Track fitting in the positron detector with GENFIT

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Abstract

We discuss the work carried out for developing a track fitting framework for the silicon tracker of the muon g-2/EDM experiment using the GENFIT package. We study the track fitting efficiency and momentum resolution of the tracker for positrons of different momentum. The p-value distribution of the fit is analyzed to evaluate the goodness of fit and to test the adequacy of the GENFIT package for track fitting in the tracker.

Introduction

Tracking is used in experiments to calculate the track parameters from the energy deposited by the particle in a detector in the form of hits.

0.1 Track reconstruction in particle physics

The task of tracking or track reconstruction is generally divided into track finding and track fitting, although, boundary between the two is fuzzy since in many cases track fitting algorithm can be used for track fitting.

0.1.1 Track finding

Track finding belongs to the broad set of mathematical problems on pattern recognition. Pattern recognition aims to categorize objects into a set of possible outcomes based on their characteristics. Pattern recognition applied in a high energy physics experiment constitutes track finding or track search. In any experiment, there are multiple particle traversing the detector simultaneously. As a result, the information collected by the detector is an assortment of hits which need to be segregated into different sets of hits, each belonging to a separate track. This task of separation of hits into possible track candidates is called track finding. Popular track finding approaches include hough transform, cellar automata.

0.1.2 Track fitting

Once the track finding algorithm separates the hits into sets of different track candidates, the job of track fitting is to estimate the vector parameters representing the state of the particle at a given point on the track, from the measurements collected in the form of detector hits. Track fitting algorithm can provide an estimate of the goodness of the fit in terms of the covariance matrix, χ^2 and hence it can also aid track finding by rejecting tracks which do not follow the expected trajectory. Different track fitting methods can be decomposed into the following set of logical steps[4]

Track parametrization

In detectors with planar geometry, a set of five parameters can describe the state of the particle. The specific choice of parameter depends on the detector geometry, for example, Cartesian coordinates are used for a detector system consisting of planar layers.

Track model

The track model is used to estimate the track parameters in a given detector layer k, given the track parameters in the detector layer i.

$$q_k = f_{k|i}(q_i) \tag{1}$$

The track model can be known analytically in simple cases such as zero magnetic field (straight line), homogeneous magnetic field (helix) but in other cases it requires a Runge-Kutta algorithm to model the particle transport in non-uniform magnetic field.

Error propagation

The covariance matrix represents the uncertainty in the estimated track parameters. It is propagated from detector level i to detector level k.

$$C_k = F_{k|i} C_i F_{k|i}^T \tag{2}$$

where C_k is the covariance matrix and $F_{k|i}$ is the Jacobian matrix of propagation from layer i to k.

Material effects

A particle traversing a medium can undergo ionization energy loss or multiple Coulomb scattering which will affect the estimated track parameters as well as the covariance matrix. In case of positrons and electrons, energy loss by bremsstrahlung also plays an important role and its cross section increases with the electron energy. On one hand bremsstrahlung energy loss affects both the estimated track parameter and its covariance matrix, while on the other hand ionization energy loss affects only the track parameters and multiple scattering affects only the covariance matrix.

Measurement model

The measurement model describes the dependence of measurements made in a detector layer k, m_k on the track parameters at the same point q_k

$$m_k = h_{kj}(q_j) \tag{3}$$

0.1.3 Track fitting methods[2]

In this section we will describe the various track fitting techniques used in high energy physics experiments. The measurements m are related to the true track parameters x_t through the measurement model H

$$m = Hx_t + \epsilon \tag{4}$$

Where ϵ describes the measurement errors which step due to effect such as limited detector resolution. The model described above is assumed to be linear in the true track parameter x_t which can also be replaced by a linear expansion if the function is non-linear $H(x_t)$. We will describe some track fitting methods-

Global least-square filter

Global least-square filters consider all the points of track in the fitting simultaneously. The estimated track parameters x are given as following-

$$x = (H^T V^{-1} H)^{-1} H^T V^{-1} m, \tag{5}$$

Where $V = Cov(\epsilon)$, is the Covariance matrix of the measurement errors ϵ. Since this method considers all the track points simultaneously, it requires inversion of a large matrices ($m \times m$ where m is the number of measurements) which has high computational cost. Another issue with this method is that the estimated track will deviate significantly from the true track in the presence of multiple scattering. Apart from that, the global nature of this method makes is unsuitable for track fitting in a system of different detectors.

Kalman Filter

Kalman filter is an iterative least square method of estimating the states of a dynamic system which works on the prediction/update scheme. The Kalman filter solves two issues associated with the global least-square filter. Firstly, the size of matrices to be inverted is small thus, saving the computational time and secondly, it can handle multiple scattering and other material related effects. The iterative nature of the filter makes it suitable for track fitting application in detector systems which consist of different sub-system. The Kalman filter algorithm can be sub-divided into two steps “filtering” and “smoothing”.

Filtering The filtering step of the filter has been divided into two steps, prediction and update. For an ideal scenario when the true track parameters $x_{t,k-1}$ are known for the detector layer $k-1$ the parameter for the k^{th} layer and can be estimated from the propagator function, keeping the errors into account.

$$x_{t,k} = F_k(x_{t,k-1}) + P_k\delta_k \tag{6}$$

Once the equation of state (eq.6) has been calculated, it can be used to predict the track parameter $(x_{k|k-1})$ and covariance matrix $(C_{k|k-1})$ in the k^{th} layer of the detector. This is called the prediction step.

$$x_{k|k-1} = F_k(x_{k-1|k-1}) \tag{7}$$

$$C_{k|k-1} = F_k C_{k-1|k-1} F_k^T + P_k Q_k P_k^T \tag{8}$$

The first term in the above equation linearly propagates the covariance matrix $(C_{k-1|k-1})$ to the k^{th} layer and $Q_k = \mathrm{Cov}(\delta_k)$ carries the information about the effect of material between the k^{th} and $k-1^{th}$ detector layer on the covariance matrix. Q_k can be calculated using an appropriate multiple Coulomb scattering model.

After predicting the state vector, the next step involves correcting the state vector after taking into account the measurements made in the k^{th} layer of the detector. This is called the update step and it is derived using the least squares method.

$$C_{k|k} = (C_{k|k-1}^{-1} + H_k^T V_k^{-1} H_k)^{-1} \tag{9}$$

$$x_{k|k} = C_{k|k}(C_{k|k-1}^{-1} x_{k|k-1} + H_k^T V_k^{-1} m_k) \tag{10}$$

Here H and V have the same meaning as in equation 5.

Smoothing The smoothing step involves running the filter from backwards from n^{th} step to 1^{st} step. The track parameters obtained at each detector layer $x_{b,k|k}$. The “smoothed” track parameter and covariance matrix are calculated from the weighted average of the forward and backward estimate as follows

$$x_{sm,k} = C_{f,k|k-1}^{-1} x_{f,k|k-1} + C_{b,k|k}^{-1} x_{b,k|k} \tag{11}$$

$$C_{sm,k} = (C_{f,k|k-1}^{-1} + C_{b,k|k}^{-1})^{-1} \tag{12}$$

Kalman filter with reference track

In the scenario where the first few hits have a bias, the propagated track can even miss a detector layer completely. This will result in fit failure even if the subsequent hits are perfectly aligned to the track. A solution to this issue is to update the filter along a reference track which is calculated by propagating the seed state (initial state) through all the detector layers, thereby saving the reference state $x_{r,k}$ in this process. Therefore, the propagation and update equations of Kalman filter are modified as follows

$$\Delta x_{k|k-1} = F_k \Delta x_{k-1|k-1} \tag{13}$$

$$\Delta x_{k|k} = \Delta x_{k|k-1} + K_k(\Delta m_k - H_k \Delta x_{k|k-1}) \tag{14}$$

Where Δ denotes the different of the states with the reference states, defined as follows

$$\Delta x_{k|k} = x_{k|k} - x_{r,k}, \quad \Delta m = m_k - H_k x_{r,k} \tag{15}$$

Although reference Kalman filter can solve the issue of first bad hits, the reference track can improve the fit only when the seed state is a good representation of the track.

Deterministic annealing filter (DAF)

A statistical estimation method is called robust if it can reject outliers. The global least square estimator and Kalman filter, both are not robust, therefore the presence of outliers can significantly affect the track parameter estimate. The Deterministic Annealing filter (DAF) can work on a sample with both bad seed hits as well as outliers. The DAF utilized a Kalman filter to estimate the track parameters and adds weights to the hits, which determines the probability of a particular hit belonging to the track. It runs a forward and backward Kalman filter followed by the smoother to calculate the weights for the states. These calculated weights are utilized in the next iteration of the filter and the procedure is repeated until a convergence criteria is reached. The DAF with a reference Kalman filter can act as a robust track fitter which can cope with bad seed states as well as outlier. The discussion of the mathematical formulation of the DAF filter is beyond the scope of this work.

0.2 Testing the goodness of fit[2]

In this section we will describe the methods used in checking the goodness of the fit.

0.2.1 χ^2 and p value test

The χ^2 distribution is used to describe the degree of agreement of the data to the track model. When the track has m measurements, and the number of parameters to be distributed are n, the χ^2 of the fit will be distributed according to a χ^2 distribution of degree of freedom $m - n$. The residual r_k at each measurement m_k point is calculated as follows.

$$r_k = m_k - H_k x_k \tag{16}$$

And the covariance matrix of r_k is calculated according to

$$R_k = V_k - H_k C_k H_k^T \tag{17}$$

where V_k is the covariance matrix of the measurements and C_k is the covariance matrix of the fit calculated by the filter. The χ_k^2 calculated for each detector layer k is

$$\chi_k^2 = r_k^T r_k^{-1} r_k \tag{18}$$

Using the χ_k^2 of individual measurements, the total χ_{tot}^2 of the track is calculated as

$$\chi_{tot}^2 = \Sigma \chi_k^2 \tag{19}$$

Since different tracks have different number of measurements, the χ^2 distribution can have different degree of freedom for different tracks. For a given value of χ_{total}^2 for the track, the p-value is calculated as

$$p = P(\chi^2 > \chi_{total}^2) \tag{20}$$

The p value is the area in the tail of the pdf of (χ^2,NDF) to the right of the value of χ_{total}^2 obtained for a given track as shown in Fig 2

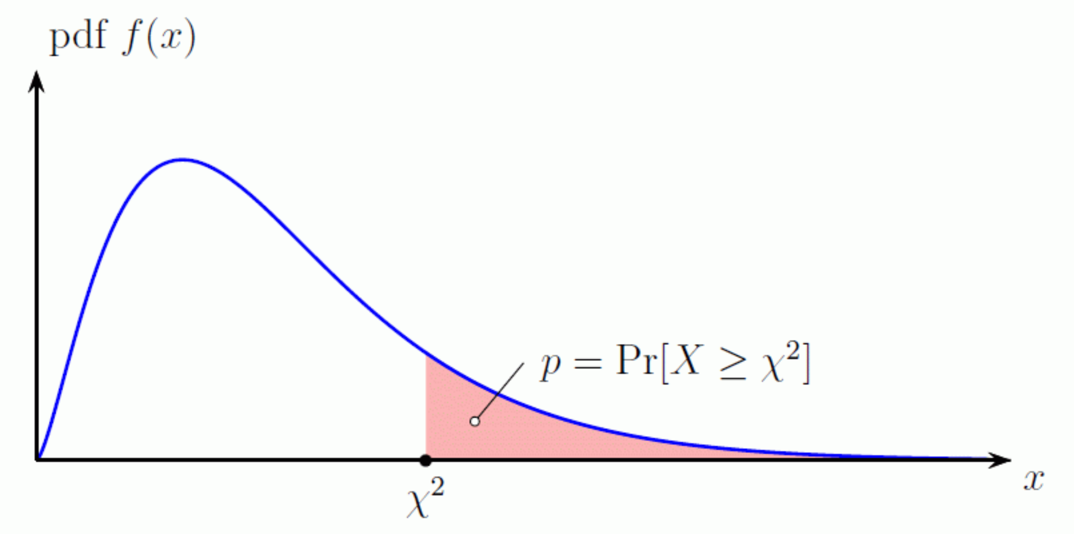


Figure 2: The relation between p value and χ^2

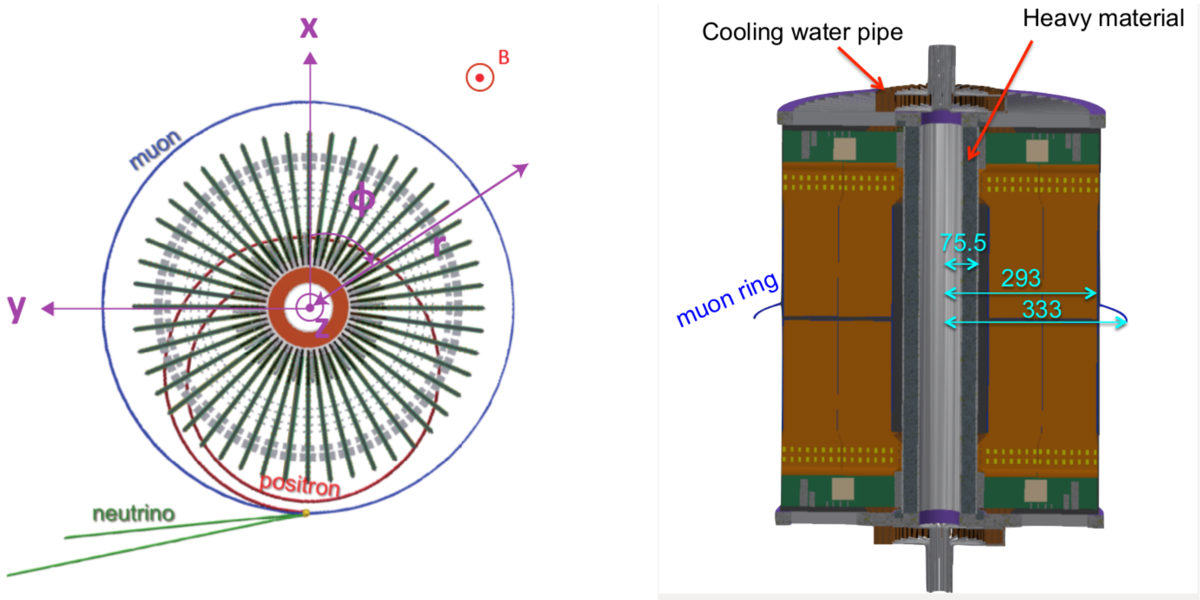
Therefore, the p-value is the probability of getting the results of the fit worse than the one that was obtained, under the condition that the null hypothesis is true. While on one hand a small p-value denotes bad fit since the χ^2 is very high on the other hand the a p-value close to 1 implies over-estimation of errors. We expect some outliers in the data which will result in p-value close to 0, however the benchmark test for ensuring a goodness of fit is that **the p-value distribution should be uniform** along with some outliers. Departure of p-value distribution from the above mentioned criteria hints towards an error in implementation.

0.3 Track reconstruction in the g-2 experiment

The muon will be transferred to the storage rings after an initial acceleration and it will subsequently decay into a e^+ along with ν_μ and $\overline{\nu}_e$ as shown in Figure 3.

0.3.1 Silicon tracker

The silicon tracker will be composed on 48 silicon strip detector vanes arranged over full azimuth, with a 3 T magnetic field for momentum measurement. The radial coordinate of the detector lies between 70 mm to 290 mm in order to detect positron with energy range (200,280) MeV. A tungsten absorber will be put at the center of the detector to stop low energy electrons from performing multiple curls.



(a) Muon decay through the tracker (b) Detector geometry

Figure 3: Silicon tracker for the g-2/EDM experiment

The Figure 4 shows the design of detector vane for the upper half of the detector. Each vane has been divided into 4 modules and each is divided into 4 sensors, with each sensor with a dimension of 98.77 × 98.77 mm.

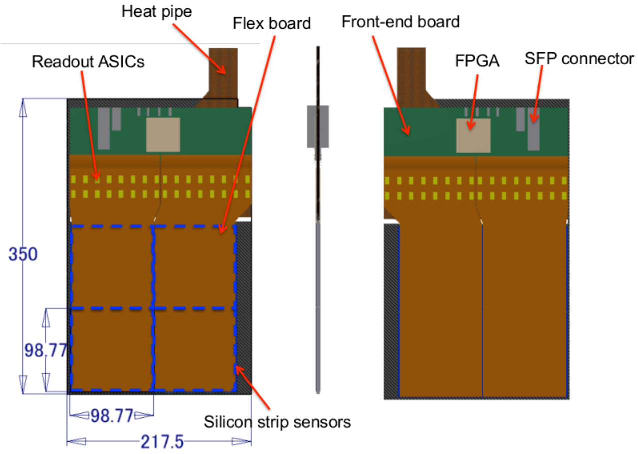


Figure 4: Design of the detector vane and module for the upper half of the detector

0.3.2 Event shape

The positron is expected to follow a helix under the influence of uniform magnetic field, however, the interaction with the silicon detector will result in departure from the ideal trajectory due to the following effects-

- Multiple scattering- Low energy positrons are more likely to undergo multiple scattering when traversing a material
- Bremsstrahlung- Bremsstrahlung cross section increases with positron energy. This will results in kinks in the positron track since the emitted photon cannot be detected by the tracker.
- Ionization energy loss- Positrons can also suffer ionization energy loss as given by the Bethe Bloch expression.

Figure 5 shows the different interaction cross section of the positron as a function of its energy.

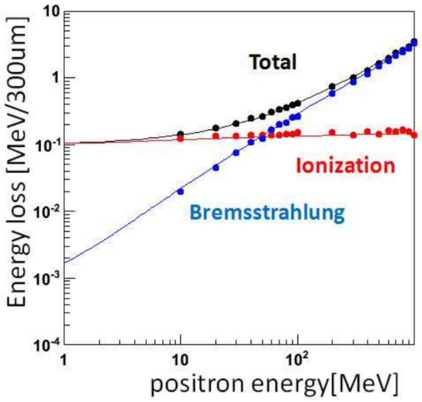
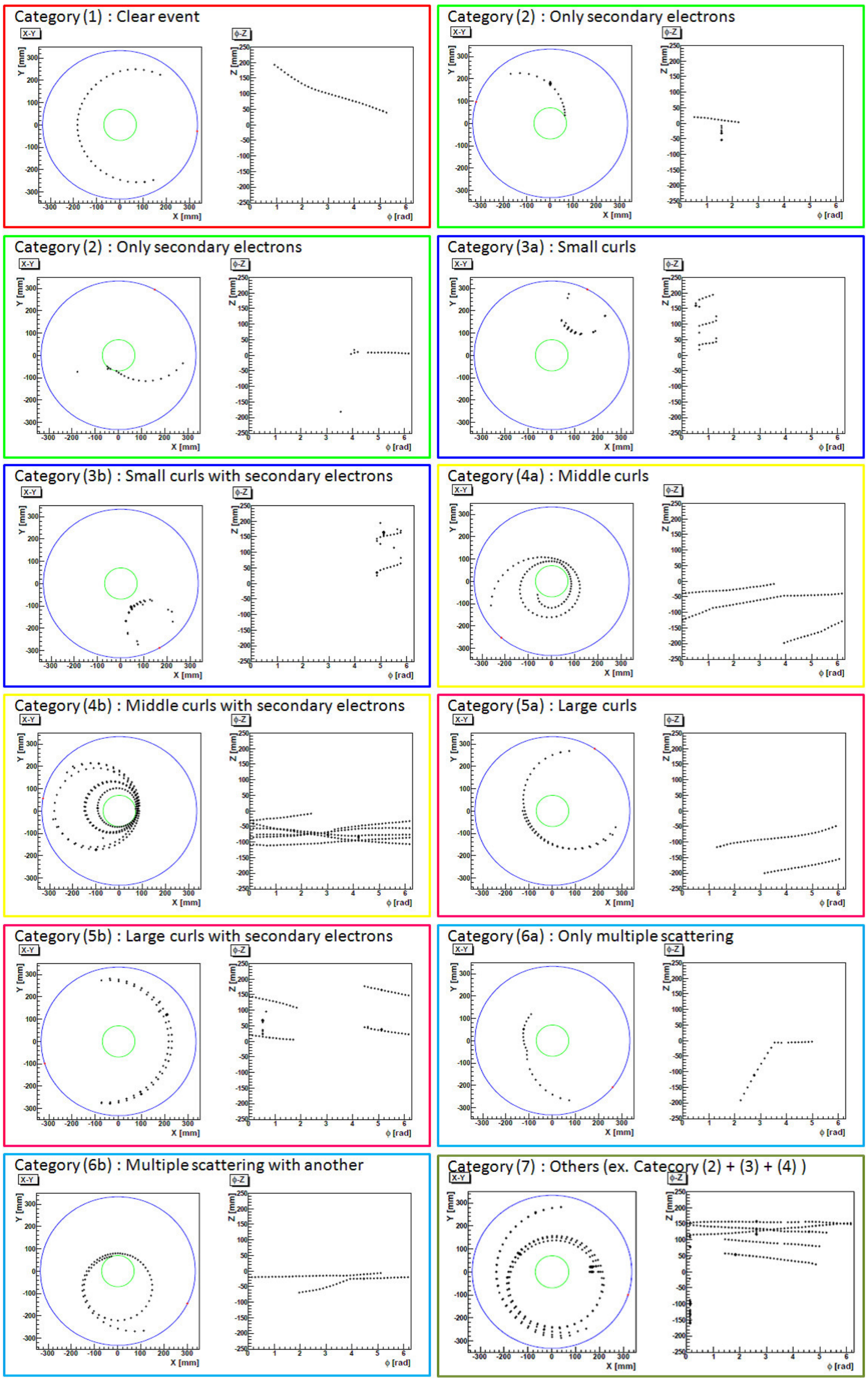


Figure 5: Interaction cross section as a function of positron energy[1]

The presence of these effects will results in a variety of event shapes as shown in Figure 6

The track fitting algorithms in GENFIT assume a Gaussian distribution of errors. However, the PDF of multiple scattering and Bremsstrahlung have a non-Gaussian component with long tail as shown in Fig 7. This will results in large number of outliers which will show a significant departure from the ideal helix trajectory.

1. Multiple scattering- Distribution of scattering angle θ has a Gaussian core with a non-Gaussian tail as shown in Figure a).
2. Bremsstrahlung- It follows Bethe-heitler distribution which is clearly non-Gaussian. Figure b) shows the PDF of $z = \frac{E_{final}}{E_{initial}}$ for different materia thickness t.



Category (1) : Clear event

Category (2) : Only secondary electrons

Category (2) : Only secondary electrons

Category (3a) : Small curls

Category (3b) : Small curls with secondary electrons

Category (4a) : Middle curls

Category (4b) : Middle curls with secondary electrons

Category (5a) : Large curls

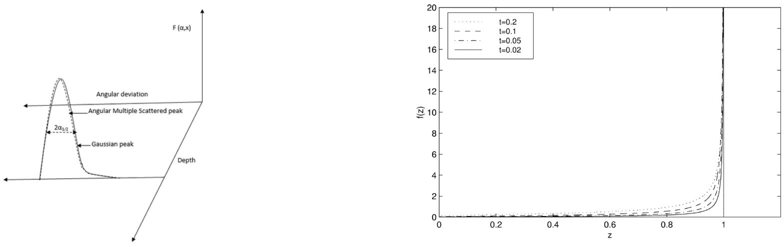
Category (5b) : Large curls with secondary electrons

Category (6a) : Only multiple scattering

Category (6b) : Multiple scattering with another

Category (7) : Others (ex. Category (2) + (3) + (4))

Figure 6: Categories of positron events as expected from simulations[1]



(a) PDF of scattering angle (b) Bremsstrahlung PDF

Figure 7: Non-Gaussian pdf of the bremsstrahlung and multiple coulomb scattering

0.3.3 Status of track reconstruction studies

Track reconstruction studies are carried out by preparing a simulation framework to generate hits followed developing by track finding framework and track fitting framework. The g-2/EDM collaboration has already prepared a GEANT/ROOT based framework to simulate events. A track finding package has been developed which implements hough transform to calculate the particle trajectory. In this work, we will discuss the work done in the direction of developing a track fitting framework, which could later be interfaced with the track finding and simulation framework in order to carry out track reconstruction studies.

0.4 Track fitting in the g-2 experiment

We will describe the track fitting studies carried out using the GENFIT track fitting package.

0.4.1 Requirements of track fitting studies

We intend to fulfill the following goals with the track fitting studies-

- Track fitting efficiency and positron momentum resolution must be known as a function of positron momentum. This will help us evaluate the detector performance for different energy positrons.
- The event shape must be analyzed to look for the occurrence of kinks in the trajectory due to effects like multiple scattering and bremsstrahlung.
- The GENFIT package has been originally designed for high energy particles, therefore, its performance must be tested for low energy electron tracking.

Once the track fitting code has been prepared and optimization has been carried out, the fitted positron track will be extrapolated to find the muon decay vertex and the decay time.

0.4.2 Track fitting using GENFIT

GENFIT[3] is am open source, experiment independent framework which implements different track fitting algorithm, thereby eliminating the need to write track fitting program for every experiment. After being used in Belle II, Panda and FOPI experiments, GENFIT has undergone significant improvements which have culminated into GENFIT2. We will briefly the steps carried out for track fitting in GENFIT2 (mentioned as GENFIT hereafter).

- Detector geometry is prepared using Root's TGeometry class and read into GENFIT.
- Electric and magnetic fields inside the detector are specified. In the simplest case, we can specify a uniform field, however, for complex geometries detailed calculation must be carried out for the field.
- An appropriate track model is selected based on the equations of motion in the magnetic/electric field. In our case, we expect the positron to follow a helix in a uniform magnetic field.
- Hit points are initialized along with the measurement errors and track fitting is carried out using a suitable algorithm.
- Goodness of fit is determined from the χ^2/NDF and the convergence of the track fitting.

0.4.3 Choice of track fitting algorithm

GENFIT offers many track fitting algorithm such as Simple Kalman Filter, Reference Kalman Filter, Deterministic Annealing Filter (DAF) with Simple Kalman, DAF with reference Kalman. These track fitting algorithm have advantages which are suited for different purposes. It was mentioned earlier that DAF filter can reject outliers which makes it a better choice for our purpose. We tested the results of track fitting on a sample of 250 simulated events with different filters. As it can be seen from table, **DAF with Reference Kalman filter** is an ideal choice for track fitting. We will therefore use this filter throughout this thesis.

Filter	Simple Kalman	Reference Kalman	Simple Kalman with Square root	Ref. Kalman with square root	DAF with Simple Kalman	DAF with Ref. Kalman
Percentage Track fitted	83.93	87.95	84.34	88.35	89.15	99.59

Table 1: Comparison of the performance of different track fitting algorithm

0.4.4 Track fitting efficiency

The figure 8 shows the track fitting efficiency as a function of initial momentum of the positron. The increase in track fitting efficiency with the positron momentum can be attributed to the decrease in multiple scattering cross section with the momentum. The track fitting efficiency is around 95 % - 98% for 180-280 MeV/c positrons, which is the designed range of momentum for the tracker.

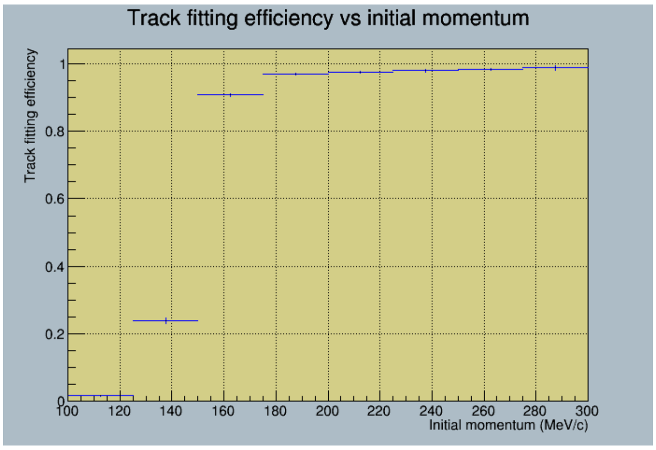


Figure 8: Track fitting efficiency vs positron momentum

0.4.5 Momentum resolution

The fitted track is extrapolated to the first positron hit and the estimated positron momentum is compared with the true momentum known from the simulations. We define momentum resolution as the σ of the distribution of $\frac{Reconstruced - True\ momentum}{True\ momentum}$. Figure 10 shows the momentum resolution histogram for different positron momentum bins. The momentum resolution improves with positron momentum as shown in Fig. 9

0.4.6 P-value distribution of χ^2/NDF

The Figure 11 shows the p-value distribution of χ^2/NDF as discussed in section 0.2.1. A uniform p-value distribution along with a few outliers around 0 is an assurance that errors have been estimated well and the data agrees to the track hypothesis. The plots

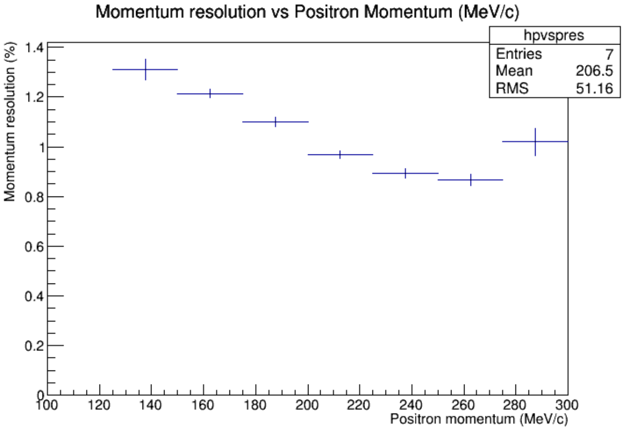


Figure 9: Momentum resolution shown improvement with positron momentum due to the effects of multiple scattering

show a peak around 0 for all the momentum bins which indicates the presence of large number of outliers. These outliers have a very high χ^2 and indicate the need to tune the GENFIT package for low momentum positron tracking and validation of the various positron cross sections used by the package.

0.4.7 Failure of track fitting

The track fitting efficiency is less than 0.5 for positrons with momentum less than 150 MeV/c. Visual inspection of the events for which the track fitting failed or did not converged led to the following two types of track topology as shown in Fig 12

1. **Kinks in e^+ trajectory**- The presence of kinks in the track can be attributed to multiple coulomb scattering and/or bremsstrahlung. These are observed throughout the entire momentum range.

2. **Multiple hits on the same vane**- These are present in the low momentum region (less than 150 MeV/c). Since, these e^+ are tangent to the vane, they end up delivering multiple hits on the same vane. The GENFIT algorithm has not been designed to handle such cases, therefore, track fitting fails or does not converge for these events.

We carried out visual analysis of the event shape to study the distribution of event shape for different momentum regime. The histogram in Fig 13 shows that the track with momentum less than 150 MeV/c show prevalence of multiple hits on the same track.

0.5 Code description

In this section we discuss few aspects of the track fitting code available on the Gitlab repository (`https://gitlab.in2p3.fr/shobhit/Genfitg-2`). The present structure of the code is described in Fig 14. It has been divided into two parts

0.5.1 Track fitting code

This part of the code deals with the track fitting algorithm and the results are saved as a TTree in a ROOT file. The code is stored as */test/minimalFittingExample/main.cc* in the Genfit code available at the Git repository. The fitting tree's variables are described as follows

Name	Description
TTrack	Final fitted track
TCovfinal	Covariance matrix of the fitted state
TFitted	Track successfully fitted or not
TFitConverged	Track fitting converged or not
TNDF	Degrees of freedom of the fit
TFNDF/TBNDF	Degrees of freedom of the forward/backward fit
TNhit	Number of hits in the first 2 quarters
TTreeno	Tree number in the chain
TEventno	Event number in the Tree
Tpval	P-value of the fit
TChi2	χ^2 of the fit
TFChi2/TBChi2	χ^2 of the forward/backward fit
Txhit/Tyhit/TZhit	x/y/z coordinates of the hits
Tpos_hit1 / Tmom_hit1	Position/momentum at hit1
Tpos_hit1_smear / Tmom_hit1_smear	Position/momentum at hit1
Txinit/Tyinit/Tzinit	x/y/z at the vertex
Txmominit/Tymominit/Tzmominit	x/y/z momentum at the vertex

0.5.2 Fitting analysis code

The fitting analysis code reads the *FittingResults.root* file produced by the Track fitting code and produces the necessary plots. The code has been written at the location */test/minimalFittingExample/AnalyzeFitting.cc* It offers the following functionality

- Track fitting efficiency vs momentum
- Momentum resolution at hit 1 vs momentum
- Event shape for various event separated by fit successful or fit not converged and the initial momentum of e^+
- Pull distribution
- P value distribution

The following "switch" variables have been created in the code to enable/disable a particular plotting function

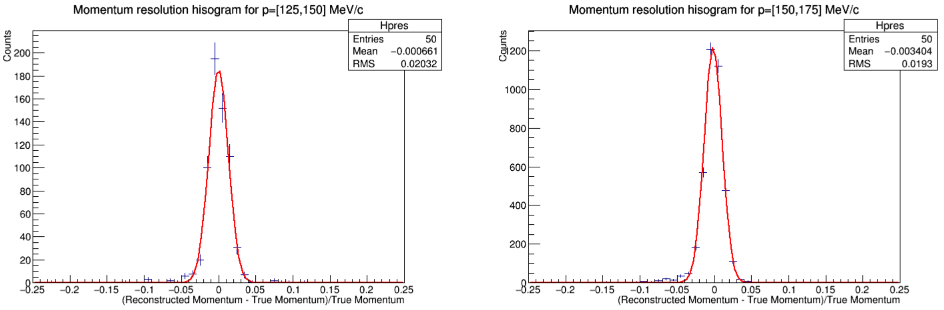
- **Printshape**- Used to enable the plotting of event shape. The event shape is is printed into 2 folders- *bin/plotes/eventshape/fitted* and *bin/plotes/eventshape/failed* depending on whether the fit converged or not. Further, each of the two folder is divided into 12 sub-folders based on the momentum bins from 100 to 300 MeV/c and the event shape is stored in the appropriate folder. Therefore, before enabling this function, appropriate directory structure must be created as described in the file README.build. The code stops after printing 200 event shapes, which can be changed by modifying the variable *maxeventprint* in the code.
- **Printgraph** Enables the printing of momentum distribution and χ^2 vs momentum distribution.

- **Plottrackeff** Used to print the track fitting efficiency vs initial momentum plot. The momentum refers to the positron's momentum at the 1^{st} hit.
- **Plotchi2dist** Enables the plotting of the p-value distribution of the fit in the bins of momentum.
- **Plotmomres** Enables the plotting of momentum resolution at hit1 vs the initial momentum of the positron.
- **Plotpull** Enables the plotting of 5 pull distribution corresponding to the variables q/p, u, u', v & v'.

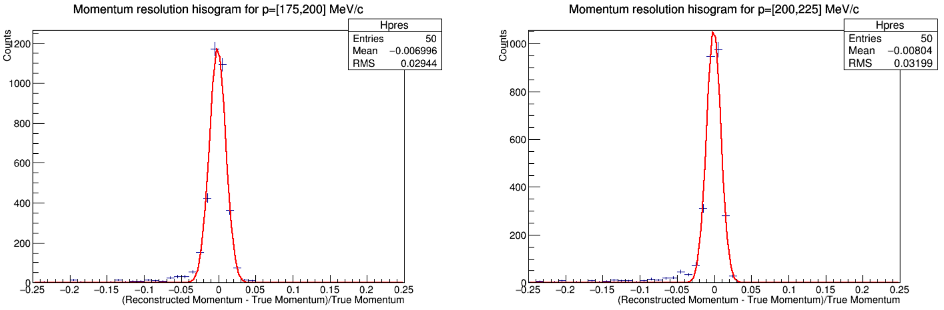
0.6 Summary and outlook

We chose DAF with ref. Kalman was chosen as the best choice owing to its capability to cope with bias initial hits and outliers. The preliminary results carrid out suggest that the momentum resolution and track fitting efficiency is good in the detector's designed range of positron momentum [200-280] MeV/c. The presence of large number of outliers in the p-value plot implies the need to test GENFIT models as well as cross sections in the low momentum regime for positrons. GENFIT's linear tracking mechanism needs to be changed to fit the track for the events with 2 hits on the same vane. The updated version of the track fitting code and analysis code has been committed to Gitlab (`https://gitlab.in2p3.fr/shobhit/Genfitg-2`). Presently the studies have been performed on the events simulated using ICEDUST (modified COMET framework) With minor modification in the reading routines, the framework could be adapted to read the Kyushu simulation file and geometry as well as interfaced with the track finding code.

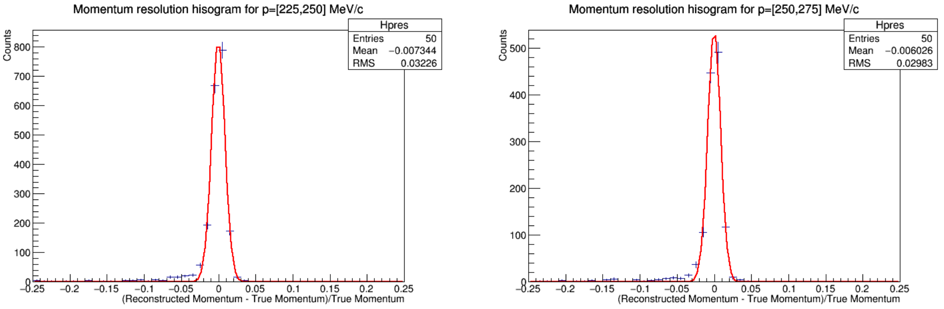
As a next step, it is important to focus on the χ^2 distribution to investigate the reason behind the presence of large number of outliers. Once done, the fitted track could be extrapolated to the μ^+ decay vertex to calculate the true momentum resolution. Further, the fitting code is still dependent on the class *measurementcreator.CC* and the dependency could be removed if it is desired to make the code compatible with the version of GENFIT post February 2017.



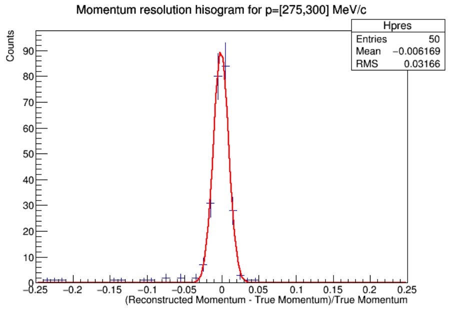
(a) [125-150] MeV/c (b) [150-175] MeV/c



(c) [175-200] MeV/c (d) [200-225] MeV/c

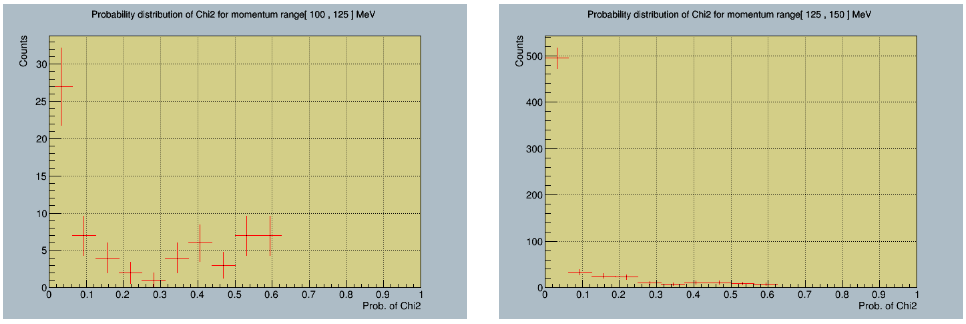


(e) [225-250] MeV/c (f) [250-275] MeV/c



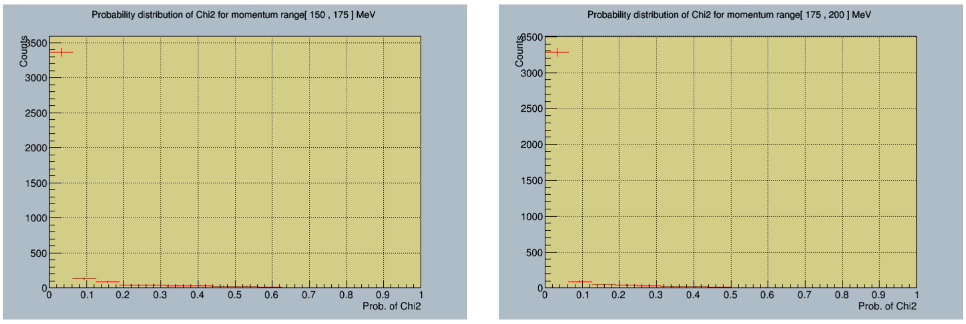
(g) [275-300] MeV/c

Figure 10: Momentum resolution histogram of the tracks for different momentum range



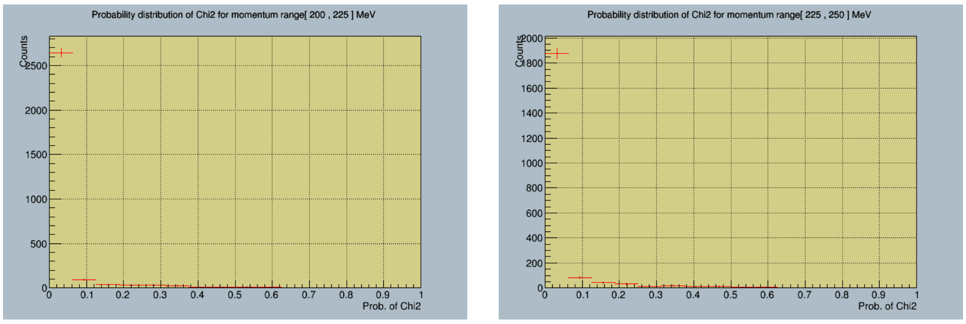
(a) [100-125] MeV/c

(b) [125-150] MeV/c



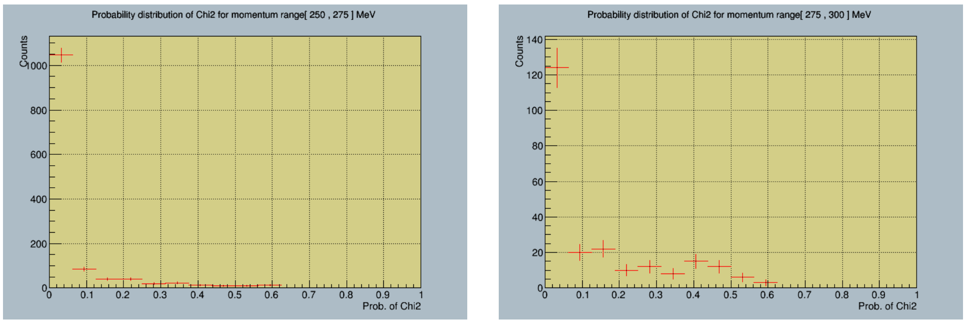
(c) [150-175] MeV/c

(d) [175-200] MeV/c



(e) [200-225] MeV/c

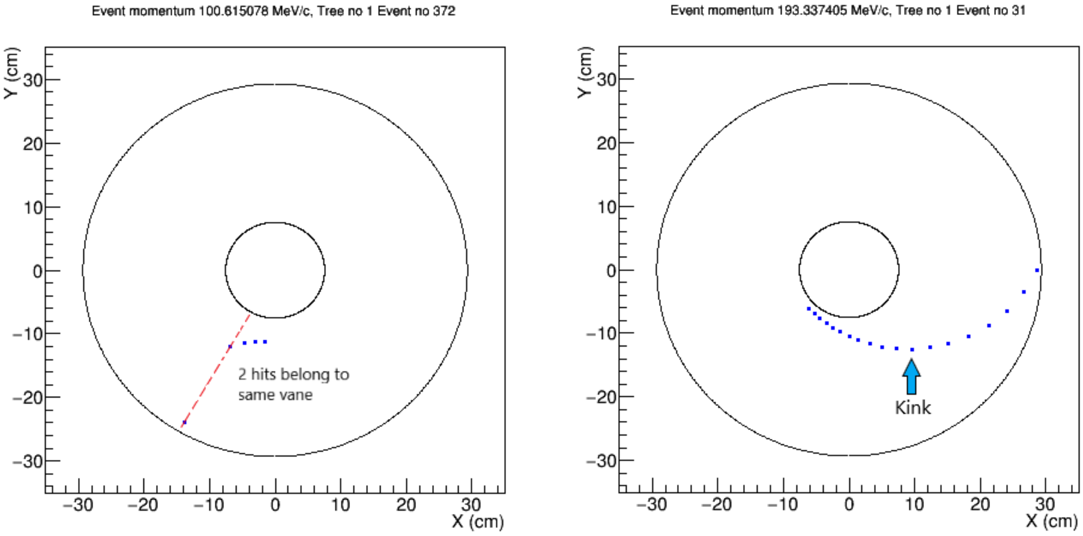
(f) [225-250] MeV/c



(g) [250-275] MeV/c

(h) [275-300] MeV/c

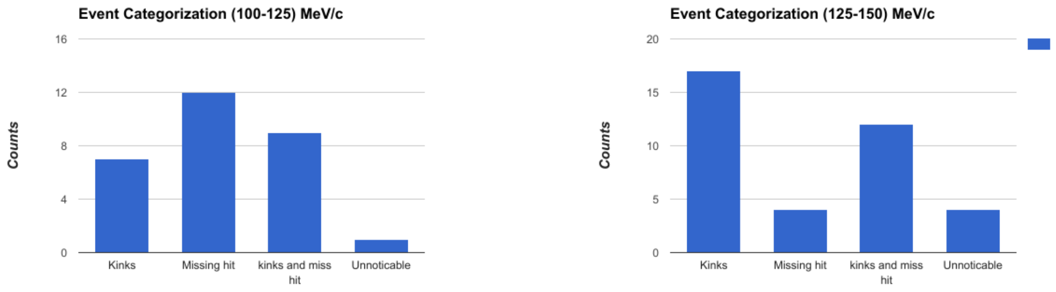
Figure 11: P-value distribution of the tracks for different momentum range



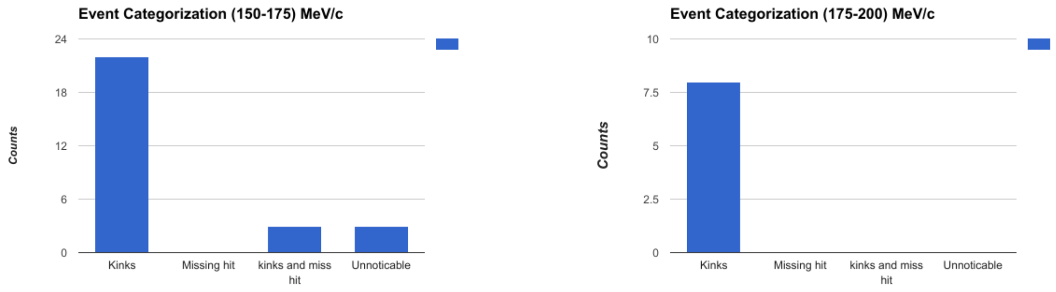
(a) 2 hits in the same vane

(b) Kink in e^+ trajectory

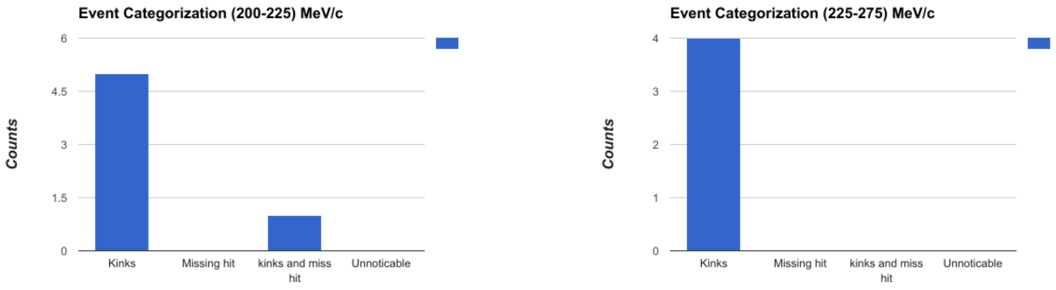
Figure 12: Event topology for which the track fitting fails



(a) [100-125] MeV/c (b) [125-150] MeV/c



(c) [150-175] MeV/c (d) [175-200] MeV/c



(e) [200-225] MeV/c (f) [225-275] MeV/c

Figure 13: Event shape distribution for different momentum range

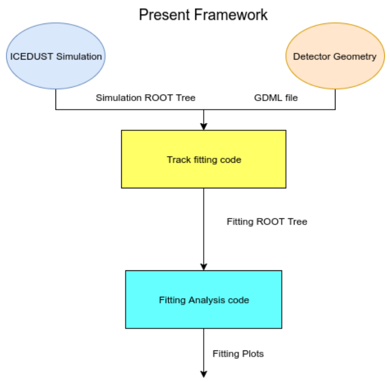


Figure 14: Structure of the track fitting framework

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