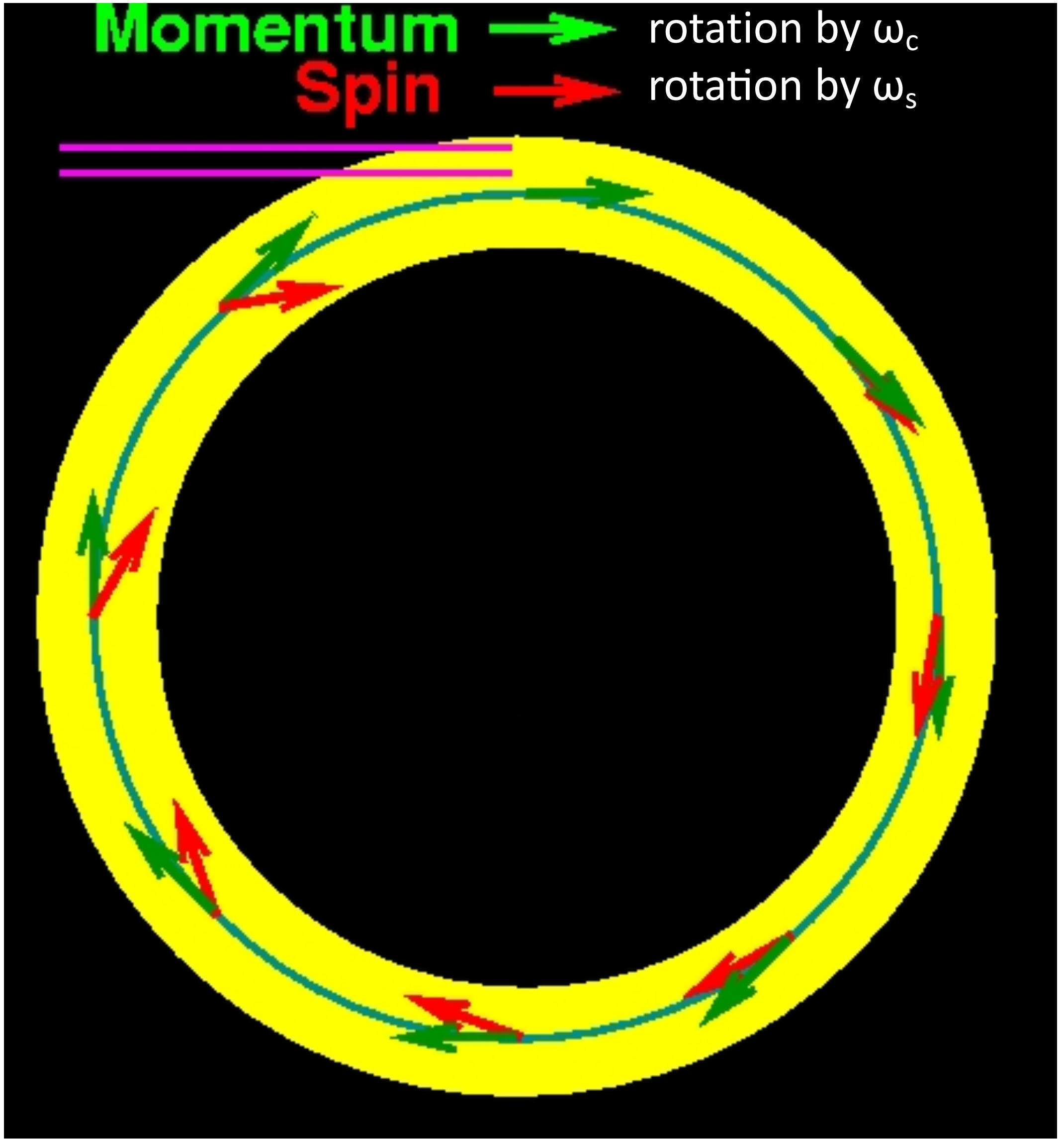
$$\frac{d\mathbf{s}}{dt} = 2\mu(\mathbf{s} \times \mathbf{B}) + 2d(\mathbf{s} \times \mathbf{E})$$

How about spin equation of motion in laboratory frame for moving particle?

J.D. Jackson, Classical Electrodynamics (3rd edition),p 561-565
T. Fukuyama, A. Silenko, Int. J. of Mod. Phys. A 28 1350147 (2013), arXiv:1308.1580
E. Won, private communications

Rotation of momentum (ω_c) and spin (ω_s)

Uniform B-field（perpendicular to the screen）



Angular rotation vectors

- Analytic expressions for ω_s, ω_c

$$\boldsymbol{\omega}_s = -\frac{e}{m}\left[\left(\frac{g-2}{2}+\frac{1}{\gamma}\right)\mathbf{B}-\left(\frac{g-2}{2}\frac{\gamma}{\gamma+1}\right)(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta}-\left(\frac{g-2}{2}+\frac{1}{\gamma+1}\right)\left(\boldsymbol{\beta}\times\frac{\mathbf{E}}{c}\right) + \frac{\eta}{2}\left(\frac{\mathbf{E}}{c}-\frac{\gamma}{\gamma+1}\left(\boldsymbol{\beta}\cdot\frac{\mathbf{E}}{c}\right)\boldsymbol{\beta}+\boldsymbol{\beta}\times\mathbf{B}\right)\right] \quad (46)$$

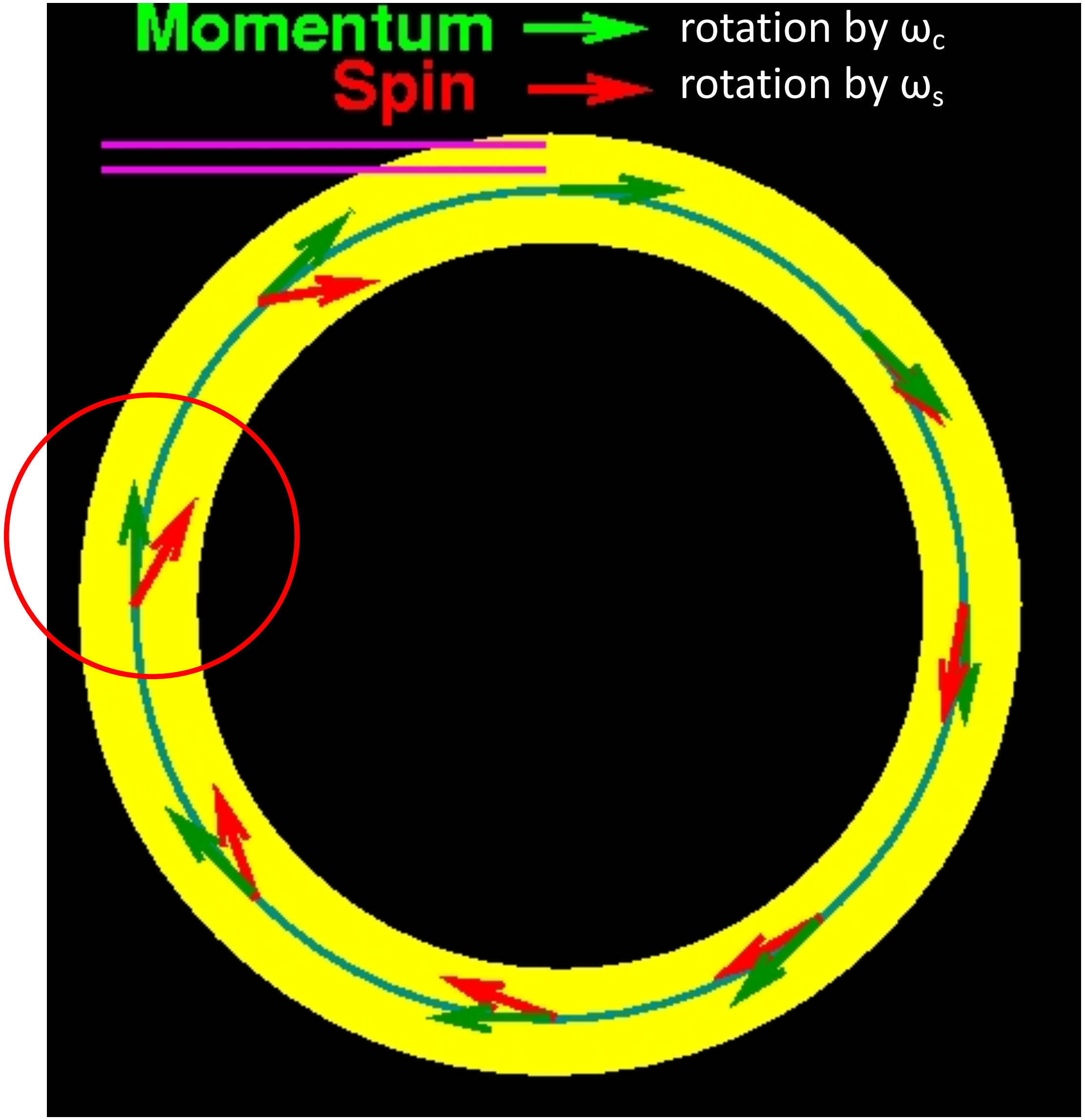
$$\boldsymbol{\omega}_c = \frac{e}{m\gamma}\left[\frac{1}{\beta}\left(\mathbf{N}\times\frac{\mathbf{E}}{c}\right)-\mathbf{B}\right]$$

a term with γ vanished!

$$\begin{aligned}\boldsymbol{\omega}_a &= \boldsymbol{\omega}_s-\boldsymbol{\omega}_c \\ &= -\frac{e}{m}\left[\left(\frac{g-2}{2}\right)\mathbf{B}-\left(\frac{g-2}{2}\frac{\gamma}{\gamma+1}\right)(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta}-\left(\frac{g-2}{2}-\frac{1}{\gamma^2-1}\right)\left(\boldsymbol{\beta}\times\frac{\mathbf{E}}{c}\right)\right. \\ &\quad \left. + \frac{\eta}{2}\left(\frac{\mathbf{E}}{c}-\frac{\gamma}{\gamma+1}\left(\boldsymbol{\beta}\cdot\frac{\mathbf{E}}{c}\right)\boldsymbol{\beta}+\boldsymbol{\beta}\times\mathbf{B}\right)\right]\end{aligned}$$

Meaning of the difference ω_s - ω_c

Uniform B-field (perpendicular to the screen)



$\omega_s - \omega_c$ is an angle between two vectors

Anomalous precession vector

- Difference of two vectors ω_s, ω_c

$$
\begin{aligned}
\boldsymbol{\omega}_a &= \boldsymbol{\omega}_s - \boldsymbol{\omega}_c \\
&= -\frac{e}{m}\left[\left(\frac{g-2}{2}\right)\mathbf{B} - \left(\frac{g-2}{2}\frac{\gamma}{\gamma+1}\right)(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta} - \left(\frac{g-2}{2} - \frac{1}{\gamma^2-1}\right)\left(\boldsymbol{\beta}\times\frac{\mathbf{E}}{c}\right)\right. \\
&+ \left.\frac{\eta}{2}\left(\frac{\mathbf{E}}{c} - \frac{\gamma}{\gamma+1}\left(\boldsymbol{\beta}\cdot\frac{\mathbf{E}}{c}\right)\boldsymbol{\beta} + \boldsymbol{\beta}\times\mathbf{B}\right)\right]
\end{aligned}
$$

- If we require $\mathbf{B}\cdot\boldsymbol{\beta} = 0, \quad \mathbf{E}\cdot\boldsymbol{\beta} = 0$
- then, we obtain

$$
\boldsymbol{\omega}_a = -\frac{e}{m}\left[a_\mu\mathbf{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\left(\boldsymbol{\beta}\times\frac{\mathbf{E}}{c}\right) + \frac{\eta}{2}\left(\frac{\mathbf{E}}{c} + \boldsymbol{\beta}\times\mathbf{B}\right)\right]
$$

here we define a_μ=(g-2)/2.

Anomalous precession vector

- If we require $\mathbf{B}\cdot\boldsymbol{\beta}=0, \quad \mathbf{E}\cdot\boldsymbol{\beta}=0$
- then, we obtain

$$\boldsymbol{\omega}_a = -\frac{e}{m}\left[a_\mu \mathbf{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\left(\boldsymbol{\beta}\times\frac{\mathbf{E}}{c}\right) + \frac{\eta}{2}\left(\frac{\mathbf{E}}{c} + \boldsymbol{\beta}\times\mathbf{B}\right)\right]$$

here we define $a_\mu=(g-2)/2$.

Beauties of this equation :

∗ "g" always appears as (g-2)

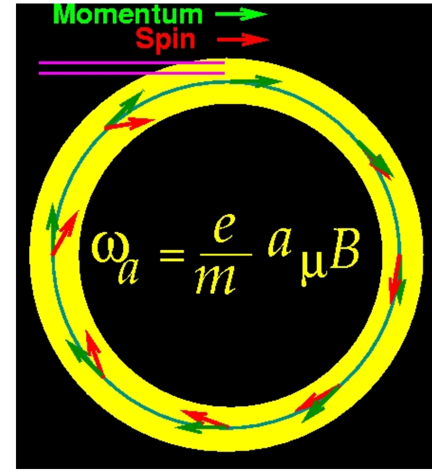
（only sensitive to g-2. g-2 << g, 1/1000）

∗ The first term doesn't include Lorentz gamma factor

（insensitive to momentum and its distribution）

∗ Allows us high-precession measurement if the 2nd term is suppressed

muon g-2 and EDM measurements



In uniform magnetic field, muon spin rotates ahead of momentum due to g-2 ≠ 0

general form of spin precession vector:

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

BNL E821 approach
γ=30 (P=3 GeV/c)

J-PARC approach
E = 0 at any γ

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)\right]$$

FNAL E989

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} + \frac{\eta}{2}\left(\vec{\beta}\times\vec{B}\right)\right]$$

J-PARC E34